Past paper questions
2.3 Trigonometry

The questions in this document have been compiled from a number of past papers, as indicated in the table below. Use these questions to formatively assess your learners' understanding of this topic.

Question	Year	Series	Paper number
2	2017	March	22
8	2013	May/June	22
2	2014	May/June	22
2	2016	May/June	21
4	2016	May/June	22
3	2013	November	22
7	2014	November	21
6(i) and (ii) only	2016	November	22
2	2017	November	21

The mark scheme for each question is provided at the end of the document.

You can find the complete question papers and the complete mark schemes (with additional notes where available) on the School Support Hub www.cambridgeinternational.org/support.

2 **(i)** Given that $\tan 2\theta \cot \theta = 8$, show that $\tan^2 \theta = \frac{3}{4}$. [3]

(ii) Hence solve the equation $\tan 2\theta \cot \theta = 8$ for $0° < \theta < 180°$. [2]

8 **(i)** Prove the identity

$$\frac{1}{\sin(x-60°)+\cos(x-30°)} \equiv \operatorname{cosec} x.$$ [3]

(ii) Hence solve the equation

$$\frac{2}{\sin(x-60°)+\cos(x-30°)} = 3\cot^2 x - 2,$$

for $0° < x < 360°$. [6]

2 Solve the equation $3 \sin 2\theta \tan \theta = 2$ for $0° < \theta < 180°$. [4]

2 Solve the equation $5\tan 2\theta = 4\cot\theta$ for $0° < \theta < 180°$. [5]

4 **(i)** Show that $\sin(\theta + 60°) + \sin(\theta + 120°) \equiv (\sqrt{3})\cos\theta$. [3]

(ii) Hence

(a) find the exact value of $\sin 105° + \sin 165°$, [2]

(b) solve the equation $\sin(\theta + 60°) + \sin(\theta + 120°) = \sec\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

3 Solve the equation $2\cot^2\theta - 5\operatorname{cosec}\theta = 10$, giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

7 The angle α lies between $0°$ and $90°$ and is such that

$$2\tan^2\alpha + \sec^2\alpha = 5 - 4\tan\alpha.$$

(i) Show that

$$3\tan^2\alpha + 4\tan\alpha - 4 = 0$$

and hence find the exact value of $\tan\alpha$. [4]

(ii) It is given that the angle β is such that $\cot(\alpha + \beta) = 6$. Without using a calculator, find the exact value of $\cot\beta$. [5]

6 **(i)** Show that $\dfrac{\cos 2\theta}{1+\cos 2\theta} \equiv 1 - \frac{1}{2}\sec^2\theta$. [2]

(ii) Solve the equation $\dfrac{\cos 2\alpha}{1+\cos 2\alpha} = 13 + 5\tan\alpha$ for $0 < \alpha < \pi$. [4]

2 Solve the equation $5\cos\theta(1+\cos 2\theta) = 4$ for $0° \leqslant \theta \leqslant 360°$. [5]

Mark schemes

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ or FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOI	Seen or implied
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

March 2017 Paper 22

Question	Answer	Marks	Guidance
2(i)	Use identity $\cot\theta = \frac{1}{\tan\theta}$	**B1**	
	Attempt use of identity for $\tan 2\theta$	**M1**	
	Confirm given $\tan^2\theta = \frac{3}{4}$	**A1**	
	Total:	**3**	
2(ii)	Obtain 40.9	**B1**	
	Obtain 139.1	**B1**	
	Total:	**2**	

May/June 2013 Paper 22

8 **(i)** Use correct $\sin(A-B)$ and $\cos(A-B)$ formula — M1
Substitute exact values for $\cos 30°$ etc. — M1
Obtain given answer correctly — A1 [3]

(ii) State $2\text{cosec}\, x = 3\cot^2 x - 2$ — B1
Use $\cot^2 x = \text{cosec}^2 x - 1$ — M1
Attempt solution of quadratic equation in cosec x or $\sin x$ — M1
$(3\text{cosec}^2 x - 2\text{cosec}x - 5 = 0$ or $5\sin^2 x = 2\sin x - 3 = 0)$
Obtain $\sin x = \frac{3}{5}$ or -1 — A1√
Obtain one correct answer for $\sin^{-1}\left(\frac{3}{5}\right)$ — A1
Obtain remaining 2 answers from 36.9°, 143.1°, 270° and no others in the range — A1 [6]
[Ignore answers outside the given range]
SC If only answer given is 270° — B1

May/June 2014 Paper 22

2 Use $\sin 2\theta = 2\sin\theta\cos\theta$ — B1
Simplify to obtain form $c_1\sin^2\theta = c_2$ or equivalent — M1
Find at least one value of θ from equation of form $\sin\theta = k$ — M1
Obtain 35.3° and 144.7° — A1 [4]

May/June 2016 Paper 21

2 Use $\cot\theta = 1 \div \tan\theta$ — **B1**
Form equation involving $\tan\theta$ only and with no denominators involving θ — **M1**
Obtain $\tan^2\theta = \frac{2}{7}$ — **A1**
Obtain 28.1 — **A1**
Obtain 151.9 — **A1** [5]
Allow other valid methods

May/June 2016 Paper 22

4 **(i)** State $\sin\theta\cos 60+\cos\theta\sin 60+\sin\theta\cos 120+\cos\theta\sin 120$ ***B1**

Use $\sin 60=\sin 120=\frac{1}{2}\sqrt{3}$ and $\cos 60=\frac{1}{2}$, $\cos 120=-\frac{1}{2}$ ***B1**

Confirm result $\sqrt{3}\cos\theta$, dependent on *B *B **DB1** [3]

(ii) **(a)** $\cos 45$ seen ***B1**

State $\sqrt{\frac{3}{2}}$ or $\frac{1}{2}\sqrt{6}$ or exact equivalent, dependent *B **DB1** [2]

(b) Carry out correct process to find at least one value of θ from $\cos^2\theta=k$ **M1**

Obtain 40.6 **A1**

Obtain 139.4 **A1** [3]

November 2013 Paper 22

3 Use trig identity correctly to obtain a quadratic in cosec θ or sin θ M1

Solve the quadratic correctly M1

Obtain sin θ = ¼ or − ⅔ A1

Obtain one correct answer A1

Carry out correct method for second answer from either root DM1

Obtain remaining 3 answers from 14.5, 165.5, 221.8, 318.2 and no others in the range A1

[Ignore answers outside the given range] [6]

November 2014 Paper 21

7 **(i)** Use $\sec^2\alpha=1+\tan^2\alpha$ B1

Confirm $3\tan^2\alpha+4\tan\alpha-4=0$ B1

Solve quadratic equation for $\tan\alpha$ M1

Obtain, finally, $\tan\alpha=\frac{2}{3}$ only A1 [4]

(ii) State or imply $\tan(\alpha+\beta)=\frac{1}{6}$ B1

State $\frac{\frac{2}{3}+\tan\beta}{1-\frac{2}{3}\tan\beta}=\frac{1}{6}$, following their value of $\tan\alpha$ B1√

Solve equation of form $\frac{a+bt}{c+dt}$ for t M1

Obtain $\tan\beta=-\frac{9}{20}$ A1

Conclude with $\cot\beta=-\frac{20}{9}$ or exact equivalent A1 [5]

November 2016 Paper 22

6 (i)	Use $\cos 2\theta = 2\cos^2\theta - 1$ appropriately twice	B1		Alternative method $\frac{1-2\sin^2\theta}{2\cos^2\theta} = \frac{1}{2}\sec^2\theta - \tan^2\theta$ or $\frac{1}{2\cos^2\theta} - \tan^2\theta$ B1
	Simplify to confirm $1-\frac{1}{2}\sec^2\theta$	B1	[2]	then as for 2nd B1
(ii)	Use $\sec^2\alpha = 1+\tan^2\alpha$	B1		
	Obtain equation $\tan^2\alpha + 10\tan\alpha + 25 = 0$ or equivalent	B1		
	Attempt solution of 3-term quadratic equation for $\tan\alpha$ and use correct process for finding value of α from negative value of $\tan\alpha$	M1		If quadratic is incorrect, need to see evidence of attempt to solve as required to obtain M1
	Obtain 1.77	A1	[4]	Allow better or in terms of π $\left(\frac{1013\pi}{1800}\right)$

November 2017 Paper 21

2	Use $\cos 2\theta = 2\cos^2\theta - 1$	**B1**	
	Obtain $10\cos^3\theta = 4$ or equivalent	**B1**	
	Use correct process to find at least one value of θ from equation of form $k_1\cos^3\theta = k_2$	**M1**	
	Obtain 42.5	**A1**	
	Obtain 317.5 and no others between 0 and 360	**A1**	
		5	