Past paper questions
5.2 Permutations and combinations

The questions in this document have been compiled from a number of past papers, as indicated in the table below. Use these questions to formatively assess your learners' understanding of this topic.

Question	Year	Series	Paper number
7	2014	June	63
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The mark scheme for each question is provided at the end of the document.

You can find the complete question papers and the complete mark schemes (with additional notes where available) on the School Support Hub www.cambridgeinternational.org/support.

7 Nine cards are numbered 1, 2, 2, 3, 3, 4, 6, 6, 6.

(i) All nine cards are placed in a line, making a 9-digit number. Find how many different 9-digit numbers can be made in this way

(a) if the even digits are all together, [4]

(b) if the first and last digits are both odd. [3]

(ii) Three of the nine cards are chosen and placed in a line, making a 3-digit number. Find how many different numbers can be made in this way

(a) if there are no repeated digits, [2]

(b) if the number is between 200 and 300. [2]

7 **(a)** Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if

(i) there are no restrictions, [2]

(ii) the number made is an even number. [4]

(b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

6 **(a)** Find the number of different ways the 7 letters of the word BANANAS can be arranged

(i) if the first letter is N and the last letter is B, [3]

(ii) if all the letters A are next to each other. [3]

(b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cannot both be in the group together. [3]

7 **(a)** Find the number of different arrangements which can be made of all 10 letters of the word WALLFLOWER if

(i) there are no restrictions, [1]

(ii) there are exactly six letters between the two Ws. [4]

(b) A team of 6 people is to be chosen from 5 swimmers, 7 athletes and 4 cyclists. There must be at least 1 from each activity and there must be more athletes than cyclists. Find the number of different ways in which the team can be chosen. [4]

7 **(a)** Eight children of different ages stand in a random order in a line. Find the number of different ways this can be done if none of the three youngest children stand next to each other. [3]

(b) David chooses 5 chocolates from 6 different dark chocolates, 4 different white chocolates and 1 milk chocolate. He must choose at least one of each type. Find the number of different selections he can make. [4]

(c) A password for Chelsea's computer consists of 4 characters in a particular order. The characters are chosen from the following.

- The 26 capital letters A to Z
- The 9 digits 1 to 9
- The 5 symbols # ~ * ? !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Chelsea can make. [4]

6 The 11 letters of the word REMEMBRANCE are arranged in a line.

(i) Find the number of different arrangements if there are no restrictions. [1]

(ii) Find the number of different arrangements which start and finish with the letter M. [2]

(iii) Find the number of different arrangements which do not have all 4 vowels (E, E, A, E) next to each other. [3]

4 letters from the letters of the word REMEMBRANCE are chosen.

(iv) Find the number of different selections which contain no Ms and no Rs and at least 2 Es. [3]

6 **(a)** Seven fair dice each with faces marked 1, 2, 3, 4, 5, 6 are thrown and placed in a line. Find the number of possible arrangements where the sum of the numbers at each end of the line add up to 4. [3]

(b) Find the number of ways in which 9 different computer games can be shared out between Wainah, Jingyi and Hebe so that each person receives an odd number of computer games. [6]

5 **(a)** Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]

(b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.

(i) Find the number of different teams the manager can choose. [2]

(ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

6 **(a)** A village hall has seats for 40 people, consisting of 8 rows with 5 seats in each row. Mary, Ahmad, Wayne, Elsie and John are the first to arrive in the village hall and no seats are taken before they arrive.

(i) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John assuming there are no restrictions? [2]

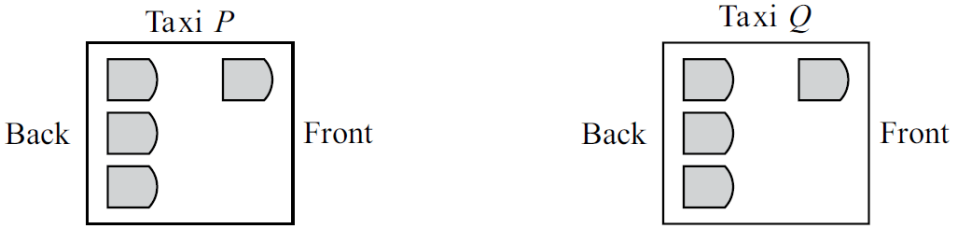
(ii) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John if Mary and Ahmad sit together in the front row and the other three sit together in one of the other rows? [4]

(b) In how many ways can a team of 4 people be chosen from 10 people if 2 of the people, Ross and Lionel, refuse to be in the team together? [4]

6 A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(a) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi.

Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(b) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

Mark schemes

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ or FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable)/ Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOI Seen or implied

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

7	(i) (a)	6! (×) 4! **OR** (×) 4 × 3 ÷ 2!2!3! **OR** ÷ 2!3! Total 720 ways	M1 M1 M1 A1	**4**	Seen in a single term expression as numerator Seen in a single term expression as numerator (denominator may be 1) Seen in a single term expression as denominator Correct ans
	(i) (b)	1*******3 = $\frac{7!}{3!2!} = 420$ 3*******1= 420 3*******3= 420 Total = 1260 ways	B1 M1 A1	**3**	$\frac{7!}{3!2!}$ seen oe Attempting to evaluate and sum at least 2 of 1***3, 3***1, 3***3 Correct ans
	(ii) (a)	5 × 4 × 3 = 60 ways ($^5P_3$)	M1 A1	**2**	$^5P_3$ or $^5C_3$ ×3! (can be implied) Correct ans
	(ii) (b)	2** in 212, 213, 214, 216, 221, 223, 224, 226, 231, 232, 233, 234, 236, 241, 242, 243, 246 261, 262, 263, 264, 266 Total = 22 ways **Alternative Methods:** $3 \times {}^4C_1 + 2 \times {}^5C_1$ **OR** $^5P_2 + {}^2C_1$ **OR** $^4P_2 + 2 \times {}^4P_1 + {}^2C_1$	M1 A1 M1 **OR** M1 **OR** M1	**2**	Listing attempt starting with 2, at least 10 correct entries Correct ans $p \times {}^4C_1 + q \times {}^5C_1$, oe $p + q > 2$ **OR** $^5P_2$ seen **OR** Any 2 terms added

7 (a) (i)	$\frac{9!}{2!2!3!}$ = 15120 ways	B1 B1 **[2]**	Dividing by 2!2!3! Correct answer
(ii)	********3 in $\frac{8!}{2!2!3!}$ = 1680 ways	B1	Correct ways end in 3
	********7 in $\frac{8!}{2!3!}$ = 3360 ways	B1	Correct ways end in 7
	Total even = 15120 – 1680 – 3360	M1	Finding odd and subt from 15120 or their **(i)**
	= 10080 ways	A1 **[4]**	Correct answer
	OR ********2 in 8!/2!3! = 3360 ways ********6 in 8!/2!2!3! = 1680 ways ********8 in 8!/2!2!2! = 5040ways Total = 10080 ways	B1 B1 M1 A1	One correct way end in even correct way end in another even Summing 2 or 3 ways Correct answer
	OR "15120" ×6/9 = 10080	M2 A2	Mult their **(i)** by 2/3 oe Correct answer
(b)	T(3) S(6) G(14) 1 1 3 in $3\times6\times{}^{14}C_3 = 6552$ 1 3 1 in $3\times{}^{6}C_3\times14 = 840$ 3 1 1 in $1\times6\times14 = 84$ 2 2 1 in $^{3}C_2\times{}^{6}C_2\times14 = 630$ 2 1 2 in $^{3}C_2\times6\times{}^{14}C_2 = 1638$ 1 2 2 in $3\times{}^{6}C_2\times{}^{14}C_2 = 4095$	M1 M1 M1 B1	Mult 3 (combinations) together assume $6 = {}^{6}C_1$etc Listing at least 4 different options Summing at least 4 different options At least 3 correct numerical options
	Total ways = 13839 (13800)	A1 **[5]**	Correct answer

May/June 2015 Paper 62

6 (a) (i)	N*****B Number of ways = $\frac{5!}{3!}$ = 20	B1 B1 B1 **3**		5! Seen in num oe or alone mult by $k \geqslant 1$ 3! Seen in denom can be mult by $k \geqslant 1$ Correct final answer
(ii)	B(AAA)NNS Number of ways = $\frac{5!}{2!}$ or ${}^5P_3$ = 60	M1 M1 A1 **3**		5! seen as a num can be mult by $k \geqslant 1$ Dividing by 2! Correct final answer
(b)	${}^{14}C_9$ total options = 2002 T and M both in ${}^{12}C_7 = 792$ Ans 2002 – 792 = 1210 OR Neither in ${}^{12}C_9 = 220$ One in ${}^{12}C_8 = 495$ Other in ${}^{12}C_8 = 495$	M1 B1 A1 **3** M1 B1		${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt ${}^{12}C_7$ (792) seen Correct final answer Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9 + {}^{12}P_8 + {}^{12}P_8$ here only Second correct option seen accept another 495 or if M1 not awarded, any correct option

May/June 2016 Paper 62

7 (a) (i)	$\frac{10!}{2!3!} = 302400$	**B1** [1]		Exact value only, isw rounding
(ii)	e.g. *W******W*, **W******W, W******W**	**M1**		8! Seen mult or alone. Cannot be embedded (arrangements of other 8 letters).
	$\frac{8!}{3!} \times 3$(for the Ws)	**M1**		Dividing by 3! (removing repeated L's)
		M1		Mult by 3 (different W positions) may be sum of 3 terms
	= 20160	**A1** [4]		
(b)	S(5) A(7) C(4) 1 3 2 : $5\times{}^7C_3\times{}^4C_2 = 1050$ 1 4 1 : $5\times{}^7C_4\times 4 = 700$ 2 3 1 : ${}^5C_2\times{}^7C_3\times 4 = 1400$ 3 2 1 : ${}^5C_3\times{}^7C_2\times 4 = 840$ (Outcomes : Options)	**M1**		Mult 3 combinations, ${}^5C_x$, ${}^7C_y$, ${}^4C_z$ (not 5 x 7 x 4)
		A1		2 correct options unsimplified
		M1		Summing only 3 or 4 correct outcomes involving combs or perms
	Total = 3990	**A1** [4]		

May/June 2017 Paper 61

7(a)	*EITHER:* e.g. xxxxx =5! for the other children	**(B1**	5! OE seen alone or mult by integer k ⩾ 1, no addition
	Put y in 6 ways, then 5 then 4 for the youngest children	**B1**	Mult by 6P3 OE
	Answer 5! × 6P3 = 14400	**B1)**	Correct answer
	OR: total – 3 tog – 2 tog = 8! – 6!3! – 6! × 2 × 5 × 3 = 14400	**(B1**	8! – 6! × k ⩾ 1seen
		B1	6!3! or 6! × 2 × 5 × 3 seen subtracted
		B1)	Correct answer
	Total:	**3**	
7(b)	D W M 2 2 1 = 6C2 × 4C2 × 1 = 90	**B1**	One correct unsimplified option
	3 1 1 = 6C3 × 4 × 1 = 80	**M1**	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied
	1 3 1 = 6 × 4C3 × 1 = 24	**M1**	Summing the correct 3 unsimplified outcomes only
	Total=194 ways	**A1**	
	Total:	**4**	
7(c)	C D S 2 1 1 = $^{26}C_2 \times 9 \times 5 \times 4!$ = 351 000	**M1**	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	1 2 1 = $26 \times {}^{9}C_2 \times 5 \times 4!$ = 112 320	**M1**	4 relevant products seen excluding 4! e.g. 26 × 9 × 8 × 5 or $26 \times {}^{9}P_2 \times 5$ for 2nd outcome, condone 26 × 9 × 5 × 37 as being relevant
	1 1 2 = $26 \times 9 \times {}^{5}C_2 \times 4!$ = 56 160	**M1**	mult all terms by 4! or 4!/2!
	Total = 519 480	**A1**	
	Total:	**4**	

November 2013 Paper 62

6	**(i)** 1663200	B1	**[1]**	
	(ii) M xxxxxxxxx M	M1		9! or 9P9 seen
	Number of ways = $\frac{9!}{3!2!}$ = 30240	A1	**[2]**	Correct answer
	(iii) 4 vowels together = 8! × 4/2!2! = 40320	M1 M1		8!/2!2! seen mult by something 4 oe 4!/3! or 4C1 etc. seen mult by something
	1663200 – 40320 = 1622880	B1	**[3]**	Correct answer SC 7!/2!2! × 8P4 or 7! × 8P4/3! Or 7!/2!2! × 8P4/3! M1
	(iv) Exactly 2 Es 4C2 = 6 Exactly 3 Es 4C1 = 4 Total = 10 ways	M1 B1 A1	**[3]**	Summing 2 options One option correct Correct answer
	OR 5C2 = 10	M2 A1		M1 for k5C2 Correct ans

November 2014 Paper 63

6	(a)	1*****3 or 3*****1 or 2*****2 $= 6^5 \times 3$	M1 M1		Mult by 6^5 (for middle 5 dice outcomes) Mult by 3 or summing 3 different combinations (for end dice outcomes)
		$= 23328$	A1	3	Correct answer accept 23 300
	(b)	W J H 1 1 7 = ${}^9C_1 \times {}^8C_1 \times 1 = 72$ 1 7 1 = ${}^9C_1 \times {}^8C_7 \times 1 = 72$ 7 1 1 = ${}^9C_7 \times {}^2C_1 \times 1 = 72$ 1 3 5 = ${}^9C_1 \times {}^8C_3 \times 1 = 504$ mult by 3! 3 3 3 = ${}^9C_3 \times {}^6C_3 \times 1 = 1680$	M1 A1 A1 M1 M1		Multiplying 3 combinations (may be implied) 1 unsimplified correct answer (72, 504, 1680, 216 or 3024) A 2nd unsimplified different correct answer Summing options for 1,1.7 or 1,3,5 oe (mult by 3 or 3!) Summing at least 2 different options of the 3
		Total 4920	A1	6	Correct ans
		If no marks gained Listing all 10 different outcomes	SCM1		If games replaced M1M1M1 max available If factorials used M0M1M1 max available

November 2016 Paper 61

5	(a)	e.g. P*N*P*P*L $= \frac{5!}{3!} \times \frac{{}^6P_4}{2!}$ $= 3600$	**M1** **M1** **M1** **A1**	[4]	Mult by 5! in num Dividing by 3! or 2! Mult by ${}^6P_4$ oe
	(b) (i)	${}^7C_5 \times {}^5C_4 \times {}^2C_1 \times {}^2C_1$ $= 420$	**M1** **A1**	[2]	Mult 4 combs of which three are correct
	(ii)	both in team ${}^6C_4 \times {}^4C_3 \times 2 \times 2 = 240$ $420 - 240 = 180$ ways **OR** Bat in bowl out + bowl in bat out + both out $= {}^6C_4 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_4 \times 2 \times 2$ $= 60 + 96 + 24 = 180$ ways **OR** Bat in bowl out + bat out $= 60 + {}^6C_5 \times {}^5C_4 \times 2 \times 2 = 60 + 120 = 180$ ways	**M1** **M1** **A1** **M1** **A1** **A1** **M1** **A1 A1**	[3]	Evaluating both in team and subtracting from (i) 240 seen can be unsimplified ft their 420, their 240 summing 2 or 3 options not both in team 2 or 3 options correct unsimplified Correct ans from correct working As above, or bowl in bat out + bowl out

6(a)(i)	$^{40}P_5$	**M1**	$^{40}P_x$ or $^{y}P_5$ oe seen, can be mult by $k \geqslant 1$
	= 78 960 960	**A1**	
		2	
6(a)(ii)	not front row e.g. WEJ** in 3× 3! = 18 ways	**B1**	3! seen mult by $k \geqslant 1$
	7 rows in 7 × 18= 126 ways	**B1**	mult by 7
	front row: e.g. *MA** in 4 × 2 = 8 ways	**M1**	attempt at front row arrangements and multiplying by the 7 other rows arrangements, need not be correct
	Total 126×8 = 1008	**A1**	
		4	
6(b)	*EITHER:* e.g. *R** in $^{8}C_3$ ways = 56 ways *L** in $^{8}C_3$ = 56 ways	**(M1**	Considering either R or L only in team
	**** in $^{8}C_4$ = 70 ways	**M1***	Considering neither in team
		DM1	summing 3 scenarios
	Total 182 ways	**A1)**	
	OR1: No restrictions $^{10}C_4$ = 210 ways	**(M1**	$^{10}C_4$ – , Considering no restrictions with subtraction
	RL = $^{8}C_2$ = 28	**M1***	Considering both in team
	210 – 28	**DM1**	subt
	= 182 ways	**A1)**	
6(b)	*OR2:* R out in $^{9}C_4$ = 126 ways L out in $^{9}C_4$ = 126 ways	**(M1**	Considering either R out or L out
	Both out in $^{8}C_4$ = 70	**M1***	Considering both out
		DM1	Summing 2 scenarios and subtracting 1 scenario
	126 + 126 – 70 = 182 ways.	**A1)**	
		4	

Mark Scheme Notes (Specimen Paper)

Marks are of the following three types.

M	Method mark, given for a valid method applied to the problem. Method marks can still be given even if there are numerical errors, algebraic slips or errors in units. However the method must be applied to the specific problem, e.g. by substituting the relevant quantities into a formula. Correct use of a formula without the formula being quoted earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, given for an accurate answer or accurate intermediate step following a correct method. Accuracy marks cannot be given unless the relevant method mark has also been given.
B	Mark for a correct statement or step.
DM or DB	M marks and B marks are generally independent of each other. The notation DM or DB means a particular M or B mark is dependent on an earlier M or B mark (indicated by *). When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT below).
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures (sf) or would be correct to 3 sf if rounded (1 decimal point (dp) for angles in degrees). As stated above, an A or B mark is not given if a correct numerical answer is obtained from incorrect working.
- Common alternative solutions are shown in the Answer column as: '**EITHER** Solution 1 **OR** Solution 2 **OR** Solution 3 …'. Round brackets appear in the Partial Marks column around the marks for each alternative solution.
- Square brackets [ ] around text show extra information not needed for the mark to be awarded.
- The total number of marks available for each question is shown at the bottom of the Marks column in bold type.

The following abbreviations may be used in a mark scheme.

AG	Answer given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
CAO	Correct answer only (emphasising that no 'follow through' from an error is allowed).
CWO	Correct working only
FT	Follow through after error (see Mark Scheme Notes for further details).
ISW	Ignore subsequent working
OE	Or equivalent form
SC	Special case
SOI	Seen or implied

6(a)	[Two in same taxi:] $^6C_2 \times {}^4C_4 \times 2$ or $^6C_2 + {}^6C_4$	1	**M1**	$^6C_4$ or $^6C_2$ OE seen anywhere
		1	**M1**	'something' × 2 only or adding 2 equal terms
	= 30	1	**A1**	
		3		
6(b)	[Mark, Jon and Sarah in taxi P:] $(^5C_1 \times 2 \times 2) \times {}^4P_4$	1	**M1**	$^5P_1$, $^5C_1$ or 5 seen anywhere
		1	**M1**	Multiply by 2 or 4 OE
		1	**M1**	Multiply by $^4P_4$ OE, e.g. 4! or $4 \times {}^3P_3$ or can be part of 5!
	= 480	1	**A1**	
		4		