Past paper questions

3.7 Vectors

The questions in this document have been compiled from a number of past papers, as indicated in the table below. Use these questions to formatively assess your learners' understanding of this topic.

Question	Year	Series	Paper number
9	2016	May/June	31
10	2015	May/June	32
6	2015	May/June	31
10	2014	May/June	32
10	2013	May/June	33
8	2016	March	32
10	2018	March	32
10	2013	May/June	32

The mark scheme for each question is provided at the end of the document.

You can find the complete question papers and the complete mark schemes (with additional notes where available) on the School Support Hub www.cambridgeinternational.org/support.

9 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

(i) Find the equation of the plane containing A, B and C, giving your answer in the form $ax + by + cz = d$. [6]

(ii) The line through D parallel to OA meets the plane with equation $x + 2y - z = 7$ at the point P. Find the position vector of P and show that the length of DP is $2\sqrt{(14)}$. [5]

10 The points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

(ii) Find the equation of the plane containing the line l and the point A. Give your answer in the form $ax + by + cz = d$. [6]

6 The straight line l_1 passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

(ii) Find the acute angle between the direction of the line l_2 and the direction of the x-axis. [3]

10 Referred to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

(i) Find the exact value of the cosine of angle BAC. [4]

(ii) Hence find the exact value of the area of triangle ABC. [3]

(iii) Find the equation of the plane which is parallel to the y-axis and contains the line through B and C. Give your answer in the form $ax + by + cz = d$. [5]

10 The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant. The plane p has equation $x + 2y + 2z = 6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to the plane p. [2]

(ii) The line l intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

(iii) The acute angle between the line l and the plane p is $\tan^{-1} 2$. [5]

8 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. The plane p has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$.

(i) Show that l is parallel to p. [3]

(ii) A line m lies in the plane p and is perpendicular to l. The line m passes through the point with coordinates $(5, 3, 1)$. Find a vector equation for m. [6]

10 The line l has equation $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$. The plane p has equation $2x - 3y - z = 4$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [3]

(iii) A second plane q is parallel to l, perpendicular to p and contains the point with position vector $4\mathbf{j} - \mathbf{k}$. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

10 The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$.

(i) Find the position vector of the point of intersection of the line through A and B and the plane p. [4]

(ii) A second plane q has an equation of the form $x + by + cz = d$, where b, c and d are constants. The plane q contains the line AB, and the acute angle between the planes p and q is $60°$. Find the equation of q. [7]

Mark schemes

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ or FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOI Seen or implied

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

9	**(i)**	*EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$	**B1**
		Use scalar product to obtain an equation in *a*, *b*, *c* e.g. $a - 2b - 3c = 0$, $a + b - c = 0$, or $3b + 2c = 0$	**M1**
		State two correct equations	**A1**
		Solve to obtain ratio $a : b : c$	**M1**
		Obtain $a : b : c = 5 : -2 : 3$	**A1**
		Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1**
		*OR*1: Substitute for two points, e.g. *A* and *B*, and obtain $a + 3b + 2c = d$ and $2a + b - c = d$	**(B1**
		Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from all three equations	**M1**
		Obtain two correct equations in three unknowns, e.g. in *a*, *b*, *c*	**A1**
		Solve to obtain their ratio	**M1**
		Obtain $a : b : c = 5 : -2 : 3$, $a : c : d = 5 : 3 : 5$, $a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$	**A1**
		Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1)**
		*OR*2: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$	**(B1**
		Obtain a second such vector and calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$	**M1**
		Obtain two correct components of the product	**A1**
		Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	**A1**
		Substitute in $5x - 2y + 3z = d$ to find *d*	**M1**
		Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1)**
		*OR*3: Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{j} + 2\mathbf{k}$	**(B1**
		Obtain a second such vector and form correctly a 2-parameter equation for the plane	**M1**
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$	**A1**
		State three correct equations in *x*, *y*, *z*, λ, μ	**A1**
		Eliminate λ and μ	**M1**
		Obtain equation $3x - 2y + 3z = 5$, or equivalent	**A1)**
			[6]

10	**(i)**	Carry out a correct method for finding a vector equation for AB	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		Equate at least two pairs of components of general points on AB and l and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$; or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$	A1	
		Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1	**[5]**
	(ii)	*EITHER*: Obtain a vector parallel to the plane and not parallel to l, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Use scalar product to obtain an equation in a, b and c, e.g. $3a + b - c = 0$	B1	
		Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for one ratio, e.g. $a : b$	M1	
		Obtain final answer $a : b : c = 1 : 4 : 7$ A1		
		Use coordinates of a relevant point and values of a, b and c in general equation and find d	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*1: Obtain a vector parallel to the plane and not parallel to l, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1	
		Obtain two correct components	A1	
		Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $x + 4y + 7z = d$, or equivalent, and find d	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*2: Obtain a vector parallel to the plane and not parallel to l, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a 2-parameter equation for the plane	M1	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(3\mathbf{i} + \mathbf{j} - \mathbf{k})$	A1	
		State 3 correct equations in x, y, z, s and t	A1	
		Eliminate s and t	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*3: Using the coordinates of A and two points on l, state three simultaneous equations in a, b, c and d, e.g. $a + b + 2c = d$, $2a - b + 3c = d$ and $4a + 2b + c = d$	B1	
		Solve and find one ratio, e.g. $a : b$	M1	
		State one correct ratio	A1	
		Obtain a correct ratio of three of the unknowns, e.g. $a : b : c = 1 : 4 : 7$, or equivalent	A1	
		Either use coordinates of a relevant point and the found ratio to find the fourth unknown, e.g. d, or find the ratio $a : b : c : d$	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*4: Obtain a vector parallel to the plane and not parallel to l, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-2 & y+1 & z-3 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0$	A1	
		Attempt to expand the determinant	M1	
		Obtain or imply two correct cofactors	A1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	**[6]**

6	(i)	Obtain $\pm\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ as direction vector of l_1	B1	
		State that two direction vectors are not parallel	B1	
		Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answers for λ and μ	A1	
		Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
	(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1	
		Use correct process for finding modulus and evaluating inverse cosine	M1	
		Obtain $79.5°$ or 1.39 radians	A1	[3]

10	**(i)**	*EITHER:*	State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form	B1	
			Using the correct processes evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, or equivalent	M1	
			Using the correct process for the moduli divide the scalar product by the product of the moduli	M1	
			Obtain answer $\frac{20}{21}$	A1	
		OR:	Use correct method to find lengths of all sides of triangle *ABC*	M1	
			Apply cosine rule correctly to find the cosine of angle *BAC*	M1	
			Obtain answer $\frac{20}{21}$	A1	**4**
	(ii)		State an exact value for the sine of angle *BAC*, e.g. $\sqrt{41}/21$	B1√	
			Use correct area formula to find the area of triangle *ABC*	M1	
			Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent	A1	**3**
			[SR: Allow use of a vector product, e.g. $\overrightarrow{AB}\times\overrightarrow{AC} = -6\mathbf{i}+2\mathbf{j}-\mathbf{k}$ B1√. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]		
	(iii)	*EITHER:*	State or obtain $b = 0$	B1	
			Equate scalar product of normal vector and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) to zero	M1	
			Obtain $a + b - 4c = 0$ (or $a - 4c = 0$)	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*1:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j})\times(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct product, e.g. $-4\mathbf{i}-\mathbf{k}$	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain $4x + z = 9$, or equivalent	A1	
		*OR*2:	Attempt to form 2-parameter equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	A1	
			State 3 equations in x, y, z, λ and μ	A1	
			Eliminate μ	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*3:	State or obtain $b = 0$	B1	
			Substitute for *B* and *C* in the plane equation and obtain $2a + c = d$ and $3a - 3c = d$ (or $2a + 4b + c = d$ and $3a + 5b - 3c = d$)	B1	
			Solve for one ratio, e.g. $a : d$	M1	
			Obtain $a : c : d$, or equivalent	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*4:	Attempt to form a determinant equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\begin{vmatrix} x-2 & y-4 & z-1 \\ 0 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 0$	A1	
			Attempt to use a correct method to expand the determinant	M1	
			Obtain two correct terms of a 3-term expansion, or equivalent	A1	
			Obtain answer $4x + z = 9$, or equivalent	A1	**5**

10	**(i)**	Equate scalar product of direction vector of l and p to zero	M1	
		Solve for a and obtain $a = -6$	A1	[2]
	(ii)	Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$	B1	
		Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ	M1	
		Obtain either $\lambda = \dfrac{2}{3}$ or $\mu = \dfrac{1}{3}$; or $\lambda = \dfrac{2}{a-1}$ or $\mu = \dfrac{1}{a-1}$; or reach $\lambda(a-4) = 0$ or $(1+\mu)(a-4) = 0$	A1	
		Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied	A1	[4]
	(iii)	Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation	M1*	
		Use $\frac{2}{\sqrt{5}}$ as sine of the angle	A1	
		State equation in any form, e.g. $\dfrac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \dfrac{2}{\sqrt{5}}$	A1	
		Solve for a	M1 (dep*)	
		Obtain answers for $a = 0$ and $a = \dfrac{60}{31}$, or equivalent	A1	[5]
		[Allow use of the cosine of the angle to score M1M1.]		

March 2016 Paper 32

8	**(i)**	*EITHER*: Substitute for **r** in the given equation of p and expand scalar product	**M1**	
		Obtain equation in λ in any correct form	**A1**	
		Verify this is not satisfied for any value of λ	**A1**	
		*OR*1: Substitute coordinates of a general point of l in the Cartesian equation of plane p	**M1**	
		Obtain equation in λ in any correct form	**A1**	
		Verify this is not satisfied for any value of λ	**A1**	
		*OR*2: Expand scalar product of the normal to p and the direction vector of l	**M1**	
		Verify scalar product is zero	**A1**	
		Verify that one point of l does not lie in the plane	**A1**	
		*OR*3: Use correct method to find the perpendicular distance of a general point of l from p	**M1**	
		Obtain a correct unsimplified expression in terms of λ	**A1**	
		Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent, for all λ	**A1**	
		*OR*4: Use correct method to find the perpendicular distance of a particular point of l from p	**M1**	
		Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent	**A1**	
		Show that the perpendicular distance of a second point is also $5/\sqrt{6}$, or equivalent	**A1**	[3]
	(ii)	*EITHER*: Calling the unknown direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ state equation $2a+b+3c=0$	**B1**	
		State equation $2a-b-c=0$	**B1**	
		Solve for one ratio, e.g. $a:b$	**M1**	
		Obtain ratio $a:b:c=1:4:-2$, or equivalent	**A1**	
		OR: Attempt to calculate the vector product of the direction vector of l and the normal vector of the plane p, e.g. $(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	**M2**	
		Obtain two correct components of the product	**A1**	
		Obtain answer $2\mathbf{i}+8\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
		Form line equation with relevant vectors	**M1**	
		Obtain answer $\mathbf{r}=5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-2\mathbf{k})$, or equivalent	**A1**√	[6]

March 2018 Paper 32

10(i)	Express general point of l in component form, e.g. $\mathbf{r}=(4+\mu)\mathbf{i}+(3+2\mu)\mathbf{j}+(-1-2\mu)\mathbf{k}$, or equivalent NB: Calling the vector $\mathbf{a}+\mu\mathbf{b}$, the **B1** is earned by a correct reduction of the sum to a single vector or by expressing the substitution as a distributed sum $\mathbf{a.n}+\mu\,\mathbf{b.n}$	**B1**
	Substitute in given equation of p and solve for μ	**M1**
	Obtain final answer $2\mathbf{i}-\mathbf{j}+3\mathbf{k}$ from $\mu=-2$	**A1**
		3

10(ii)	Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1
	Obtain answer 10.3° (or 0.179 radians)	A1
		3
10(iii)	*EITHER*: State $a + 2b - 2c = 0$ or $2a - 3b - c = 0$	(B1
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 8 : 3 : 7$, or equivalent	A1
	Substitute a, b, c and given point and evaluate d	M1
	Obtain answer $8x + 3y + 7z = 5$	A1)
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	(M1
	Obtain two correct components of the product	A1
	Obtain correct product, e.g. $8\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	A1
	Use the product and the given point to find d	M1
	Obtain answer $8x + 3y + 7z = 5$, or equivalent	A1)
	*OR*2: Attempt to form a 2-parameter equation with relevant vectors	(M1
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) + \mu(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$	A1
	State 3 equations in x, y, z, λ and μ	A1
	Eliminate λ and μ	M1
	State answer $8x + 3y + 7z = 5$, or equivalent	A1)
		5

10	**(i)**	Carry out a correct method for finding a vector equation for AB	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
		Substitute components in equation of p and solve for λ or for μ	M1	
		Obtain $\lambda = \frac{3}{2}$ or $\mu = -\frac{1}{2}$ and final answer $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
	(ii)	Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
		Obtain $3 + b - c = 0$, or equivalent	A1	
		Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\frac{1}{2}$, or form horizontal equivalent	M1*	
		Obtain correct equation in any form, e.g. $\dfrac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\frac{1}{2}$	A1	
		Solve simultaneous equations for b or for c	M1 (dep*)	
		Obtain $b = -4$ and $c = -1$	A1	
		Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent (The f.t. is on b and c.)	A1√	[7]