Past paper questions

6.1 The Poisson Distribution

The questions in this document have been compiled from a number of past papers, as indicated in the table below. Use these questions to formatively assess your learners' understanding of this topic.

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The mark scheme for each question is provided at the end of the document.

You can find the complete question papers and the complete mark schemes (with additional notes where available) on the School Support Hub www.cambridgeinternational.org/support.

7 The number of planes arriving at an airport every hour during daytime is modelled by the random variable X with distribution Po(5.2).

(i) State two assumptions required for the Poisson model to be valid in this context. [2]

(ii) (a) Find the probability that the number of planes arriving in a 15-minute period is greater than 1 and less than 4, [3]

(b) Find the probability that more than 3 planes will arrive in a 40-minute period. [2]

(iii) The airport has enough staff to deal with a maximum of 60 planes landing during a 10-hour day. Use a suitable approximation to find the probability that, on a randomly chosen 10-hour day, staff will be able to deal with all the planes that land. [4]

2 The number of phone calls arriving in a 10-minute period at a switchboard is modelled by the random variable X which has the distribution $\text{Po}(4.1)$. Use an approximating distribution to find the probability that more than 90 calls arrive in a 4-hour period. [5]

1 It is known that 1.2% of rods made by a certain machine are bent. The random variable X denotes the number of bent rods in a random sample of 400 rods.

(i) State the distribution of X. [2]

(ii) State, with a reason, a suitable approximate distribution for X. [2]

(iii) Use your approximate distribution to find the probability that the sample will include more than 2 bent rods. [2]

6 Calls arrive at a helpdesk randomly and at a constant average rate of 1.4 calls per hour. Calculate the probability that there will be

(i) more than 3 calls in $2\frac{1}{2}$ hours, [3]

(ii) fewer than 1000 calls in four weeks (672 hours). [4]

4 The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X, having the gene is found.

(i) State the distribution of X and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $P(X \leqslant 3)$. [2]

6 A publishing firm has found that errors in the first draft of a new book occur at random and that, on average, there is 1 error in every 3 pages of a first draft. Find the probability that in a particular first draft there are

(i) exactly 2 errors in 10 pages, [2]

(ii) at least 3 errors in 6 pages, [3]

(iii) fewer than 50 errors in 200 pages. [4]

7 In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable X denotes the number of prize-winning tickets that have been sold.

(i) State, with a justification, an approximating distribution for X. [3]

(ii) Use your approximating distribution to find $P(X < 4)$. [3]

(iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geqslant 1$. [4]

7 **(a)** A large number of spoons and forks made in a factory are inspected. It is found that 1% of the spoons and 1.5% of the forks are defective. A random sample of 140 items, consisting of 80 spoons and 60 forks, is chosen. Use the Poisson approximation to the binomial distribution to find the probability that the sample contains

(i) at least 1 defective spoon and at least 1 defective fork, [3]

(ii) fewer than 3 defective items. [3]

(b) The random variable X has the distribution $\mathrm{Po}(\lambda)$. It is given that

$$\mathrm{P}(X = 1) = p \quad \text{and} \quad \mathrm{P}(X = 2) = 1.5p,$$

where p is a non-zero constant. Find the value of λ and hence find the value of p. [4]

3 1% of adults in a certain country own a yellow car.

(i) Use a suitable approximating distribution to find the probability that a random sample of 240 adults includes more than 2 who own a yellow car. [4]

(ii) Justify your approximation. [2]

5 **(i)** A random variable X has the distribution Po(42).

(a) Use an appropriate approximating distribution to find $P(X \geqslant 40)$. [4]

(b) Justify your use of the approximating distribution. [1]

(ii) A random variable Y has the distribution B(60, 0.02).

(a) Use an appropriate approximating distribution to find $P(Y > 2)$. [3]

(b) Justify your use of the approximating distribution. [1]

Mark schemes

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ or FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOI	Seen or implied
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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7(i)	Planes arrive at constant mean rate	**B1**	
	Planes arrive at random	**B1**	or Planes arrive independently Must be in context
	Total:	**2**	
7(ii)(a)	(λ =) 5.2 ÷ 4	**M1**	
	$e^{-1.3}(\frac{1.3^2}{2}+\frac{1.3^3}{3!})$	**M1**	Allow any λ , allow one end error
	= 0.330 (3 sfs)	**A1**	Accept 0.33
	Total:	**3**	
7(ii)(b)	$1-e^{-3.467}\times(1+3.467+\frac{3.467^2}{2!}+\frac{3.467^3}{3!})$	**M1**	Allow any λ except 5.2 or 1.3, allow one end error
	= 0.456 (3 sfs)	**A1**	
	Total:	**2**	
7(iii)	N(52, 52) stated or implied	**B1**	
	$\frac{60.5-52}{\sqrt{52}}$ (= 1.179)	**M1**	ft their mean and var. Allow wrong or no cc or no √
	Φ("1.179")	**M1**	
	= 0.881 (3 sf)	**A1**	
	Total:	**4**	

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2	λ = 98.4	**B1**	
	N(98.4, 98.4) seen or implied	**B1**	
	$\frac{90.5-"98.4"}{\sqrt{"98.4"}}$ (= –0.796)	**M1**	allow with wrong or no cc. No sd/var mix.
	ϕ("0.796")	**M1**	Correct area consistent with working
	= 0.787 (3 sf)	**A1**	
		5	

May/June 2013 Paper 72

1	**(i)**	Binomial	B1	
		$n = 400$, $p = 0.012$	B1 [2]	Both. Not $p = 1.2\%$ Or B(400, 0.012): B1B1
	(ii)	Poisson	B1	
		n large and mean = 4.8, which is < 5	B1 [2]	n large, p small
	(iii)	$1 - e^{-4.8}(1 + 4.8 + \frac{4.8^2}{2})$	M1	P(X = 0, 1, 2); allow any λ; allow one end error
		= 0.857/0.858	A1 [2]	(Normal/Binomial in (ii) can score M1 only)
			[Total: 6]	

May/June 2013 Paper 73

6	**(i)**	$\lambda\ (= 1.4 \times 2.5) = 3.5$ $1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$ = 0.463 (3 sf)	B1 M1 A1 3	Any λ allow one end error
	(ii)	$(\lambda = 672 \times 1.4 = 940.8)$ N(940.8, 940.8) $\frac{999.5 - 940.8}{\sqrt{940.8}}$ (= 1.914) Φ('1.914') = 0.972 (3 sf)	B1 M1 M1 A1 4	Seen or implied Allow with wrong or no cc . no sd/var mixes
			[Total: 7]	

May/June 2014 Paper 71

4	**(i)**	B(3500, 0.001) Poisson with mean = 3.5 $n > 50$ and $np < 5$	B1 B1 B1 [3]	or Po(3.5) Both. Or $n > 50$ and $\lambda < 5$ or $3.5 < 5$
	(ii)	$e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$ = 0.537 (3 dp)	M1 A1 [2]	Allow any λ
			[Total: 5)	

May/June 2015 Paper 71

6 (i)	$e^{-\frac{10}{3}} \times \frac{\left(\frac{10}{3}\right)^2}{2}$	M1	P(2), allow any λ
	$= 0.198$ (3 sf)	A1 [2]	
(ii)	$1 - e^{-2}\left(1 + 2 + \frac{2^2}{2}\right)$	M1 M1	M1 allow any λ and/<u>or</u> 1end error Correct expression , correct λ
	$= 0.323$ (3 sf)	A1 [3]	
(iii)	$N\left(\frac{200}{3}, \frac{200}{3}\right)$	M1	seen or implied
	$\frac{49.5 - \frac{200}{3}}{\sqrt{\frac{200}{3}}}$ $(= -2.102)$	M1	For standardising allow <u>either</u> wrong or no cc No sd/var mix
	$\Phi('-2.102') = 1 - \Phi('2.102')$ $= 0.0178$ (3 sf)	M1 A1 [4]	For finding area consistent with their working
		[Total: 9]	

May/June 2015 Paper 72

7 (i)	Poisson (Actually binomial with) $n > 50$ and np (or λ) $(= 2.1)$ which is < 5	B1 B1 B1 3	 Allow without "binomial" Accept n large Accept p small ($p < 0.1$)
(ii)	$\lambda = 2.1$ $e^{-2.1}\left(1 + 2.1 + \frac{2.1^2}{2} + \frac{2.1^3}{3!}\right)$ $= 0.839$ (3 sf)	B1 M1 A1 3	 Attempt P(0,1,2,3) any λ allow 1 end error SR_1 Ft Normal N(2.1,2.1) B1 standardising M1 0.833 A1 SR_2 Ft Binomial B(10500,0.0002) B1 calculating binomial prob P(0,1,2,3) M1 = 0.8386 A1
(iii)	$P(X \geqslant 1) = 1 - e^{-2.1}$ $(= 0.87754)$	M1	Any λ
	$P(X = 1,2,3) = e^{-2.1}\left(2.1 + \frac{2.1^2}{2} + \frac{2.1^3}{3!}\right)$ $(= 0.71619)$	M1	Or '0.839' – $e^{-2.1}$ Any λ
	$\frac{P(X = 1,2,3)}{P(X > 1)}$ $\left(= \frac{0.71619}{0.87754}\right)$	M1	Allow any attempted $\frac{P(X = 1,2,3)}{P(X > 1)}$ Any λ
	$= 0.816$ (3 sf)	A1 4	SR_1 Ft Normal P(>0.5)=0.86523 M1 P(1,2,3)=0.698 M1 0.698/0.86523 = 0.807 M1A1 SR_2 FT Binomial M1 M1 M1 A1
	Total	**10**	

May/June 2016 Paper 71

7 (a) (i)	0.01×80 and 0.015×60 $(1 - e^{-0.8}) \times (1 - e^{-0.9})$ $= 0.327$ (3 sf)	M1 M1 A1 [3]	$(1 - e^{-\lambda}) \times (1 - e^{-\mu})$ any λ, μ $(\lambda \neq \mu)$ allow one end error
(ii)	$\lambda = 0.02 \times 40 + 0.015 \times 60$ $e^{-1.7} \times (1 + 1.7 + \frac{1.7^2}{2})$ $= 0.757$ (3 sf)	M1 M1 A1 [3]	or their 0.8 + 0.9
(b)	$e^{-\lambda} \times \lambda = p$ and $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5p$ $\lambda = 3$ $p = e^{-3} \times 3$ $= 0.149$ (3 sf)	M1 A1 M1 A1 [4]	or $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5 \times e^{-\lambda} \times \lambda$ seen or implied their λ

May/June 2016 Paper 73

3 (i)	Use of Poisson Mean = 2.4 $1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2})$ $= 0.43(0)$ (3 sf)	B1 B1 M1 A1 [4]	Allow any λ *(Allow one end error)* Final answer SR Use of binomial: B1 for ans 0.431 (3 sf)
(ii)	$240 > 50$ or $n>50$ $240 \times 0.01 = 2.4 < 5$ or $np<5$ or $p<0.1$	B1 B1 [2]	SR n large, p small: B1

May/June 2017 Paper 73

5(i)(a)	$X \sim N(42, 42)$	**B1**	stated or implied
	$\frac{39.5 - "42"}{\sqrt{"42"}}$ $(= -0.386)$	**M1**	allow with wrong or no cc
	$1 - \phi("-0.386") = \phi("0.386")$	**M1**	correct area consistent with their working
	$= 0.65(0)$ (3 sf)	**A1**	
	Total:	**4**	
5(i)(b)	$42 >$ (e.g. 15) or mean is large	**B1**	$\lambda > 15$ or higher, λ = large ignore subsequent work if not undermining what already written
	Total:	**1**	
5(ii)(a)	$Y \sim Po(1.2)$	**B1**	stated or implied
	$1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$	**M1**	allow any λ allow one end error
	$= 0.121$ (3 sf)	**A1**	Using binomial: 0.119 SR **B1**
	Total:	**3**	
5(ii)(b)	$60 \times 0.02 = 1.2 < 5$ or mean is small	**B1FT**	or large n small p FT Poisson only
	Total:	**1**	