Past paper questions
1.1 Quadratics

The questions in this document have been compiled from a number of past papers, as indicated in the table below. Use these questions to formatively assess your learners' understanding of this topic.

Question	Year	Series	Paper number
1	2017	March	12
8i	2013	June	11
3	2013	June	13
2	2014	June	11
8	2014	June	13
5	2015	November	11
11i & 11ii	2015	June	12
1	2015	June	13
6b	2016	June	11
1	2016	November	13

The mark scheme for each question is provided at the end of the document.

You can find the complete question papers and the complete mark schemes (with additional notes where available) on the School Support Hub www.cambridgeinternational.org/support.

1 **(i)** Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

(ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

8 **(i)** Express $2x^2 - 12x + 13$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

3 **(i)** Express the equation $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. [2]

(ii) Solve the equation $2\cos^2\theta = \tan^2\theta$ for $0 \leqslant \theta \leqslant \pi$, giving solutions in terms of π. [3]

2 **(i)** Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

8 **(i)** Express $2x^2 - 10x + 8$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

5 A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

11 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $\mathrm{f}(x) = p$ has no real roots. [3]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $\mathrm{g}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

1 Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

6 **(a)** Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

1 Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

Mark schemes

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ or FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOI	Seen or implied
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

May/June 2017 Paper 12

1(i)	Coefficient of $x = 80(x)$	B2	Correct value must be selected for both marks. SR +80 seen in an expansion gets **B1** or −80 gets **B1** if selected.
	Total:	2	
1(ii)	Coefficient of $\frac{1}{x} = -40\left(\frac{1}{x}\right)$	B2	Correct value soi in (ii), if powers unsimplified only allow if selected. SR +40 soi in (ii) gets **B1**.
	Coefficient of $x = (1 \times \text{their } 80) + (3 \times \text{their } -40) = -40(x)$	M1 A1	Links the appropriate 2 terms only for **M1**.
	Total:	4	

May/June 2013 Paper 11

8 **(i)** $2(x-3)^2 - 5$ or $a = 2$, $b = -3$, $c = -5$	**B1B1B1** **[3]**	

May/June 2013 Paper 13

3	$2\cos^2\theta = \tan^2\theta$			
(i)	$\rightarrow 2\cos^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$	M1		Use of $t^2 = s^2 \div c^2$ or alternative. Correct eqn.
	$\rightarrow$ Uses $c^2 + s^2 = 1 \rightarrow 2c^4 = 1 - c^2$	A1	[2]	
(ii)	$(2c^2 - 1)(c^2 + 1) = 0 \quad \rightarrow c = \pm\dfrac{1}{\sqrt{2}}$	M1		Method of solving for 3-term quadratic.
	$\rightarrow \theta = \frac{1}{4}\pi$ or $\frac{3}{4}\pi$.	A1 A1√	[3]	(in terms of π). √ for $\pi - 1^{st}$ ans. Cannot gain A1√ if other answers given in the range.

May/June 2014 Paper 11

2 **(i)** $(2x-3)^2 - 9$	**B1B1** **[2]**	For −3 and −9
(ii) $2x - 3 > 4 \quad 2x - 3 < -4$	**M1**	At least one of these statements
$x > 3\frac{1}{2}$ (*or*) $x < -\frac{1}{2}$ cao Allow $-\frac{1}{2} > x > 3\frac{1}{2}$	**A1**	Allow 'and' $3\frac{1}{2}$, $-\frac{1}{2}$ soi scores first M1
OR $4x^2 - 12x - 7 \rightarrow (2x-7)(2x+1)$	**M1**	Attempt to solve 3-term quadratic
$x > 3\frac{1}{2}$ (*or*) $< -\frac{1}{2}$ cao Allow $-\frac{1}{2} > x > 3\frac{1}{2}$	**A1** **[2]**	Allow 'and' $3\frac{1}{2}$, $-\frac{1}{2}$ soi scores first M1

May/June 2014 Paper 13

8 $2x^2 - 10x + 8 \rightarrow a(x+b)^2 + c$		
(i) $a = 2,\ b = -2\frac{1}{2},\ c = -4\frac{1}{2}$	3 × B1	Or $2\left(x - 2\frac{1}{2}\right)^2 - 4\frac{1}{2}$
$\rightarrow$ min value is $-4\frac{1}{2}$ Allow $(2\frac{1}{2}, -4\frac{1}{2})$	B1√	Can score by sub $x = 2\frac{1}{2}$ into original but not by differentiation
	[4]	
(ii) $2x^2 - 10x + 8 - kx = 0$		Sets equation to 0 and uses
Use of "$b^2 - 4ac$"	M1	discriminant correctly
$(-10-k)^2 - 64 < 0$ or $k^2 + 20k + 36 < 0$	M1	Realises discriminant < 0. Allow ⩽
$\rightarrow k = -18$ or -2	A1	co Dep on 1st M1 only
$-18 < k < -2$	A1	co
	[4]	

November 2015 Paper 11

5 **(i)**	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao	**B1B1**	
	$\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	**B1**	
		[3]	
(ii)	$-\frac{8}{x^2} + 2 = 0 \rightarrow 2x^2 - 8 = 0$	**M1**	Set = 0 and rearrange to quadratic form
	$x = \pm 2$	**A1**	
	$y = \pm 8$	**A1**	If A0A0 scored, SCA1 for just (2, 8)
	$\frac{d^2y}{dx^2} > 0$ when $x = 2$ hence MINIMUM	**B1√**	Ft for "correct" conclusion if $\frac{d^2y}{dx^2}$ incorrect or any valid method inc. a good sketch
	$\frac{d^2y}{dx^2} < 0$ when $x = -2$ hence MAXIMUM	**B1√** [5]	

May/June 2015 Paper 12

11	$f: x \mapsto 2x^2 - 6x + 5$		
(i)	$2x^2 - 6x + 5 - p = 0$ has no real roots	M1	Sets to 0 with p on LHS.
	Uses $b^2 - 4ac \rightarrow 36 - 8(5-p)$	DM1	Uses discriminant.
	Sets to 0 $\rightarrow p < \frac{1}{2}$	A1 [3]	co – must be "<", not "⩽".
(ii)	$2x^2 - 6x + 5 = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	3 × B1 [3]	co

May/June 2015 Paper 13

1	$2(x-3)^2-11$	**B1B1B1** **[3]**	For 2, $(x-3)^2$, −11. Or a=2, b= 3, c= 11

May/June 2016 Paper 11

6 (a)	$y = 2x^2 - 4x + 8$ Equates with $y = mx$ and selects a, b, c Uses $b^2 = 4ac$ $\rightarrow m = 4$ or -12.	**M1** **M1** **A1** [3]	Equate + solution or use of dy/dx Use of discriminant for both.
(b) (i)	$f(x) = x^2 + ax + b$ Eqn of form $(x-1)(x-9)$ $\rightarrow a = -10, b = 9$ (or using 2 sim eqns **M1 A1**)	**M1** **A1** [2]	Any valid method allow $(x+1)(x+9)$ for **M1** must be stated
(ii)	Calculus or $x = \frac{1}{2}(1+9)$ by symmetry $\rightarrow (5, -16)$	**M1** **A1** [2]	Any valid method

November 2016 Paper 13

1	$kx^2 - 3x = x - k \Rightarrow kx^2 - 4x + k(=0)$ $(-4)^2 - 4(k)(k)$ soi $k > 2$, $k < -2$ cao Allow $(2, \infty)$ etc. Allow $2<k<-k$	**M1** **M1** **A1**	[3]	Eliminate y and rearrange into 3-term quad $b^2 - 4ac$.