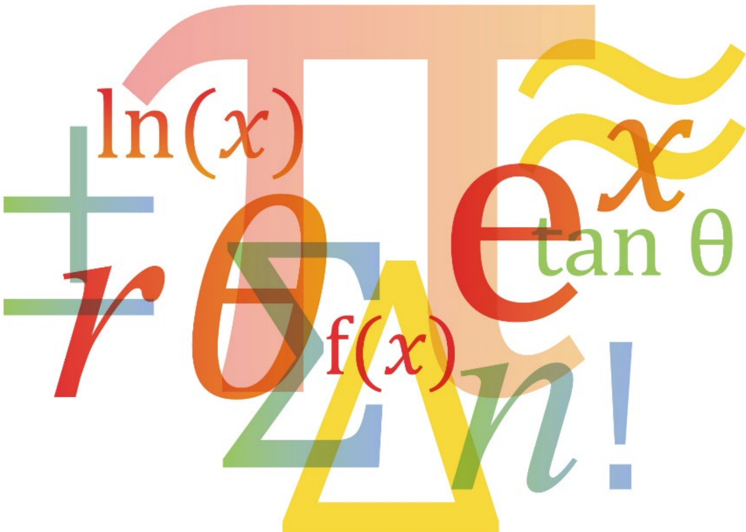
Cambridge Assessment International Education

Teaching Pack

1.1 Quadratics

Cambridge International AS & A Level Mathematics 9709



Version 1

Cambridge Pathway

In order to help us develop the highest quality resources, we are undertaking a continuous programme of review; not only to measure the success of our resources but also to highlight areas for improvement and to identify new development needs.

We invite you to complete our survey by visiting the website below. Your comments on the quality and relevance of our resources are very important to us.

www.surveymonkey.co.uk/r/GL6ZNJB

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www.cambridgeinternational.org/cambridge-for/teachers/teacherconsultants/

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Introduction

This pack will help you to develop your learners' skills in mathematical thinking and mathematical communication, which are essential for success at AS & A Level and in further education.

Mathematical thinking and communication will be developed by focusing on:

1. Conceptual understanding – the 'why' behind the 'what'
2. Strategic competence – forming and solving problems
3. Adaptive reasoning – explanations, justifications and deductive reasoning

Throughout all activities, the learners will also develop:

- procedural fluency – know when, how and which rules to use
- positive disposition – believe maths can be learned, applied and is useful
- their skills in writing mathematically – writing working & proofs

These link to the course Assessment Objectives (AOs) which you can find in detail in the syllabus:

A01 Knowledge and understanding

A02 Application and communication

Each *Teaching Pack* contains one or more lesson plans and associated resources, complete with a section of preparation and reflection.

Each lesson is designed to be an hour long but you should adjust the timings to suit the lesson length available to you and the needs of your learners.

Important note
Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other topics.

This content is designed to give you and your learners the chance to explore a more active way of engaging with mathematics that encourages independent thinking and a deeper conceptual understanding. It is not intended as specific practice for the examination papers.

The *Teaching Packs* are designed to provide you with some example lessons of how you might deliver content. You should adapt them as appropriate for your learners and your centre. A single pack will only contain at most five lessons, it will **not** cover a whole topic. You should use the lesson plans and advice provided in this pack to help you plan the remaining lessons of the topic yourself.

Lesson preparation

This *Teaching Pack* will cover the following syllabus content:

Candidate should be able to:	Notes and examples
• carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form	e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph
• solve quadratic equations, and quadratic inequalities, in one unknown	By factorising, completing the square and using the formula.
• recognise and solve equations in x which are quadratic in some function of x	e.g. $x^4 - 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$, $\tan^2 x = 1 + \tan x$.

The remaining *two* bullet points for topic 1.1 Quadratics are not covered in this *Teaching Pack* (see the syllabus for detail). You will need to write your own lesson plans for these items.

Candidate should be able to:	Notes and examples
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant	e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term 'repeated root' is included.
• solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic	e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.

Prior knowledge and skills

For all lessons, it is assumed that learners have already completed Cambridge IGCSE™ Mathematics 0580, or a course at an equivalent level. See the syllabus for more details of the expected prior knowledge for taking Cambridge International AS & A Level Mathematics 9709.

When planning any lesson, make a habit of always asking yourself the following questions about your learners' prior knowledge and skills:

- Do I need to re-teach this or do learners just need some practice?
- Is there an interesting activity that will efficiently achieve this?

Learners will need to be confident with the terminology used for algebra and algebraic equations from their IGCSE (or equivalent) course. This terminology is also identified below.

If learners cannot derive and solve simultaneous equations in two unknowns they will be at a disadvantage when building on these ideas when solving a pair of simultaneous equations of which one is linear and one is quadratic.

Learners will need to be able to expand products of algebraic expressions and factorise quadratic expressions, otherwise they will be at a disadvantage when using this knowledge to solve all types of quadratic equations and quadratic inequalities. For quadratic equations in different variables, learners will need to be able to solve simple trigonometric equations e.g. $\tan x = 3$.

Even if learners had mastered the areas above during their IGCSE (or equivalent) course, it is best not to assume that they are still fluent in this topic as it can lead to learners struggling to solve problems at AS and A Level.

Key learning objectives

The following list represents the main underlying concepts that you should make sure your learners have understood by the end of this topic.

- All quadratic expressions can be put in the form $a(x+b)^2+c$ and we can use this form to sketch quadratic functions.
- The discriminant of a quadratic polynomial allows you to determine how many real roots a quadratic equation will have.
- You can use a sketch of a quadratic function to determine the solution set for a quadratic inequality in one variable.
- You can use the techniques for solving quadratic equations when an equation can be rearranged into the form $ax^2+bx+c=0$ where a, b, c are real numbers.

Why this topic matters

Learners will be required to solve quadratic equations throughout the course and developing these skills now will allow them to concentrate on the new skill acquisition in later areas.

Key terminology and notation

Your learners will need to be confident with the following terminology and notation:

discriminant	for a quadratic expression of the form ax^2+bx+c this is b^2-4ac.
parabola	this is the name given to the graph of $y = ax^2+bx+c$
quadratics in some function of x	any expression of the ax^2+bx+c where a, b, c are real numbers and x is a function of x.
quadratic polynomial	this is a polynomial of the form ax^2+bx+c
repeated root	when a quadratic equation has only one solution
vertex	this is the maximum or minimum point for a parabola

Insights video

There is an Insights video linked to this *Teaching Pack*:

- **1.1 Quadratics** – watch this video which will show you how to help your learners to better understand how to solve quadratic equations using the 'completing the square' technique.

Teacher tutorials

There are *four* sets of Teacher tutorials linked to this *Teaching Pack*:

- **Completing the square** – review this tutorial before teaching Lesson plan 1; this will show you how to highlight the important parts of the completing the square process.
- **Solving quadratic equations** – review this tutorial before teaching Lesson plan 2; this will show you how to determine whether an equation is a quadratic in a function of x.

- **Sketching quadratic functions** – review this tutorial before teaching Lesson plan 3; this will show you how to highlight the important features required to sketch a quadratic function.
- **Solving quadratic inequalities** – review this tutorial before teaching Lesson plan 4; this will show you how to connect solving a quadratic inequality with sketching a quadratic function.

Lesson progression

Lesson 1 covers completing the square. Lesson 2 focuses on the solving of quadratic equations. The content from these two lessons is then used in Lesson 3. Lesson 4 builds on the ideas explored in all three lessons and applies them to solving quadratic inequalities.

Going forward

This topic links to all aspects of the syllabus content through the solving of quadratic equations and inequalities.

Lesson plan 1: Completing the square

Preparation	• Review the *Completing the square* Teacher tutorial for a series of questions and responses for the resources used in this lesson plan

Resources	• Paper, Mini whiteboards or other writing materials • Lesson slides: *Completing the square* • Worksheet A: *Completing the square (squares/algebra)* • Worksheet B: *Completing the square* • Worksheet C: *Quadratic expressions* • Worksheet D: *Completing the square variations* • Worksheet E: *Completing the square – spot the errors*

Learning objectives	By the end of the lesson: • ***all*** learners should recognise the completed square form • **most** learners should be able to confidently apply the process of completing the square on $x^2 + bx + c$ • **some** learners should be able to confidently apply the process of completing the square on $ax^2 + bx + c$.

Dependencies

Learners need to know how to expand perfect squares e.g. expand $(x + 3)^2$. Learners will need their algebraic manipulation skills from IGCSE (or equivalent).

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Expanding $(x + b)^2$ incorrectly e.g. $(x + 3)^2 = x^2 + 9$.	If learners believe this then they will not be able to engage with the completing the square process correctly.	Using cognitive conflict should help learners be more aware. Using the image of a square of length $x + b$ and asking how many terms are generated by the calculation $(x + b)^2$ may help learners break their misconceptions.

Timings	Activity
10 min	**Starter/Introduction** Worksheet A: *Completing the square (squares/algebra)* Give out Worksheet A. Learners work on Part 1, or display this with a projector and let learners use mini whiteboards. This will allow you to review each learner's work quickly. This activity should be a refresher on expanding $(x + b)^2$ and enable learners to make connections to the language of a perfect square. Example questions to ask:

Timings	Activity
	• Why do you think we use the phrase 'a perfect square' to describe this set of quadratics? • Looking at your answer, can you tell what the 'side length' of the square will be when the quadratic is in the form $x^2 + bx + c$? Learners should be able to expand each bracket. Some learners may have forgotten how to or have the misconception highlighted above. You could use the example way to resolve the misconception to aid both issues in this case. Now ask learners to work on Worksheet A Part 2. Ask: Are there any connections between the two parts? Learners should be able to link the previous part on perfect squares to filling in the required information to answer Part 2. **Support:** Give another example to aid learners. Start with $x^2 + 6x + \square$ and ask the same two questions. This part links directly with Part 1 Question 1. **Challenge:** Ask learners to try Part 3. *What does this question say about all quadratic expressions?*
  10 min	**Main lesson** Lesson slides: *Completing the square* Worksheet B: *Completing the square* Play the video on slide 2 and use Worksheet B. Ask learners to describe what is happening. Pause the video at about 47 seconds which is the end of (i) – then review the questions below before moving to the next section of the video. Pause again at 1min 47 and repeat the process. Example questions to ask as you pause the video after each new slide: • What is being identified first? • What are the steps in this procedure? • Which terms appear to be the most important? Collect learners' ideas on a board visible to the whole class. Learners should begin to identify that the most important terms are the x^2 and x terms. They should also be able to spot that (for the case of x^2 coefficient 1) we are halving the value of the coefficient of the x term.
20 min	Worksheet C: *Quadratic expressions* Play the final part of the video. Ask learners to use Worksheet C to apply the completing the square process to the set of examples. Example questions to ask: • What form will each expression take? • Are some of the expressions harder than others to complete the square on?

Timings	Activity
	• Are there any questions that are not like the quadratics we have seen so far? Learners should be able to complete questions 1 and 2 without many issues. Learners will find questions 3, 5 and 6 more challenging as the coefficient of x is odd. Learners will find questions 4, 7 and 8 most challenging and may require the support of other learners. **Support:** If learners require additional support, use the following scaffolding: Quadratic expression is … Coefficient of x is … Half of the coefficient of x is … Perfect square is … Subtract the square of … Simplify the constants. **Challenge:** Ask learners to work on questions 4, 7 and 8, and write down their solution as a guide to help other learners with this process. After 15 minutes ask learners to collect their solutions together in pairs and share their work. Lesson slides: *Completing the square* Show learners the slide 3. Ask the pairs to make any corrections they need to and highlight the errors they made. Collect the main ideas and issues each pair finds and highlight them to the class.
10 min	Worksheet D: *Completing the square variations* Give out Worksheet D – What can we vary in this situation? Example questions to ask: • What can we change in the first line and how will that affect subsequent lines of working? • What types of variation would we expect to see when applying the completing the square process? • Can you come up with an example of your suggested variation? Learners may find it difficult to engage with this task as they may not be used to thinking about the variation in a mathematical object. It may help if you model this process through the first image on Answer sheet D.
10 min	**Plenary** Worksheet E: *Completing the square – spot the errors* Use this task to get learners to review their learning by spotting common errors and misconceptions. Use 'Think-pair-share' but the pair will share with another pair, rather than the whole class (see *How to engage your learners* guide for details of this technique). Each time an expression is discussed one learner is chosen to highlight the error or misconception. Circulate the room and listen to each group. Handout the answers at the end and make sure each group of four has seen and understood the errors.

Reflection Reflect on your lesson; use the **Lesson reflection** notes to help you.

Lesson plan 2: Solving quadratic equations

Preparation	• Review Teacher tutorial *Solving quadratic equations*

Resources	• Paper, Mini whiteboards or other writing materials including:
	  • A large sheet of paper per pair (ideally A3 size)
	  • A small sheet of paper per learner (use A4 and cut into four)
	• Worksheet C: *Quadratic expressions*
	• Lesson slides *Solving quadratic equations*
	• Worksheet F: *Spot the quadratic equation*

Learning objectives	By the end of the lesson: • ***all*** learners should be able to solve a quadratic equation when told which method to use • **most** learners should be able to choose an appropriate method for solving a given quadratic equation • **some** learners should be able to spot when a given equation is a quadratic equation in a function of x

Dependencies

Learners need to remember the content on quadratic equations from IGCSE (or equivalent) and how to complete the square.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
There is a difference between factorising and completing the square on an expression, and solving using each of these processes.	Learners may try and solve an expression or only factorise a quadratic equation and not find the values of x.	Give learners a selection of quadratic expressions and equations. Ask learners what process you can perform on each object.
The a, b and c coefficients can be linked to any term in the quadratic without affecting the solution using the quadratic formula	Not identifying a, b and c correctly will lead to incorrect solutions.	Give learners a selection of quadratic equations written in completely different ways. Ask learners to rearrange the questions into the correct form and identify a, b and c.

Timings	Activity
5 min	**Starter/Introduction** Worksheet C: *Quadratic expressions* Give a piece of A3 paper to each pair of learners. Using Worksheet C, learners in pairs factorise questions 1 to 4 and complete the square on questions 5 to 8. They write these expressions down on their A3 piece of paper. Learners should be able to access this introduction as they will have recently completed a lesson on completing the square.
20 min	**Main lesson** Use the expressions learners have from the introductory activity. Ask 'What can we do to turn each of these expressions into equations?' Highlight that we need each expression to be equal to something e.g. a number, another expression etc. In this case, we will let each expression equal zero and then solve. Model question 1 and question 8 with learners. Be aware that question 7 has no solutions. Ask learners to complete their questions in pairs on their A3 sheet of paper. Lesson slides: *Solving quadratic equations* After 10 minutes reveal the answers (slides 2-4) and ask learners to identify the next steps for solving the quadratics given in: • factorised form, parts 1 to 4 • completing the square form, parts 5 to 8. Choosing an equation solved for each form, learners can then collect this information and write their own worked example with notes highlighted in the class discussion. **Support:** Some learners may need some additional support. If you see this, encourage pairs to come together to form a stronger group of learners who can assist each other.
5 min	Recap the quadratic formula by selecting quadratic equations • $3x^2 + 4x + 2 = 0$ • $3x^2 - 4x + 2 = 0$ • $3x^2 + 4x - 2 = 0$ • $2x^2 + 4x + 3 = 0$ and asking learners to identify a, b and c in each case. Highlight that the quadratic equation must be written in this form $ax^2 + bx + c = 0$ in order to use the quadratic formula e.g. ask learners to solve $2x^2 = 3x - 1$. **Challenge:** Ask learners to complete the square on $ax^2 + bx + c = 0$ and derive the quadratic formula.
20 min	Lesson slides: *Solving quadratic equations* Worksheet F: *Spot the quadratic equation* Change the pairs of learners in the room (use your knowledge of the way learners have worked so far to help decide your pairs). Play the video *Solving quadratic equations* on slide 5 and give out Worksheet F. Ask learners to identify what function

Timings	Activity
	in x we have as our quadratic equation. Ask them to rearrange the equations so that they can be solved e.g. replace x^2 with Y. After 10 minutes, ask learners to try and solve the quadratic equations they have formed. **Support:** Ask learners to start with questions 1, 5, 2, 3 and 6. Learners may find questions 1 and 5 easy, but struggle with questions 2, 3 and 6. Allow other learners in the room to help and get them to clarify, when they are helping, how they were able to identify the function x which created a quadratic equation. Circulate the room and discuss this activity with learners: *How did you decide which function was a quadratic equation?*
10 min	**Plenary** Ask learners to identify a quadratic equation that they would give as an example for: • factorising as a process to help solve • completing the square as a process to help solve • the quadratic formula. **Challenge:** Can learners identify from the coefficients a, b and c if a quadratic equation will factorise? Can learners identify from the coefficients a, b and c if a quadratic question can be solved? **Homework/Additional activity** – Ask learners to produce a quadratic equation in a function of x to be solved on one side of their A6 sheet of paper. They place their solution on the back. Collect these questions in and the class can use these questions as a revision resource for future retrieval practice in lessons.

Reflection	Reflect on your lesson; use the **Lesson reflection** notes to help you.

Lesson plan 3: Sketching quadratic functions

Preparation	• Review the Teacher tutorial *Sketching quadratic functions* • Access Desmos online (or another graphing package)
Resources	• Paper, Mini whiteboards or other writing materials • *Sketching quadratic functions* Lesson slides • Desmos online (or another graphing package) • Worksheet G: *Graphs of quadratic functions* • A6 sheet of paper (created by cutting up A4 into four)
Learning objectives	By the end of the lesson: • ***all*** learners should be able to identify the features of a parabola: shape, intersections, turning point • **most** learners should be able to sketch a quadratic function using factorisation and/or complete the square • **some** learners should be able to determine a quadratic function given a graph of any parabola

Dependencies

Learners need to know how to factorise and complete the square on quadratic functions.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Learners only focus on the *x*-intercepts (roots) of the quadratic function when sketching.	If learners believe this then they will not be able to sketch or identify stretches of quadratic functions.	From algebra to geometry: using a graphing package, such as Desmos, and varying k for y=k(x+1)(x-4). Ask learners to describe what they see. From geometry to algebra: using Worksheet G in this lesson can enable learners to understand the need for another point of the parabola in order to determine stretches.

Timings	Activity
10 min	**Starter/Introduction** Using mini whiteboards or other writing materials, ask learners to sketch $y = x^2$. Highlight to learners that sketching is not the same as plotting. When we sketch a graph we need to highlight the key features: **shape, intersects, turning points**. Learners will need to recall this basic shape from IGCSE (or equivalent). They may need some support in doing this.

Timings	Activity
	Support: Ask learners to create a table of values for $y = x^2$ if they are not able to recall the shape of the graph. *What are the key features of this graph? Do we need to plot all the points to show these key features?* Ask learners about the shape they have created. Define this shape as a **parabola**. Lesson slides: *Sketching quadratic functions* Ask learners to sketch the following, using their first graph $y = x^2$ as a starting point: 1) $y = (x-2)^2$ 2) $y = 2(x-2)^2$ 3) $y = -2(x-2)^2$ 4) $y = (x-2)^2 - 9$ Use slide 2 to show each parabola (starting with $y = x^2$). **Support:** Key questions corresponding to each part to aid learners are: 1) What value of x will give $y = 0$? What does this mean for the position of the parabola? 2) What has happened to the y-coordinates of the graph of equation 1 to create equation 2? 3) What has happened to the y-coordinates of the graph of equation 1 to create equation 3? 4) What has happened to the y-coordinates of the graph of equation 1 to create equation 4? Ask learners what form part 4, $y = (x-2)^2 - 9$ takes. Highlight to learners that this is in a completed square form and it allows them to 'see' where the minimum of this parabola is. We call this point (and the corresponding maximum point on negative coefficients of x^2) the **vertex** of the parabola. **Make it explicit that $(x-2)^2$ will always be greater than or equal to zero**. Therefore, when $x = 2$ this will give the minimum value of zero. **Support:** Learners will find this challenging if they did not meet transformations explicitly in IGCSE (or equivalent). Encourage learners who are confident to help other learners with their understanding. Allow them to move around the room to help each other. This will allow you to listen to their language and make any suggestions about how they are interpreting the sketching of the graphs.

Timings	Activity
	Main lesson

20 min

Lesson slides: *Sketching quadratic functions*

Display slide 3 *Which parabola?* Ask learners to read the instructions and then to work in pairs and determine the equations of the parabolas.

After 5 minutes, bring the class together to share one of the ideas they have used or discovered so far. Collect these ideas on a board visible to the whole class. You can use the Which parabola? teachers notes (included in the Lesson slides notes) to help structure this part of the lesson.

> **Support:** Some learners will initially find the image overwhelming and will require more confident learners to help them persevere. You could give learners a hint: We have already looked at a method of 'seeing' where the graph of a quadratic function will be in the xy-plane by using a form for its algebraic equation.

Allow learners then to continue for another 5 minutes. Then ask learners to meet up with a second pair and evaluate their solution and their strategy for finding the solution. Share findings as a class.

> **Support:** Ask learners to complete the square on the two equations. *Where might these graphs be, given their vertex?*

20 min

Worksheet G: Graphs of quadratic functions

Give out Worksheet G to each pair. Ask learners to start with graphs A and B. *Can you determine the equation of the quadratic function that gives each graph?*

Learners will most likely now focus on the vertex of the parabola, whose coordinates are not marked on the graph.

> **Support:** *If you know the roots of the parabola, where will the x-coordinate of the vertex be? How can you use this information to find an equation for the parabola? If you know the roots of the parabola, how else can you write an equation for the quadratic function?*

Some learners may spot that you can find all four equations by setting up a series of simultaneous equations for each parabola, using the three points and the general equation for a quadratic function $y = ax^2 + bx + c$ to find a, b and c. Highlight to learners that this will work, but is inefficient.

After 15 minutes, collect ideas together from the class. Split the responses into ideas that are specific to the values used in the graph and bigger ideas that could be used for any problem of this form.

> **Challenge:** Learners can attempt graphs C and D. Learners can also sketch the quadratic functions given on Worksheet C.

Timings	Activity
	Plenary

Timings	Activity
10 min	Give out the A6 paper to all learners. Ask learners to create their own problem based on the ideas in this lesson on one side of the A6 paper. You could give the following list of concepts for learners to choose from: using completing the square, using factorisation, creating a new parabola given an equation of a similar parabola, finding an equation of a parabola given a graph. Ask learners to write down their solution on the back of the paper. Collect in the A6 pieces of paper and use as class retrieval practice in a future lesson.

Reflection	Reflect on your lesson; use the **Lesson reflection** notes to help you.

Lesson plan 4: Solving quadratic inequalities

Preparation	• Review the Teacher tutorial *Quadratic inequalities*

Resources	• Worksheet H: *Review of linear inequalities* • Worksheet I: *Quadratic graphs* • Worksheet J: *Review questions (quadratic inequalities)* • Lesson slides: *Quadratic inequalities* • Paper, Mini whiteboards or other writing materials

Learning objectives	By the end of the lesson: • ***all*** learners should have developed procedural fluency in solving quadratic inequalities of the form $ax^2 + bx + c$ ■ 0, where ■ represents one of $<, >, \leq, \geq$. • **most** learners should be able to link the quadratic inequalities of the form $ax^2 + bx + c$ ■ 0, where ■ represents $<, >, \leq, \geq$ to their graphical representation • **some** learners should be able to confidently link any quadratic inequality to a graphical representation and identify the part of the horizontal axis that corresponds to the solution set

Dependencies

Learners need to know how to solve quadratic equations (Lesson plan 2) and sketch quadratic functions (Lesson plan 3). Learners should also be familiar with solving linear inequalities.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Inequality signs are interchangeable	These misconceptions might exist before starting this lesson/material. This will make building on previous learning challenging in order to solve quadratic inequalities.	Using the introductory activity, you can highlight the correct use of the inequality symbols.
Inequality signs are not affected by manipulations involving negative numbers.		Giving a simple statement, such as 6 < 7, and asking learners to multiple/divide by negative one. Any incorrect solutions learners provide will result in some cognitive conflict at seeing the incorrect statement, such as -6 < -7.
All inequality solution sets are written as one combined inequality in x. e.g. $-1 > x > 4$	If learners believe this they will not be able to find the correct solutions sets where there are two regions.	Allow learners to say in words first what the set of solutions is, e.g. *x* is less than -1 but greater than 4. Ask the question, is it possible to be the same value of *x* that satisfies both of these conditions?

Timings	Activity
10 min	**Starter/Introduction** Lesson slides: Quadratic inequalities Worksheet H: *Review of linear inequalities* From the images on slide 3-6/Worksheet H, ask learners to write down on mini whiteboards or other writing materials the correct inequalities in x which describe the sets of numbers identified on the number line, then choose learners to share their solutions with the class. The purpose of this activity is to review inequality notation and symbols. Learners may notice that the solution sets for Worksheet A are the answers to the inequalities on Worksheet H. From Worksheet H, ask learners in pairs to solve the linear inequalities and write down the solution sets. The purpose of the activity is to draw out any misconceptions learners may have when it comes to the use of inequality signs. You could confirm this to one pair and allow this information to filter to other pairs. This gives the pairs of learners a chance to confirm their own answers to Worksheet H with other pairs. Some learners may have made errors based on misconceptions of inequalities from previous learning. Knowing what the answers need to be will help learners to look back over their answers in their pairs and check with another pair to determine the error in their steps. This will most likely be poor use of inequality signs or issues with negative numbers. Learners will see the error and correct it themselves, with the help of their peers.
5 min	**Main lesson** Lesson slides: Quadratic inequalities Worksheet I: *Quadratic graphs* Display slides 6-9, or give out hard copies of Worksheet I to each **pair** of learners. *Each image consists of a parabola, a straight line and an identified region on the x-axis. Take two minutes and make a note in your pairs of anything you notice about each image.* Learners will seem a little confused at first as there are four images containing a lot of information. Encourage learners to focus on just one of the diagrams. *What functions/graphs can we see in the image?* *What might the highlighted areas represent?* **Support:** You could add some values to the x-axis of image A and ask learners to explicitly state what has been highlighted on the x-axis. You could ask the follow-up question:

Timings	Activity
	What output values do we get from our quadratic function if we input these x-values?
5 min	After five minutes, join pairs together to form groups of four. Ask learners to compare what they identified and come up with a list of things they noticed and write this down.
	Learners will give a range of things they have noticed. They may include: *There is a parabola (quadratic graph) intersecting the x-axis.* *There is a straight line intersecting a parabola.* *There is a set of values on the x-axis that are identified using the open and closed dots we have seen for inequalities.* *The red line has something to do with the intersections of the parabolas with various lines.*
	As you circulate the room, ask students to provide ideas that are interesting for the purpose of showing a solution set for a quadratic inequality involving a parabola and a straight line.
15 min	Use learners' ideas of what they have noticed to draw their attention to what each image is showing. Image A: Here we have a parabola intersecting the *x*-axis. The *x*-values which give *y*-coordinates less than or equal to zero are identified. Image B: Here we have a parabola intersecting a straight line. The *x*-values which give *y*-coordinates of the parabola greater than or equal to y-coordinate of the line are identified. Image C: Here we have a parabola intersecting a straight line. The *x*-values which give *y*-coordinates of the parabola greater than the *y*-coordinate of the line are identified. Image D: Here we have a parabola intersecting a straight line. The *x*-values which give *y*-coordinates of the parabola less than the *y*-coordinate of the line are identified. Focusing on Image A on Worksheet I: *Can you suggest an equation for a parabola that would look like this? The set of x-values identified are when the parabola's y-coordinate is less than or equal to zero. What would that look like as an inequality for your own created quadratic expression? This is a quadratic inequality.*
	Challenge: Create equations for each of the graphs in images B, C and D. Use these equations to help describe inequalities whose set of *x*-values that satisfy each are the identified red regions on the *x*-axis.
	Support: State that the parabola in Image A is defined by $= x^2 - 3x + 2$. *What is the region identified in red on the x-axis? What is true about the y-coordinate of the parabola at for these values of x? How could we write this as an inequality involving the expression* $x^2 - 3x + 2$*?*

Timings	Activity
10 min	*Try and suggest another parabola equation for Image B and this time an equation for the straight line. The set of x-values identified are when the parabola's y-coordinate is greater than or equal to the y-coordinate of the line, what would that look like as an inequality using your examples?* Lesson slides: *Quadratic inequalities* [Modelling process] Use the video on slide 10 to model the process for solving a quadratic inequality. Highlight the key features: rearranging the inequality, sketching the quadratic, highlighting the region require, writing the inequality down. Now learners can repeat the process with: $x^2 + 10x + 21 \geq 0$.
10 min 5 min	**Plenary** Worksheet J: *Review questions (quadratic inequalities)* Using Worksheet J, ask learners to think about the image they would expect to see for each quadratic inequality. i.e. what would the quadratic graph look like and how many regions of the x-axis would we expect to be highlighted. Now try and solve each quadratic inequality. **Support:** Select questions (a), (d) and (e) to do first, as they are inequalities in *x*. Did their original suggestion for the type of solution set for each inequality make sense when compared to their final answer? If not, where could the error be? Where is it most likely to be?

Reflection	Reflect on your lesson; use the **Lesson reflection** notes to help you.

Planning your own lessons

You now need to plan lessons to cover the following bullet points:

Candidate should be able to:	Notes and examples
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant	e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term 'repeated root' is included.
• solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic	e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.

Follow the structure of the *Teaching Pack*, and use techniques from the 'How to' guides, to create your own engaging lessons to cover these bullet points. Consider what preparation you need for each lesson: what prior knowledge is needed, what are the key objectives, what are the dependencies, what common misconceptions are there, and so on.

Below, we have provided an outline of some activities and approaches you might like to try.

Lesson 5: Discriminant of a quadratic polynomial
Common misconceptions: Not extracting the correct value or expression for a, b and c from $ax^2 + bx + c = 0$.
Starter: You could try different arrangements of a quadratic equation and ask learners to pick out the "a", "b" and "c". You can use this activity as a diagnostic to see which learners require additional support, and it could be extended to use algebraic coefficients.
Main: You could use Underground Mathematics Quadratics resource *Discriminating and* have a class discussion as outlined in the Teacher Support.
Plenary: You could try combining the ideas from this lesson with the ideas from Lesson 4, quadratic inequalities, with questions similar to Underground Mathematics Quadratics Review question R6828.

Lesson 6: Solving simultaneous equations (quadratic and linear terms)
Common misconceptions: Not substituting correctly rearranged equations to create a single quadratic equation in one variable.
Starter: You could try a review of the different ways to solve a pair of linear simultaneous equations (elimination and substitution). You could then highlight why substitution will work when you have quadratic terms and linear terms in your equations.
Main: You could use old exam paper questions and have a class discussion about the different forms the question could take.
Plenary: You could let learners create their own simultaneous equation questions mimicking the different forms they found in the main lesson activity.

You may also find it useful to use the Underground Mathematics Quadratics *Pick a card* activity. to review the main concepts on the topic of quadratics.

You will find some other activity suggestions in the Scheme of Work.

Lesson reflection

As soon as possible after the lesson you need to think about how well it went.

One of the key questions you should always ask yourself is:

Did all learners get to the point where they can access the next lesson? If not, what will I do?

Reflection is important so that you can plan your next lesson appropriately. If any misconceptions arose or any underlying concepts were missed, you might want to use this information to inform any adjustments you should make to the next lesson.

It is also helpful to reflect on your lesson for the next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time. There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.

To help you reflect on your lesson, answer the most relevant questions below.

Were the lesson objectives realistic?
What did the learners learn today? Or did they learn what was intended? Why not?
What proportion of the time did we spend on the most important topics?
Were there any common misconceptions?
What do I need to address next lesson?
What was the learning atmosphere like?
Did my planned differentiation work well?
How could I have helped the lowest achieving learners to do more?
How could I have stretched the highest achieving learners even more?
Did I stick to timings?
What changes did I make from my plan and why?

Summary evaluation

What two things went really well? (Consider both teaching and learning.)

What two things would have improved the lesson? (Consider both teaching and learning.)

What have I learned from this lesson about the class or individuals that will inform my next lesson?

Worksheets and answers

Worksheet A: Completing the square (squares/algebra)

Part 1:
1) Expand $(x+3)^2$

2) Expand $(x+10)^2$

3) Expand $(x+8)^2$

4) Expand $(x-8)^2$

Part 2:
1) Given

$$x^2 + 10x + \quad =$$

What number must be hidden for the expression to be a perfect square? What are the dimensions of the square?

2) Given

$$x^2 + 8x + \quad =$$

What number must be hidden for the expression to be a perfect square? What are the dimensions of the square?

3) Given

$$x^2 - 8x + \quad =$$

What number must be hidden for the expression to be a perfect square? What are the dimensions of the square?

Part 3:
1) Find the values of *A* and *B* for which $x^2 - 12x + 5 \equiv (x+A)^2 + B$

2) Find the values of *A*, *B* and *C* for which $5 + 12x - x^2 \equiv A(x+B)^2 + C$

3) Find the values of *A*, *B* and *C* for which $2x^2 - 12x + 5 \equiv A(x+B)^2 + C$

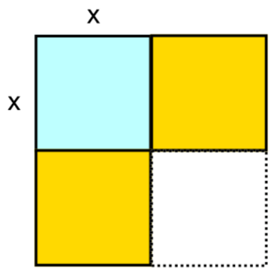
4) Find the values of *A*, *B* and *C* for which $4x^2 - 12x + 5 \equiv A(x+B)^2 + C$

Worksheet B: Completing the square

1) Factorise by completing the square

$x^2 - 10x =$

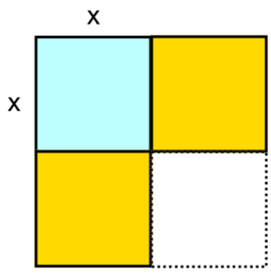
What constant needs to be subtracted to compensate for the completed corner of the square?



2) Factorise by completing the square

$x^2 - 22x =$

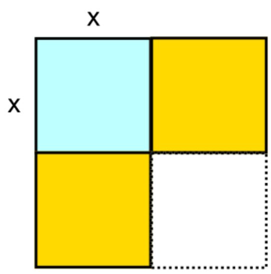
What constant needs to be subtracted to compensate for the completed corner of the square?



3) Factorise by completing the square

$x^2 - bx =$

What constant needs to be subtracted to compensate for the completed corner of the square?



Worksheet C: Quadratic expressions

1) $x^2 + 2x + 1$

2) $x^2 - 4x - 5$

3) $x^2 + 3x + 2$

4) $2x^2 + 5x - 3$

5) $x^2 + 2x - 9$

6) $x^2 - 7x - 1$

7) $3x^2 - 2x + 7$

8) $4x^2 - 4x - 3$

Worksheet D: Completing the square variations

$$x^2 + 8x + 1$$

$$(x + 4)^2 - 16 + 1$$

$$(x + 4)^2 - 15$$

Worksheet E: Completing the square – spot the errors

1)

$$x^2 + 4x + 3$$
$$\equiv (x+2)^2 + 4 + 3$$
$$\equiv (x+2)^2 + 7$$

2)

$$x^2 - 3x + 6$$
$$\equiv (x-3)^2 - 9 + 6$$
$$\equiv (x-3)^2 - 3$$

3)

$$2x^2 + 4x - 3$$
$$\equiv (2x+2)^2 - 4 - 3$$
$$\equiv (2x+2)^2 - 7$$

4)

$$2x^2 + 6x + 4$$
$$\equiv 2(x^2 + 3x) + 4$$
$$\equiv 2\left((x+\tfrac{3}{2})^2 - \tfrac{9}{4}\right) + 4$$
$$\equiv 2(x+\tfrac{3}{2})^2 - \tfrac{9}{4} + 4$$

Worksheet F: Spot the quadratic equation

1) $x^4 - 3x^2 - 4 = 0$

2) $x - 4\sqrt{x} + 3 = 0$

3) $x^{\frac{1}{4}} - 2x^{\frac{1}{2}} + 1 = 0$

4) $3\sin^2 x + 2\sin x - 1 = 0$

5) $x^4 = 10x^2 - 9$

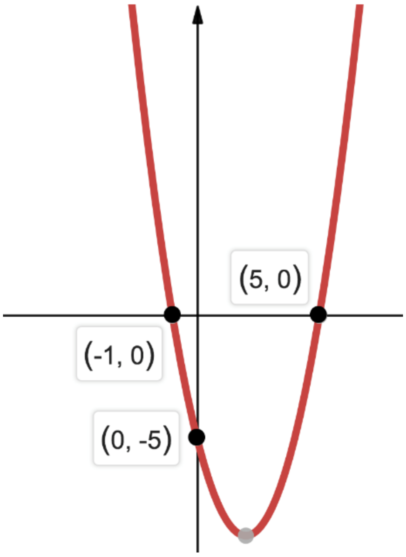
6) $\frac{4}{x} + \frac{3}{x^2} + 1 = 0$

7) $\cos^2 x - 2\cos x = 8$

8) $3\tan x = 3 - \tan^2 x$

Worksheet G: Graphs of quadratic functions

A



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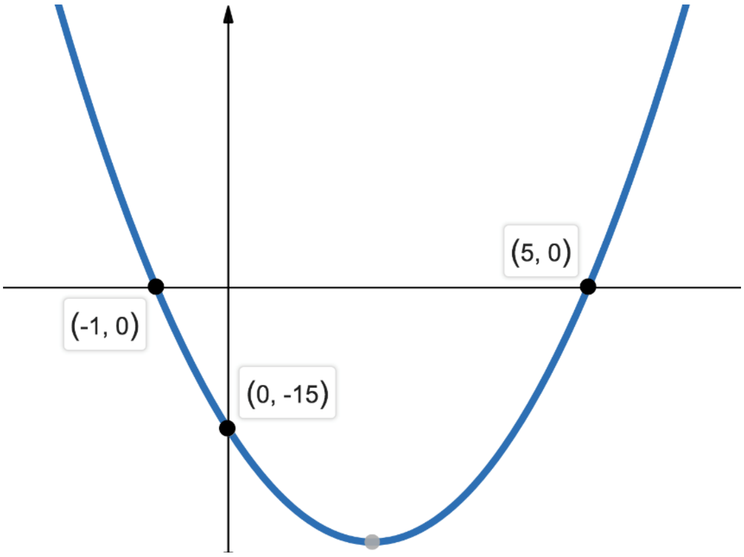

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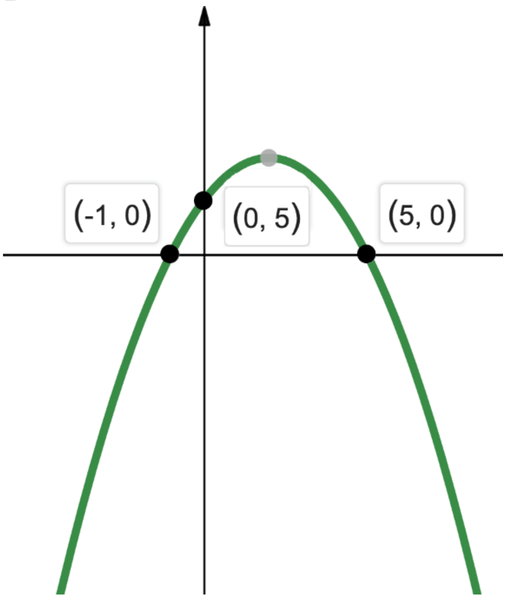
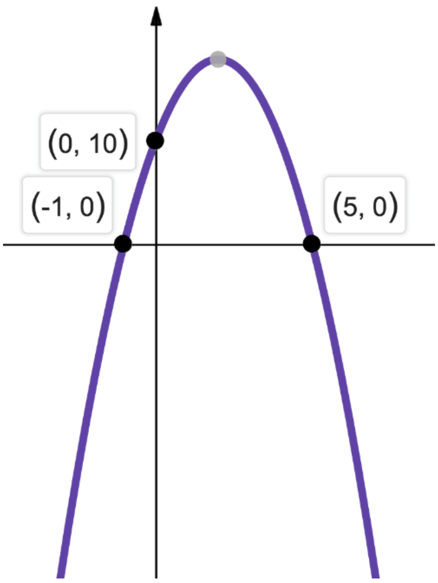
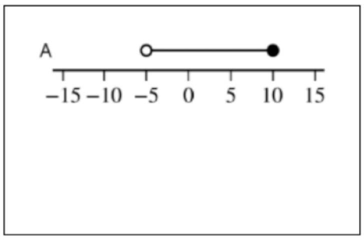


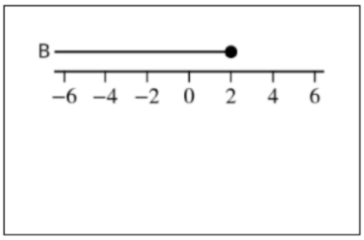
D

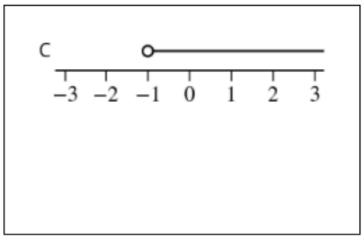


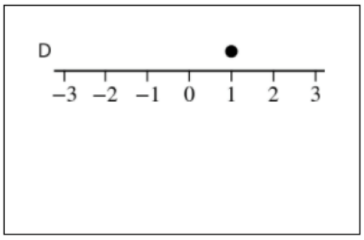
Worksheet H: Review of linear inequalities

Part A
Write down the correct inequalities in *x* which describe the sets of numbers identified on the number line.









Part B
Solve the linear inequalities and write down the solution sets.

(1) $2(x-1) > x-3$

(2) $3x+7 > x-3 \ and -2x \geq -20$

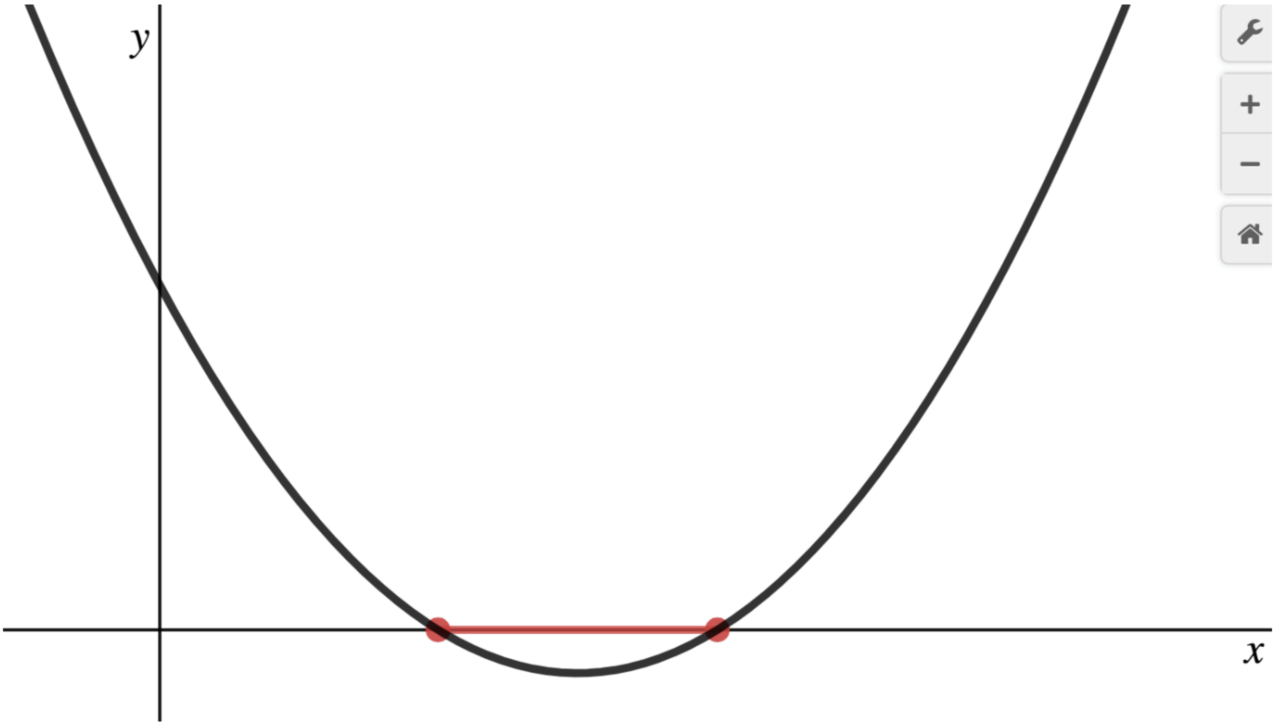
(3) $3(x+1) \leq 2x+5$

(4) $2(x+2) \leq 6 \ and \ 2(x+4) \geq 11-x$

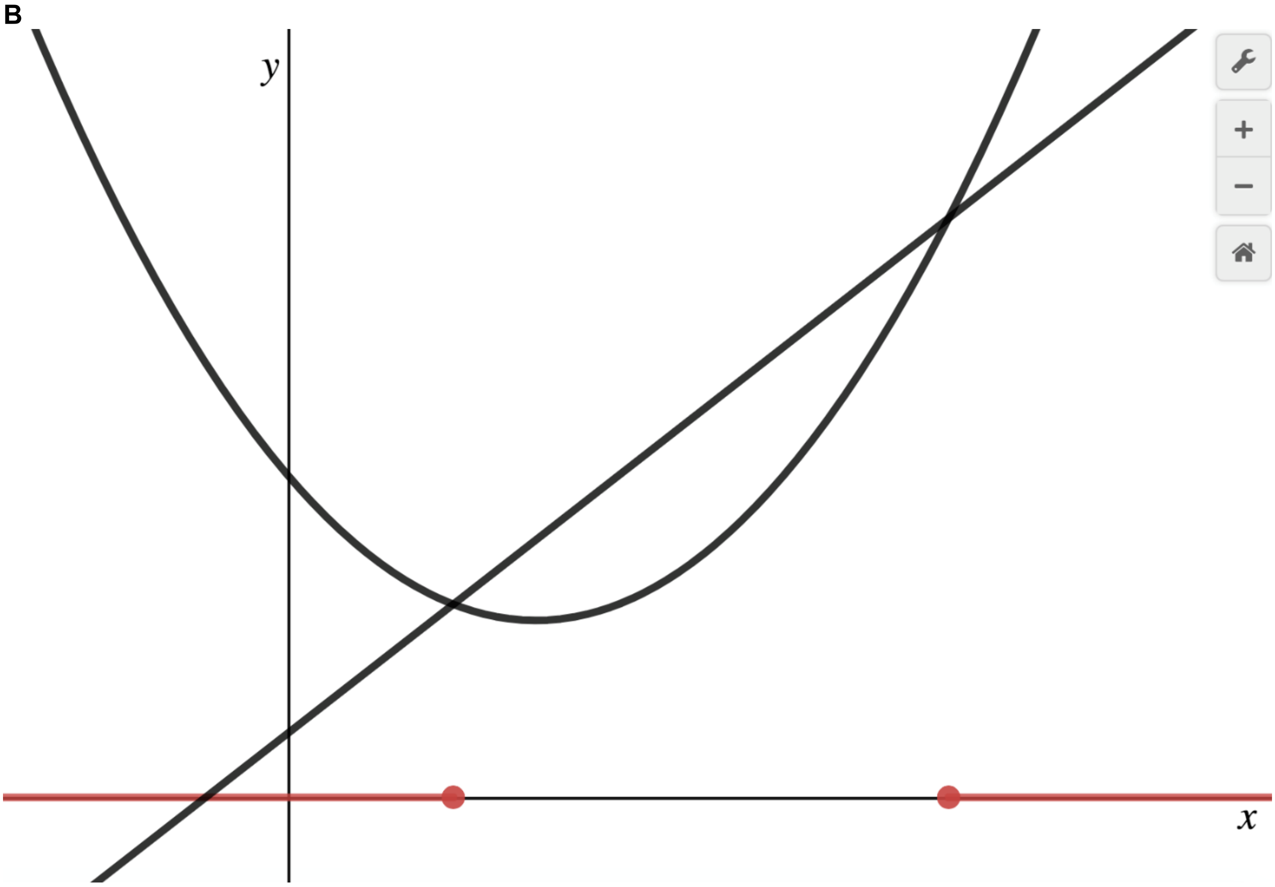
Worksheet I: Quadratic graphs

Each image consists of a parabola, a straight line and an identified region on the x-axis. What do you notice about each image? Write down your ideas on this sheet.

A

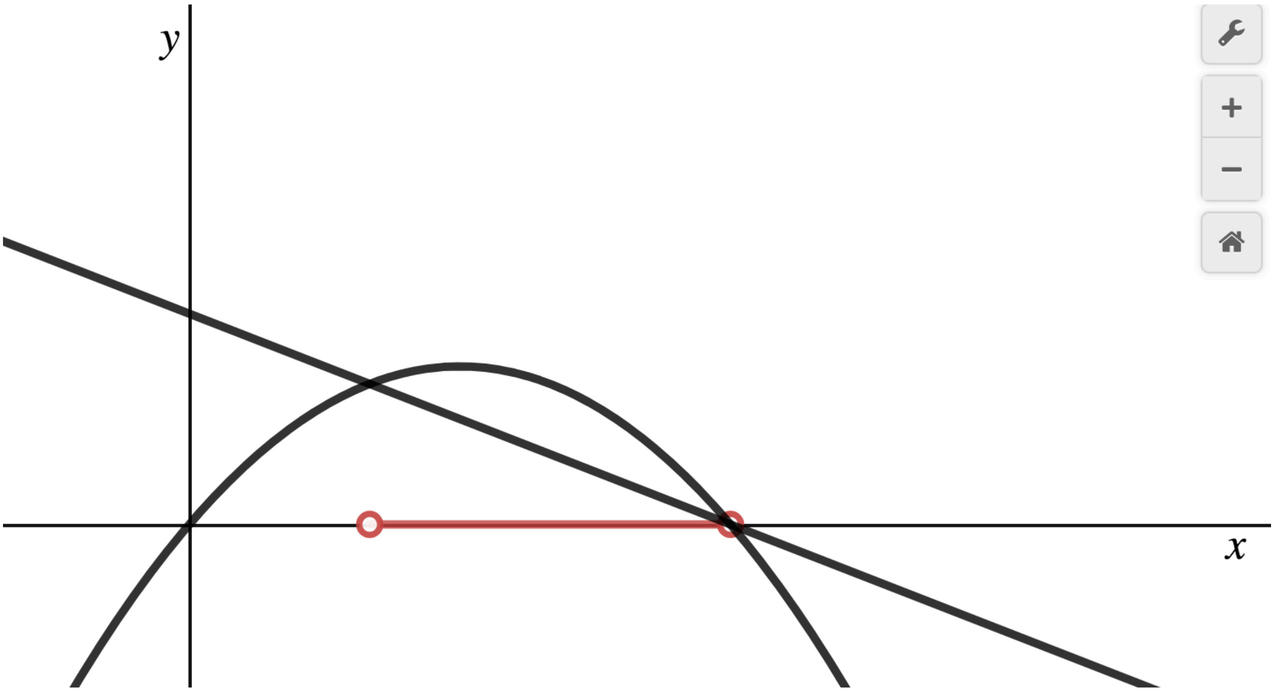


B



Worksheet I: Quadratic graphs continued

C



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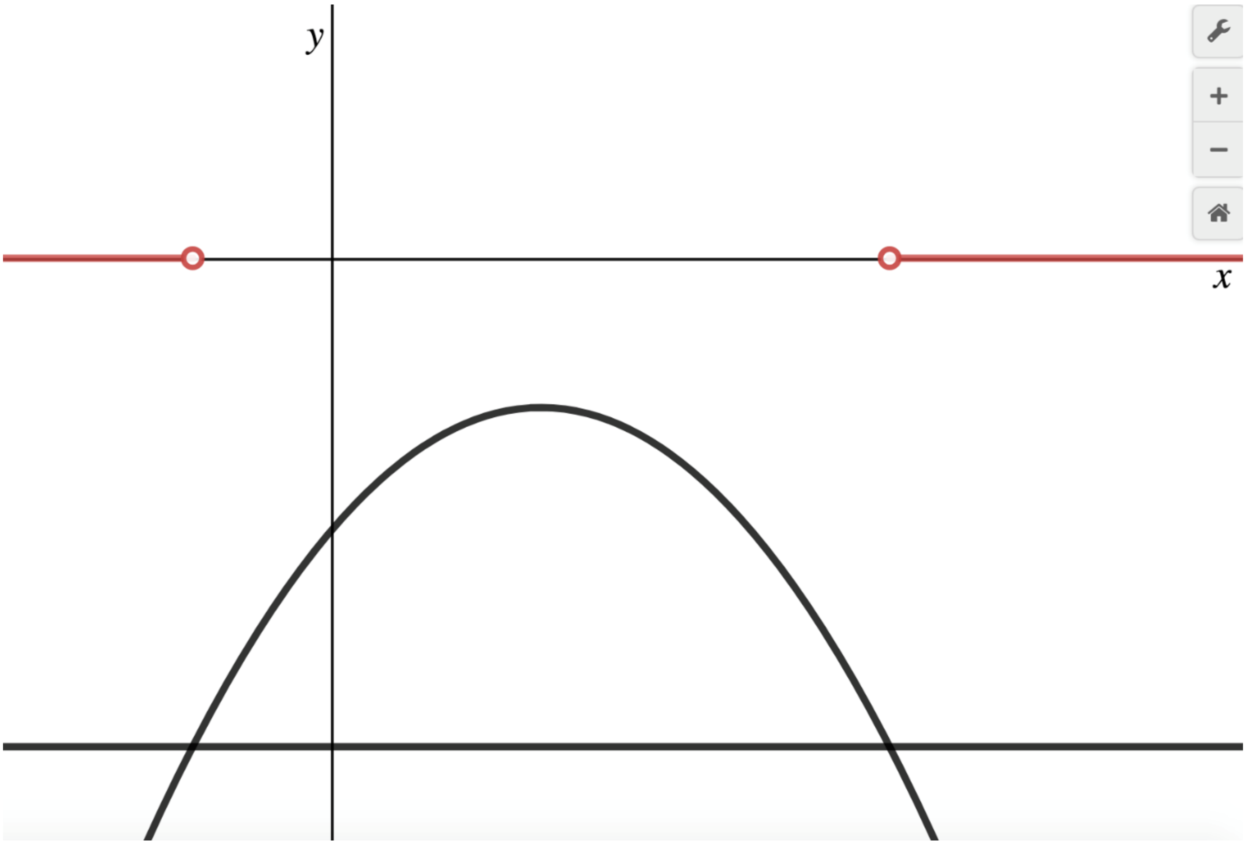


Worksheet J: Review questions (quadratic inequalities)

First, think about the image each of the following quadratic inequalities would give i.e. what region of the input-axis do you expect to see highlighted for the solution set.

Now solve each quadratic inequality.

Compare your final solution sets to the ones that you originally thought would occur.

(a) $x^2 - 5x + 4 < 0$

(b) $k^2 - 5k + 4 \geq 0$

(c) $y(y - 1) \leq 20$

(d) $3x^2 + 5x - 2 < 0$

(e) $4x - 3 \geq x^2$

(f) $10t^2 > t + 3$

Worksheet A: Answers

Part 1:

1) $x^2 + 6x + 9$
2) $x^2 + 20x + 100$
3) $x^2 + 16x + 64$
4) $x^2 - 16x + 64$

Part 2:

1) The number hidden is 25. The dimensions of the square are $x + 5$ and $x + 5$.

2) The number hidden is 16. The dimensions of the square are $x + 4$ and $x + 4$.

3) The number hidden is 16. The dimensions of the square are $x - 4$ and $x - 4$.

Part 3:

1) A = -6, B = -31.

2) A = -1, B = -6, C = 41 .

3) A = 2, B = -3, C = -13.

4) A =4 , B = -3/2, C = 11/4.

Worksheet B: Answers

1) We need to subtract 25, so that $x^2 - 10x = (x-5)^2 - 25$.

2) We need to subtract 121, so that $x^2 + 22x = (x+11)^2 - 121$.

3) We need to subtract $\frac{b^2}{4}$, so that $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4}$.

Worksheet C: Answers

Completing the square (for use with Lesson 1):

1) $x^2 + 2x + 1 = (x + 1)^2$

2) $x^2 - 4x - 5 = (x - 2)^2 - 9$

3) $x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

4) $2x^2 + 5x - 3 = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$

5) $x^2 + 2x - 9 = (x + 1)^2 - 10$

6) $x^2 - 7x - 1 = \left(x - \frac{7}{2}\right)^2 - \frac{53}{4}$

7) $3x^2 - 2x + 7 = 3\left(x - \frac{1}{3}\right)^2 + \frac{20}{3}$

8) $4x^2 - 4x - 3 = 4\left(x - \frac{1}{2}\right)^2 - 4$

Worksheet C: Answers (Lesson 2)

Factorise and complete the square (for use with Lesson 2):

1) $x^2 + 2x + 1 = (x + 1)^2$

2) $x^2 - 4x - 5 = (x - 5)(x + 1)$

3) $x^2 + 3x + 2 = (x + 2)(x + 1)$

4) $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

5) $x^2 + 2x - 9 = (x + 1)^2 - 10$

6) $x^2 - 7x - 1 = \left(x - \frac{7}{2}\right)^2 - \frac{53}{4}$

7) $3x^2 - 2x + 7 = 3\left(x - \frac{1}{3}\right)^2 + \frac{20}{3}$

8) $4x^2 - 4x - 3 = 4\left(x - \frac{1}{2}\right)^2 - 4$

Factorise and solve when equal to zero (for use with Lesson 2):

1) If $0 = x^2 + 2x + 1 = (x + 1)^2$ then $x = -1$.

2) If $0 = x^2 - 4x - 5 = (x - 5)(x + 1)$ then $x = 5$ or $x = -1$.

3) If $0 = x^2 + 3x + 2 = (x + 2)(x + 1)$ then $x = -2$ or $x = -1$.

4) If $0 = 2x^2 + 5x - 3 = (2x - 1)(x + 3)$ then $x = \frac{1}{2}$ or $x = -3$.

Complete the square and solve when equal to zero (for use with Lesson 2):

5) If $0 = x^2 + 2x - 9 = (x + 1)^2 - 10$ then $x = -1 \pm \sqrt{10}$.

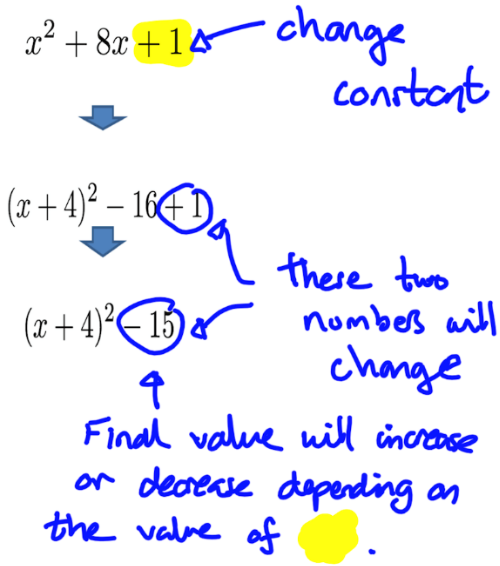
6) If $0 = x^2 - 7x - 1 = \left(x - \frac{7}{2}\right)^2 - \frac{53}{4}$ then $x = \frac{7 \pm \sqrt{53}}{2}$.

7) If $0 = 3x^2 - 2x + 7 = 3\left(x - \frac{1}{3}\right)^2 + \frac{20}{3}$ then no solutions.

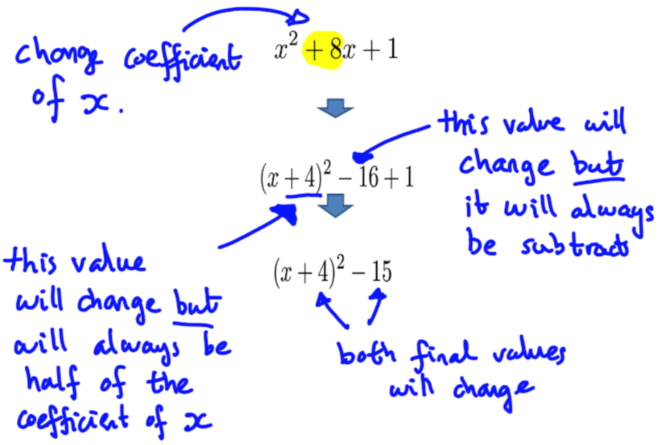
8) If $0 = 4x^2 - 4x - 3 = 4\left(x - \frac{1}{2}\right)^2 - 4$ then $x = \frac{3}{2}$ or $x = -\frac{1}{2}$.

Worksheet D: Answers

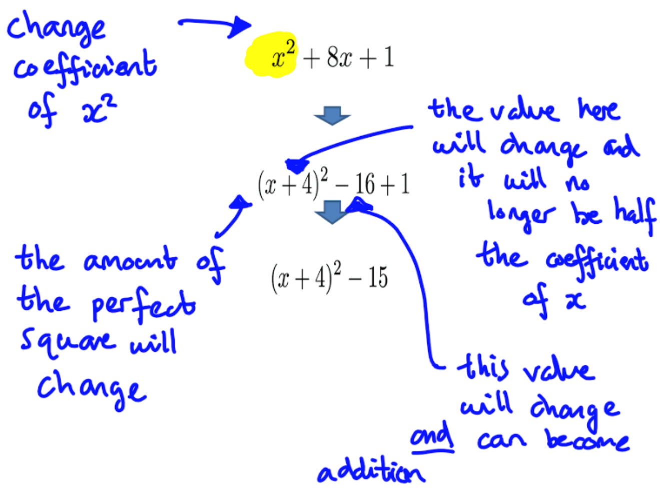
Variation 1:



Variation 2:



Variation 3:



Worksheet E: Answers

1)

$x^2 + 4x + 3$

$\equiv (x+2)^2 + 4 + 3$ — should be -4

$\equiv (x+2)^2 + 7$ ✗ $(x+2)^2 - 1$

2)

$x^2 - 3x + 6$

$\equiv (x-3)^2 - 9 + 6$ — should be $-\frac{3}{2}$; should be $-9/4$

$\equiv (x-3)^2 - 3$ ✗

$(x-\frac{3}{2})^2 + \frac{15}{4}$

3)

$2x^2 + 4x - 3$

$\equiv (2x+2)^2 - 4 - 3$ — incorrect should be $2(x^2+2x)$ then $2((x+1)^2-1)$

$\equiv (2x+2)^2 - 7$ ✗

$2(x+1)^2 - 5$

4)

$2x^2 + 6x + 4$

$\equiv 2(x^2 + 3x) + 4$

$\equiv 2((x+\frac{3}{2})^2 - \frac{9}{4}) + 4$

$\equiv 2(x+\frac{3}{2})^2 - \frac{9}{4} + 4$ ✗ — should be $-9/2$

$2(x+\frac{3}{2})^2 - 1/2$

Worksheet F: Answers

1) This is a quadratic in x^2. Let $Y = x^2$, then $Y^2 - 3Y - 4 = 0$.

2) This is a quadratic in $\sqrt{x}$. Let $Y = \sqrt{x}$, then $Y^2 - 4Y + 3 = 0$.

3) This is a quadratic in $x^{\frac{1}{4}}$. Let $Y = x^{\frac{1}{4}}$, then $Y - 2Y^2 + 1 = 0$.

4) This is a quadratic in $\sin x$. Let $Y = \sin x$, then $3Y^2 + 2Y - 1 = 0$.

5) This is a quadratic in x^2. Let $Y = x^2$, then $Y^2 = 10Y - 9$.

6) This is a quadratic in x^{-1}. Let $Y = x^{-1}$, then $4Y + 3Y^2 + 1 = 0$.

7) This is a quadratic in $\cos x$. Let $Y = \cos x$, then $Y^2 - 2Y = 8$.

8) This is a quadratic in $\tan x$. Let $Y = \tan x$, then $3Y = 3 - Y^2$.

Worksheet G: Answers

A $y = x^2 - 4x - 5$

B $y = -x^2 + 4x + 5$

C $y = 3x^2 - 12x - 15$

D $y = -2x^2 + 8x + 10$

Worksheet H: Answers

Part A:

A $-5 < x \leq 10$

B $x \leq 2$

C $-1 < x$

D $x = 1$

Part B:

1) $-1 < x$

2) $-5 < x \leq 10$

3) $x \leq 2$

4) $x = 1$

Worksheet J: Answers

a) $1 < x < 4$

b) $k < 1$ or $k > 4$

c) $-4 \leq y \leq 5$

d) $-2 < x < \frac{1}{3}$

e) $1 \leq x \leq 3$

f) $t < -\frac{1}{2}$ or $t > \frac{3}{5}$

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