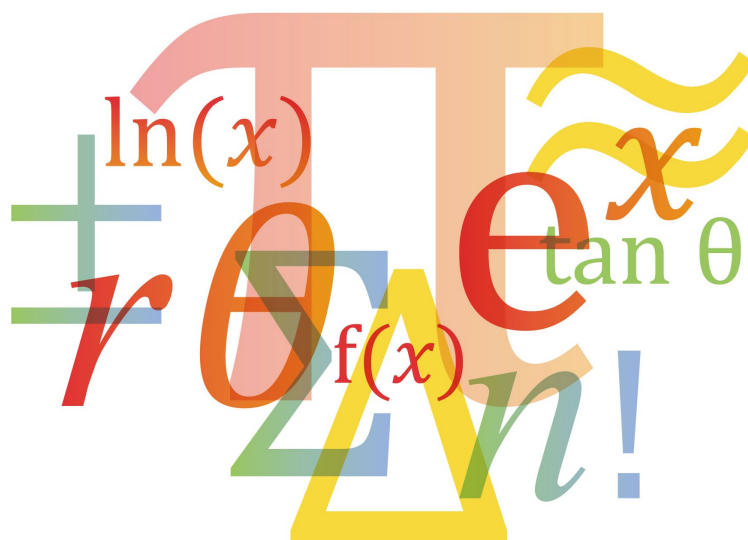


Teaching Pack

5.2 Permutations and Combinations

Cambridge International AS & A Level Mathematics 9709



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Contents

Contents	3
Introduction	4
Lesson preparation	5
Lesson 1: Arrangements	8
Lesson 2: More arrangements.....	11
Lesson 3: Permutations and combinations	14
Planning your own lessons.....	17
Lesson reflection.....	18
Worksheets and answers	19

Icons used in this pack:



Teacher preparation



Lesson plan



Lesson resource



Lesson reflection



Video

Introduction

This pack will help you to develop your learners' skills in mathematical thinking and mathematical communication, which are essential for success at AS & A Level and in further education.

Mathematical thinking and communication will be developed by focusing on:

1. Conceptual understanding – the 'why' behind the 'what'
2. Strategic competence – forming and solving problems
3. Adaptive reasoning – explanations, justifications and deductive reasoning

Throughout all activities, the learners will also develop:

- procedural fluency – know when, how and which rules to use
- positive disposition – believe maths can be learned, applied and is useful
- their skills in writing mathematically – writing working & proofs

These link to the course Assessment Objectives (AOs) which you can find in detail in the syllabus:

A01 Knowledge and understanding

A02 Application and communication

Each *Teaching Pack* contains one or more lesson plans and associated resources, complete with a section of preparation and reflection.

Each lesson is designed to be an hour long but you should adjust the timings to suit the lesson length available to you and the needs of your learners.

Important note

Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other topics.

This content is designed to give you and your learners the chance to explore a more active way of engaging with mathematics that encourages independent thinking and a deeper conceptual understanding. It is not intended as specific practice for the examination papers.

The *Teaching Packs* are designed to provide you with some example lessons of how you might deliver content. You should adapt them as appropriate for your learners and your centre. A single pack will only contain at most five lessons, it will **not** cover a whole topic. You should use the lesson plans and advice provided in this pack to help you plan the remaining lessons of the topic yourself.



Lesson preparation

This *Teaching Pack* will cover the following syllabus content:

Candidate should be able to:	Notes and examples
understand the terms permutation and combination, and solve simple problems involving selection	
- solve problems about arrangements of objects in a line, including those involving - repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') - restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).	Questions may include cases such as people sitting in two (or more) rows. Questions about objects arranged in a circle will not be included.

All bullet points for topic 5.2 are covered in this *Teaching Pack*. However, you should include a fourth lesson to consolidate and use all the skills learners have acquired to practise examination style questions. You will need to write your own lesson plan for this lesson.

Prior knowledge and skills

For all lessons, it is assumed that learners have already completed Cambridge IGCSE™ Mathematics 0580, or a course at an equivalent level. See the syllabus for more details of the expected prior knowledge for taking Cambridge International AS & A Level Mathematics 9709.

When planning any lesson, make a habit of always asking yourself the following questions about your learners' prior knowledge and skills:

- Do I need to re-teach this or do learners just need some practice?
- Is there an interesting activity that will efficiently achieve this?

Key learning objectives

The following list represents the main underlying concepts that you should make sure your learners have understood by the end of this topic:

- The product rule for counting is used to find a number of arrangements
- The general rules for ${}^n\text{P}_r$ and ${}^n\text{C}_r$ apply when choosing r objects from n distinct objects
- When objects are chosen, they are selected and then may be arranged and so selections are not ordered whereas arrangements are ordered.
- A permutation is a special case of an arrangement.
- A combination is a special case of a selection.

Why this topic matters

As well as being a topic that is assessed in its own right, the skills developed in this *Teaching Pack* are used elsewhere in statistics, such as when solving probability problems, as well as in other areas of maths.

Key terminology and notation

Your learners will need to be confident with the following terminology and notation:

$n - 1$, $n - 2$...	The value that is one less than n ; the value that is 2 less than n ...
arrangement	a selection of objects that is ordered
consonant	the letters of the alphabet that are not vowels
distinct	objects that are distinct are all different from each other
product	the result of multiplying two or more numbers. For example, the product of $5 \times 4 \times 3 = 60$
repeat	an object is repeated if it occurs more than once. For example, the word ASSESS has 4 identical letters, S. In this word the letters are not all different, some are repeated
repetition	this happens when something is repeated. For example, when making numbers using a set of digits, if repetition is allowed the digits may be repeated
restriction	this is a condition that must be taken into account when solving a problem. For example, finding the number of arrangements of the letters of the word HOPEFUL if the middle letter must be H. The restriction is that the middle letter is H
separated	this means 'not together'. If 10 pupils stand in a line but the two oldest pupils are separated, this means they are not together
selection	a choice of objects that is not ordered
vowel	the letters of the alphabet A, E, I, O, U



Insights video

There is an Insights video linked to this *Teaching Pack*:

- **5.2 Permutations and combinations** – use this video before teaching Lesson 1, which will look at arrangements, permutations and combinations and some of the challenges learners face in solving these kind of problems.



Teacher tutorials

There are *three* tutorials linked to this *Teaching Pack*:

- **Arrangements** – Review this tutorial before teaching Lesson plan 1; this gives an overview of the lesson and shows how to solve problems where objects need to be kept together.
- **More arrangements** – Review this tutorial before teaching Lesson plan 2; this gives an overview of the lesson and shows how to solve problems where objects must not be next to each other.
- **Permutations and combinations** – Review this tutorial before teaching Lesson plan 3; this gives an overview of the lesson and shows how to solve a problem on selection of groups for cars in a funfair ride.

Lesson progression

Lesson 1 covers an introduction to arrangements of distinct objects, part of the second bullet point of the syllabus content. Lesson 2 builds on this understanding and develops further ideas involving objects where some are repeated; it also considers more complex restrictions. Lesson 3 introduces

the ideas of permutations and combinations and simple problems on selection. Understanding the product rule for counting (the multiplication rule for counting) in Lesson 1 allows learners to develop a deeper understanding of the methodology needed to solve problems on arrangements in Lesson 2 and leads into permutations and combinations in Lesson 3.

Going forward

This topic links with the binomial distribution in section 5.4 Discrete random variable and section 1.6 Series. The topic also supports applications at a higher level such as Number Theory and Group Theory.



Lesson 1: Arrangements

Preparation

- Review the Teacher tutorial *Introduction to arrangements*
- Read Worksheet C: *Arrangements investigation teacher notes*
- Make sets of cards A, B, C, D, E, F, G, H, I, J for investigation
- Review Lesson slides *Arrangements*

Resources

- Paper, Mini whiteboards or other writing materials
- Worksheet A: *Arrangements investigation task 1*
- Worksheet B: *Arrangements investigation task 2*
- Worksheet C: *Arrangements investigation teacher notes*
- Lesson slides *Arrangements*
- Worksheet D: *Home study Lesson 1*

Learning objectives

By the end of the lesson:

- **all** learners should be able to solve problems about the arrangement of distinct objects in a line
- **most** learners should be able to solve problems about the arrangement of distinct objects in a line when simple restrictions are given
- **some** learners should be able to solve problems about the arrangement of distinct objects in rows

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
That, when counting, as there are, for example, 6 ways to fill this position AND 5 ways to fill the next position, there are $6 + 5$ ways to fill both positions	If learners believe this then they will not be able to successfully solve problems	The arrangements investigation included in this lesson guides learners in their understanding of the product principle for counting.

Timings	Activity
	Starter/Introduction Lesson slides Arrangements (slides 2-8) The quiz builds on the area of listing outcomes. All learners should be familiar with this from IGCSE. A box starts to be drawn as the slide is opened and takes 30 seconds to complete. Either discuss after each slide or discuss at the end of the quiz: • 'Here, the number of outcomes or ways is found by listing. • How efficient is it to list outcomes if all you need to know is the number of outcomes? • Is there a better way?'

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Timings	Activity
	Support: For learners who need more support it will be important not to move through the basics too quickly. Give more numerical examples. Give more examples with small numbers of objects so that learners can list them and be confident in their calculations. Although listing outcomes should not generally be encouraged, it is a good starting point and should help weaker learners build their skills.
	Plenary How many 5-digit numbers are multiples of 5? Your learners should now be able to answer this question. Ask for volunteers to give an answer and explain how they found it to the rest of the class. Check to see if the rest of the class agrees.

Reflection	Reflect on your lesson; use the <u>Lesson reflection</u> notes to help you.
How were your timings?	
Did you keep to your plan?	
Did learners engage with the investigation task?	
What could be done to improve upon it?	



Lesson 2: More arrangements

Preparation	- Review the Teacher tutorial <i>More arrangements</i> - Prepare a statement and envelope for the Plenary activity
Resources	- Paper, Mini whiteboards or other writing materials - Worksheet E: <i>You are the teacher: mark this work</i> - Lesson slides <i>More arrangements</i> - Worksheet F: <i>More arrangements – practice questions</i> - Worksheet G: <i>Home study Lesson 2</i>
Learning objectives	By the end of the lesson: all learners should be able to solve simple problems involving repetition and simple restrictions most learners should be able to solve problems involving repetition and restrictions such as people who must or must not stand together some learners should be able to solve more challenging problems involving rows and multiple restrictions

Dependencies

Learners need to know the meaning of the factorial notation, the product rule for counting and have already studied an introduction to arrangements.

Timings	Activity
 10 min	Starter/Introduction Worksheet E: You are the teacher: mark this work Lesson slides More arrangements (slides 2-4) In this activity, learners (working in pairs) mark a sample of work to identify errors. This makes them think more actively about what is acceptable and about the rubric on the front of the exam paper, and allows you to assess how well they have understood the topic so far. Learners find the errors in the work presented and explain what is wrong. They must decide the number of marks to award and make 3 comments praising the work and set 1 target for improvement. Use slides 2, to give instructions to the class. Use slides 3-4 as a basis for discussion once the 5 minutes is over.
 15 min	Main lesson Arranging objects that are not all different Lesson slides More arrangements (slides 5-11) Question & Answer session with the whole class using slides 5 – 11. Where discussion points arise, allow time for learners to think, answer and discuss. (This will deepen learning.) Slide 12 introduces Worksheet F.

Timings	Activity
  	<u>Worksheet F: More arrangements – practice questions</u> Learners work in pairs or small groups on Practice question 1; ask a group of learners to present their solution to the class, or they hand in their work for later marking. More restrictions <u>Lesson slides More arrangements (slides 13-18)</u> Question & Answer session with the whole class using slides 13 – 18. Where discussion points arise, allow time for learners to think, answer and discuss. <u>Worksheet F: More arrangements – practice questions</u> Learners work in pairs or small groups on Practice questions 2 - 3; ask a group of learners to present their solution to the class. <u>Worksheet G: Independent study Lesson 2</u> This can be used to support learners (in this case also give them the model answers) or as homework. Challenge: Some extension questions are given in Worksheet F: <i>Practice Questions 1 _ 2</i>. It is essential that learners experience many contexts that need interpretation. This is not rote learning. Support: More practice of the simple skills and encouraging learners to draw diagrams if possible should help a great deal. It will be important to teach learners in English. Learners who are struggling will benefit from more practice using simple contexts and restrictions.
	Plenary Read my mind! Write the statement ‘ For $n!$, n must be a positive integer’ on a piece of paper and place it in an envelope. Tell your learners that you have written a statement about $n!$ and put it in the envelope. They can have 3 guesses at what you have written. If they do not succeed, you win, if they guess in 3 tries or less, they win. This simple activity recaps the basics of the factorial notation which is essential to this topic. The responses made by learners can be written down on the board or on a flip chart (a learner could do this for you). Examples of what they may say: $n!$ is the number of ways n objects can be arranged $n!$ is $n(n - 1)(n - 2)(n - 3)..... 3 \times 2 \times 1$ $n!$ is a product If n is 3, $n!$ is $3 \times 2 \times 1$

Reflection	Reflect on your lesson; use the <u>Lesson reflection</u> notes to help you.
Did your learners engage with the presentation?	
Did you allow them enough time to think before prompting them?	
Did you ask them open-ended questions or closed questions with single answers?	



Lesson 3: Permutations and combinations

Preparation

- Review the Teacher tutorial *Permutations and combinations*
- Prepare cards for Worksheet I: *Arrangements or selections sorting activity*

Resources

- Paper, Mini whiteboards or other writing materials
- Worksheet H: *Connections*
- Lesson slides *Permutations and combinations*
- Worksheet I: *Arrangements or selections sorting activity*
- Worksheet J: *Permutations and combinations practice questions*

Learning objectives

By the end of the lesson:

- **all** learners should understand the terms permutation and combination, and recognise a situation that represents either of these
- **most** learners should know the difference between an arrangement and a selection and apply them to solve simple problems choosing r objects from n distinct objects
- **some** learners should be able to solve simple problems choosing r objects from n objects that are not all distinct

Dependencies

Learners need to know the meaning of the factorial notation, the product rule for counting and have already studied the material in Lessons 1 and 2.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
When asked to 'Find the number of different selections...' learners find the number of different arrangements.	If learners believe this then they will not be able to successfully solve problems	Connecting the words 'arrangement' and 'permutation' will come naturally through the development of ideas in Lessons 1 and 2. Connecting the words 'selection' and 'combination' will be an extension of those ideas. Learners should practise looking for and using these keywords in as many contexts as possible so that they have a more natural understanding of the language.

Timings	Activity
	Starter/Introduction Worksheet H: Connections A simple recap where learners connect a question to its answer. Questions are all based on arrangements of r objects from n . This is preparation for the work on

Timings	Activity
	permutations which is to follow. The sheets can be printed out for each learner to link the Question and Answer by joining them with lines or arrows. Alternatively, the sheets can be made into cards to be shared between small groups. Allow 5 minutes for the matching to be done. When the time is up, select different learners to explain each link. Allow time for other learners to ask questions to clear up any misunderstandings.
   	Main lesson <u>Lesson slides <i>Permutations and Combinations</i> (slides 2 – 7)</u> Build on the ideas used in the starter activity and help learners to develop the general rule for ${}^n\text{P}_r$ by considering a numerical example. Learners will need to be confident that this rule is generally applicable without necessarily looking at the algebraic factorials. Once this has been established, the general rule for ${}^n\text{C}_r$ is developed, again using a specific example, and by understanding the difference between an arrangement and a selection. <u>Worksheet I: <i>Permutations and combinations sorting activity</i></u> Learners work in small groups in this simple activity to focus on the possible language they will meet when answering questions requiring a permutation or a combination of objects, before they have to concern themselves with the actual mathematics of doing so. It is vital to give learners as much opportunity as possible to identify the difference between an arrangement of objects and a simple selection of objects. Allow 5 minutes for the sorting and 5 minutes to discuss the results afterwards. Support can be given with the prompt asking which links to arrangements. <u>Lesson slides <i>Permutations and Combinations</i> (slides 8 – 14)</u> These 3 worked examples should be a good basis from which your learners can develop their problem-solving skills using permutations and combinations. Challenge: Example 4 (slides 15-16) could be used as a challenge for more able learners or as a start for the next lesson. <u>Worksheet J: <i>Permutations and combinations practice questions</i></u> Working in small groups, learners practise the skills they have just acquired. The ability to discuss each question with their peers will be very important in developing understanding in this topic. Learners who have a deeper understanding perform better when assessed as they are more easily able to apply their skills. Some learners will prefer to answer some of the permutations questions using arrangements, as they have to this point. That is fine and it is useful to know the relationship and understand that the process is the same. The practice questions can be completed at home if necessary. Questions 13 to 15 are combinations where the items are not different. These could be used for extending

Timings	Activity
	more able learners or as a starting point for Lesson 4. Challenge: Very able learners may be able to develop the rules for ${}^n\text{P}_r$ and ${}^n\text{C}_r$ algebraically and should be able to work on permutation and combination problems that require multiple skills more quickly. Ask learners to explain why $0! = 1$. Support: Weaker learners may need to focus on understanding and applying the basic skills more comprehensively before moving on to more challenging material. Try to use as many simple contexts and examples as possible to help support the language skills and logic skills needed.
	Plenary Three key points Instruction to learners: Write down 3 key points that you have learned in today's lesson. Learners will write down some common points but many will vary. Make a list; all the important points from the lesson should arise, generated by the learners.

Reflection	Reflect on your lesson; use the <u>Lesson reflection</u> notes to help you.
How effective was the starter at focusing attention on arrangements?	
Did learners benefit from the sorting activity?	
Was the timing correct for the sorting activity or should it be used as a plenary?	

Planning your own lessons



You now need to plan a lesson to practise all the advanced counting skills that your learners have acquired in the first three lessons.

Follow the structure of the *Teaching Pack*, and use techniques from the 'How to' guides, to create your own engaging lessons to do this. Consider what preparation you need for this lesson: what prior knowledge is needed, what are the key objectives, what are the dependencies, what common misconceptions are there, and so on.

Below, we have provided an outline of some activities and approaches you might like to try.

Lesson 4: Solving problems about arrangements and selections

Common misconceptions: This should have been covered in prior lessons

Starter: You could try a quick true/false quiz, using a PowerPoint to see if your learners can identify whether a problem could be solved using arrangements, permutations or combinations

Main: You could use past paper questions and create your own bank of questions, graded in level of difficulty. You should allow group working so that learners can discuss the most efficient method of solution. You could try peer marking/presentation of solutions as part of the process.

Plenary: You could try an ordering task. Give your learners a question and the steps of the answer that have been cut into strips. Ask your learners to order the solution (can be done in teams or as a class).

You will find some other activity suggestions in the Scheme of Work.



Lesson reflection

As soon as possible after the lesson you need to think about how well it went.

One of the key questions you should always ask yourself is:

Did all learners get to the point where they can access the next lesson? If not, what will I do?

Reflection is important so that you can plan your next lesson appropriately. If any misconceptions arose or any underlying concepts were missed, you might want to use this information to inform any adjustments you should make to the next lesson.

It is also helpful to reflect on your lesson for the next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time. There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.

To help you reflect on your lesson, answer the most relevant questions below.

Were the lesson objectives realistic?

What did the learners learn today? Or did they learn what was intended? Why not?

What proportion of the time did we spend on the most important topics?

Were there any common misconceptions?

What do I need to address next lesson?

What was the learning atmosphere like?

Did my planned differentiation work well?

How could I have helped the lowest achieving learners to do more?

How could I have stretched the highest achieving learners even more?

Did I stick to timings?

What changes did I make from my plan and why?

Summary evaluation

What two things went really well? (Consider both teaching and learning.)

What two things would have improved the lesson? (Consider both teaching and learning.)

What have I learned from this lesson about the class or individuals that will inform my next lesson?

Worksheets and answers

	Worksheet	Answers
For use with Lesson 1:		
A: Arrangements investigation task 1	20-21	34-35
B: Arrangements investigation task 2	22-23	36-37
C: Arrangements investigation: teacher notes	24	
D: Independent study lesson 1	25	39-39
For use with Lesson 2:		
E: You are the teacher: mark this work	26	
F: More arrangements – practice questions	27-28	40-41
G: Independent study lesson 2	29	42-44
For use with Lesson 3:		
H: Connections	30	45
I: Permutations and combinations sorting activity	31	46
J: Permutations and combinations practice questions	32-33	47-49

Worksheet A: Arrangements investigation task 1



Complete this task with your group. You have **20 minutes** and will need to present your findings at the end.

Q1 Each letter can be used once only.

(a) Complete the table to show the number of arrangements of the letters given.

Letters	Possible arrangements	Number of letters	Number of arrangements
D	D	1	1
CD	CD, DC	2	2
BCD			

(b) By listing only the arrangements that start with A and using the results for 3 letters, find the number of possible arrangements of these 4 letters.

ABCD			
------	--	--	--

(c) Show how you found your answer.

.....

.....

(d) Complete each of these statements with a number.

The number of arrangements with 4 letters is times the number of arrangements with 3 letters.

The number of arrangements with 5 letters is times the number of arrangements with 4 letters.

Worksheet A: Arrangements investigation task 1 continued



(e) Complete this statement.

There are arrangements of the 10 letters ABCDEFGHIJ
when each letter can be used once only.

(f) Show how you found your answer.

.....

.....

Q2 In how many different ways can the eight characters

6 7 8 9) (* &

be arranged to make an 8-character pattern when

(a) each character can be used once only

.....

.....

(b) each character can be repeated?

.....

.....

Q3 You do not have to evaluate your expressions in this question.

Halima is arranging 50 different characters to make a 50-character
pattern.

(a) Write an expression for the number of ways
Halima can do this when each character can be
repeated. Simplify your answer.

You do not have to write
all the missing terms of
the expression, just the
first 3 and the last 3.

$50 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \dots \dots \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

(b) Write an expression for the number of ways Halima can do this when each character can
be used once only.

$50 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \dots \dots \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Worksheet B: Arrangements investigation task 2



Complete this task with your group. You have **20 minutes** and will need to present your findings at the end.

Q1 In this question, each letter can be used once only.

(a) Complete the table to show the number of arrangements of the letters given.

Letters	Possible arrangements	Number of letters	Number of arrangements
A	A	1	1
AB	AB, BA	2	2
ABC			

(b) By listing only the arrangements that start with A and using the results for 3 letters, find the number of possible arrangements of these 4 letters.

ABCD			
------	--	--	--

(c) Show how you found your answer.

.....

.....

(d) Complete each of these statements with numbers.

The number of arrangements with 4 letters is times the number of arrangements with letters.

The number of arrangements with 5 letters is times the number of arrangements with letters

Worksheet B: Arrangements investigation task 2

continued



(e) Complete this statement.

There are arrangements of the 10 letters ABCDEFGHIJ
when each letter is used exactly once.

(f) Show how you found your answer.

.....

.....

Q2 In how many different ways can the 8 characters

6 7 8 9) (* &

be arranged to make an 8-character pattern when

(a) each character can be used once only

.....

.....

(b) each character can be repeated?

.....

.....

Q3 Complete the following expressions for the number of different ways n different characters can be arranged to make a pattern that is n characters long when

(a) each character can be repeated

$n \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \dots \dots \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$

You cannot write all the missing terms of the expression, so just write the first 3 and the last 3.

(b) each character can be used once only.

$n \times \quad \times \quad \times \quad \times \dots \dots \times \quad \times \quad \times \quad =$

Worksheet C: Arrangements investigation teacher notes



The aim of this investigation is to develop logical thinking and allow learners to understand the basis of the product rule for counting in the context of arranging objects.

This is built upon in the Lesson slides *Arrangements*. It addresses the common error of adding values when values should be multiplied and is, therefore, worth spending the extra time on as this underpins all the work that follows.

Make sets of 10 cards with the letters ABCDEFGHIJ on them as some learners will benefit from being able to physically move the cards about.

Task 1 is more specific and is to be used for learners who need some support.

Task 2 is more general and is to be used for learners who are more able.

The two tasks have some questions in common.

Learners should be split into small groups by ability and given Task 1 or Task 2 as appropriate.

Q1 Initially, learners concentrate on arranging a whole set of different objects with no repetition.

Q2 Learners think about arranging a small set of different objects with and without repetition.

Q3 Learners think about arranging a large set of different objects with and without repetition in Task 1. In Task 2 learners find algebraic expressions for arranging n objects.

During the task, move around the class and observe. If learners need to be prompted, some possible questions to ask are:

- Can you connect the number of arrangements to the number of letters?
- How many different starting letters are there?
- How many numbers will there be in your list?
- How can you write that in a neater way?
- Can you explain why you did that?
- What is your strategy for getting started here?
- How did you work out your answer?
- Do you think this rule will always work?
- What difference has the option to repeat a character made to your answer?
- Is that expression easy to write down? Why not?

When learners have had enough time to complete the tasks (about 20 mins):

- **either** have a class discussion to discuss the findings (and then possibly use the *Arrangements investigation summary* PowerPoint to bring all the results together)
- **or** use the *Arrangements investigation summary* as part of a class Question and Answer session to bring all the results together.

The summary is important as all learners need to discuss all the answers to both tasks, and should last about 10 minutes.

Worksheet D: Independent study lesson 1



Arrangements

- 1 Find how many different numbers can be made by arranging all 8 digits of the number 24 571 983 if

- (i) there are no restrictions
- (ii) the number made is an even number.

- 2 Find the number of different ways the 6 letters of the word PLANED can be arranged if:

- (i) the first letter is A and the last letter is E
- (ii) the letters L, A and N are next to each other.

- 3 Here are 7 cards.



These cards are placed in a line to make a 7-digit number.
How many of these 7-digit numbers:

- (i) have the even digits together
- (ii) have both the first number and the last number even.

- 4 Find the number of arrangements that can be made using all 10 letters of the word WHITEBOARD if:

- (i) there are no restrictions
- (ii) there are exactly 6 letters between the T and the R.

- 5 A town hall has seats for 20 people, consisting of 4 rows with 5 seats in each row. When Anya, Bob, Kim, Tomas and Lee arrive at the town hall, all the seats are empty.

- (i) How many possible arrangements are there of seating Anya, Bob, Kim, Tomas and Lee if there are no restrictions?
- (ii) How many possible arrangements are there of seating Anya, Bob, Kim, Tomas and Lee if Bob, Kim and Tomas sit together in the back row and the other two sit together in one of the other rows?



Worksheet E: You are the teacher: mark this work

Find the errors in the work presented and explain what is wrong. Decide the number of marks to award and make 3 comments praising the work and set 1 target for improvement.

15 pop groups apply to star in a show. Sasha chooses 5 of these pop groups to appear on the show.

- (i) Sasha makes a list to show the order in which they will sing on the show. How many different lists can she make? [2]

$$(i) \quad 360\,000$$

- (ii) Of the original 15 pop groups, 10 have a drummer and 5 have no drummer. Find the number of lists in which
the first group has no drummer,
the second group has a drummer,
the third group has no drummer,
the fourth group has a drummer and
the fifth group has no drummer. [2]

$$(ii) \quad \frac{10}{ND} \times \frac{5}{D} \times \frac{9}{ND} \times \frac{4}{D} \times \frac{8}{ND} = 14\,400$$

Mark scheme

- | | | |
|------|---|--------|
| (i) | $15 \times 14 \times 13 \times 12 \times 11$ | 1 mark |
| | Correct answer 360 360 | 1 mark |
| (ii) | Multiplies 5 relevant numbers e.g. $5 \times 10 \times 4 \times 9 \times 3$ | 1 mark |
| | Correct answer 5400 | 1 mark |

Worksheet F: More arrangements – practice questions



Practice questions 1

In the English language, the letters A, E, I, O, U are vowels.
All other letters are consonants.

- 1 Find the number of ways all 8 letters of the word AVOCADOS can be arranged if:
 - (i) there are no restrictions
 - (ii) the first letter is O and the last letter is O
 - (iii) all the Os are together.

- 2 Find the number of different ways the 10 letters of the word BREADBOARD can be arranged in a line if:
 - (i) there are no restrictions
 - (ii) all the vowels are next to each other.

- 3 Find the number of ways the 11 letters of the word MISSISSIPPI can be arranged
 - (i) if all the letters I are together
 - (ii) if M is at one end and a P is at the other end.

Extension questions

- 4 Find the number of ways in which all 9 letters of the word AMARANTHS can be arranged in each of the following cases:
 - (i) the N, T and H are together
 - (ii) the letter M is in the middle and each end is occupied by one of the other consonants.

- 5 Five fair tetrahedral dice, marked 1, 2, 3, 4 are rolled.
They are then placed in a line.

Find the number of possible arrangements where the difference between the score on the first dice and the last dice in the line is 2.



Worksheet F: More arrangements – practice questions continued

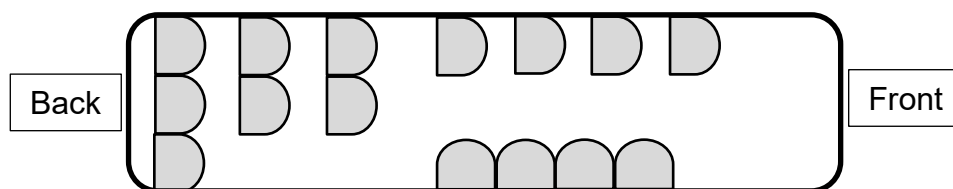


Practice questions 2

- 1 Find the number of different ways that 6 different black cows and 4 different white cows can stand in a line if:
 - (i) all 6 black cows are next to each other
 - (ii) no white cow is next to another white cow.
- 2 A quiz team consists of 1 girl and 5 boys. In how many different ways can the team members be arranged in a line if the girl is not at either end?
- 3 Find the number of different ways of arranging all nine letters of the word ALLOWABLE if
 - (i) there are no restrictions
 - (ii) no vowel is next to another vowel.
- 4 A cycle rack has space for 10 cycles, arranged in a line. On one day, there are 3 different cycles stored in randomly chosen positions and 7 empty spaces.
 - (i) Find the number of possible arrangements of the 3 cycles in the rack.
 - (ii) Find the number of arrangements where the 3 cycles are not next to each other.

Extension question

- 5 The diagram shows the 15 passenger seats in a bus.



Eight people get on the bus, Mr and Mrs Brown, Mr and Mrs Green, 3 students and a policeman.

- (i) How many possible seating arrangements are there for the 8 people?
- (ii) The 3 students sit in the back row. The policeman sits in a single seat. Mr and Mrs Brown sit in the two seats directly in front of the students. Mr and Mrs Green sit next to each other. How many possible seating arrangements are there?

Worksheet G: Independent study lesson 2



More arrangements

- 1** Find the number of ways all 9 letters of the word FOOTSTOOL can be arranged if:
 - (i) there are no restrictions
 - (ii) the first letter is T and the last letter is S
 - (iii) all the Os are together.

- 2** The 11 letters of the word ACCELERATED are arranged in a line.
 - (i) Find the number of different arrangements if there are no restrictions.
 - (ii) Find the number of different arrangements which start and finish with the letter C.
 - (iii) Find the number of different arrangements which do not have all 5 vowels (A, E, E, A, E) next to each other.

- 3** A team of 5 men and 3 women stand in a line. Find the number of ways this can be done if none of the women stand next to each other.

- 4** Numbers are made using some or all of the digits 1, 2, 3, 4. No digit is used more than once.
 - (i) Show that there are 12 different, even, 3-digit numbers.
 - (ii) Find the total number of even numbers that can be made.

- 5** In the English language, the letters A, E, I, O, U are vowels. All other letters are consonants.

Find the number of ways all 10 letters of the word ADVERTISER can be arranged so that

 - (i) the vowels are together and the consonants are together
 - (ii) the Es are not next to each other.



Worksheet H: Connections

Connect each question to its answer.

How many different 4-digit even numbers can be formed using 4 of the 6 digits
1, 2, 3, 4, 5, 6
if no digit is used more than once?

$$5 \times 4 \times 3 = 60$$

How many different 4-digit numbers can be formed using 4 of the 6 digits
3, 4, 5, 6, 7, 9
if no digit is used more than once?

$$8 \times 7 \times 6 \times 5 \times 4 = 6720$$

Five empty chairs are arranged in a row.
3 boys are to be seated in this row.
How many possible seating arrangements are there?

$$6 \times 5 \times 4 \times 3 = 360$$

A 1st prize, 2nd prize, 3rd prize and 4th prize are to be awarded to 4 different members of a group of 8 people. In how many ways can this be done?

$$5 \times 4 \times 3 \times 3 = 180$$

A woman has 8 different trees.
5 of these trees are to be planted in a line.
How many possible arrangements are there?

$$8 \times 7 \times 6 \times 5 = 1680$$

Worksheet I: Permutations and combinations sorting activity



In your group:

- Cut out the cards.
- Sort the cards into two piles: **Permutations and Combinations**
- At the end of the activity, you will be asked to share your findings and compile a master list with the rest of the class.

The number of 5-character passwords that can be made from a list of 10 different characters.

The number of ways a captain and vice-captain can be chosen from a squad of 12 sportsmen.

The number of hands of 7 randomly-chosen cards that can be dealt to one player in a card game.

The number of ways a committee of 3 senior and 3 junior students can be selected from a group of 10 senior and 8 junior students.

The number of teams of 3 people that can be selected from a group of 12 people.

The number of ways 5 out of 7 paintings can be arranged in a line on a wall.

The number of ways 2 boys and 1 girl can be chosen from a group of 5 boys and 6 girls.

5 comedians are chosen from a list of 12 to star in a show. The names of the comedians are written on a list. How many different possible lists are there?

The number of ways 3 of the 8 runners in a race can finish 1st, 2nd and 3rd.

A selection of 3 letters from the word PEARL is made. How many different selections are possible?

Worksheet J: Permutations and combinations practice questions



- 1 Find the number of different selections of 5 letters from the 10 letters of the word BLACKSMITH.
- 2 A college minibus has 12 passenger seats. How many possible seating arrangements are there for 8 passengers?
- 3 A class of 25 students has 17 boys and 8 girls. The teacher is to select 2 students for a prize. Find the number of selections which have at least one boy.
- 4 The head girl and deputy-head girl of a school are to be chosen from a list of 5 girls and the head boy and deputy-head boy are to be chosen from a list of 6 boys. In how many different ways can this be done?
- 5 In an exam, candidates must answer 8 questions. They must select at least 3 from the 5 questions in section A and at least 4 from 7 questions in section B. Find the number of selections a candidate can make.
- 6 3 girls and 4 boys wish to sit in a row of 10 seats.



The 3 girls sit in the seats on the left and the 4 boys sit on the right.
How many possible seating arrangements are there?

- 7 Find how many numbers between 3000 and 4000 can be formed using the digits 1, 2, 3, 4, 5, 6 if
 - (a) digits are allowed to be repeated
 - (b) digits must not be repeated.
- 8 Anil and Banhi have 33 new music tracks:
 - 18 are rock
 - 9 are pop
 - 6 are metal.
 - (a) From these tracks, Anil makes a selection of 4 rock, 4 pop and 2 metal. How many different possible selections can he make?
 - (b) Banhi makes a playlist using 8 of the 18 rock tracks. How many different possible playlists can she make?
- 9 Find the number of ways of choosing a quiz team of 5 players from 6 men and 8 women if:
 - (a) there are more women than men in the team
 - (b) 3 of the men are brothers and are either all **in** the team or all **not** in the team.

Worksheet J: Permutations and combinations practice questions continued



- 10** Terry has 10 books, including one book of poetry. He chooses 3 of these books to take with him on a trip.
- (a) In how many ways can he choose 3 of his books?
- (b) How many of these choices will not include the book of poetry?
- 11** A committee of 3 people, a chairperson, secretary and treasurer, are to be chosen from the 30 members of a gardening club.
- Find the number of ways this can be done if:
- (a) there are no restrictions
- (b) one of the members, Abu, refuses to be on the committee if another member, Nikki, is on the committee.
- 12** An art gallery is planning to display 12 paintings in a line along a wall.
- 2 paintings are by Adam (A).
 - 2 paintings are by Basu (B).
 - 3 paintings are by Chen (C).
 - 4 paintings are by De Witt (D).
 - 1 painting is by Easterlefa (E).
- (a) Find the number of possible arrangements of these 12 paintings.
- (b) 4 of the 12 paintings are to be sold at an auction. Exactly one of the paintings must be by Basu and exactly one must be by Chen.
Find the number of ways in which these 4 paintings can be selected.
- 13** Three letters from the 9 letters of the word FOOTSTOOL are selected.
- (a) Find the number of selections which contain no Os and exactly one T.
- (b) Find the number of selections which contain no Os.
- 14** Four letters from the 11 letters of the word ACCELERATED are chosen.
- Find the number of different selections which contain no Cs and no As and at least 2 Es.
- 15** Four letters are selected from the 10 letters of the word ADVERTISER.
- Find the number of different selections if the four letters must contain the same number of Es and Rs with at least one of each.

Worksheet A: Answers



Q1 In this question, each letter can be used once only.

(a) Complete the table to show the number of arrangements of the letters given.

Letters	Possible arrangements	Number of letters	Number of arrangements
D	D	1	1
CD	CD, DC	2	2
BCD	BCD, BDC CBD, CDB DBC, DCB	3	6

(b) By listing only the arrangements that start with A and using the results for 3 letters, find the number of possible arrangements of these 4 letters.

ABCD	ABCD, ABDC ACBD, ACDB ADBC, ADCB	4	24
------	---	---	----

(c) Show how you found your answer.

$$4 \times 6 = 24$$

(d) Complete each of these statements with a number.

The number of arrangements with 4 letters is4..... times the number of arrangements with 3 letters.

The number of arrangements with 5 letters is5..... times the number of arrangements with 4 letters

(e) Complete this statement.

There are3628800..... arrangements of the 10 letters ABCDEFGHIJ when each letter can be used once only.

Worksheet A: Answers continued



- (f) Show how you found your answer.

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

Q2 In how many different ways can the eight characters

6 7 8 9) (* &

be arranged to make an 8-character pattern when

- (a) each character can be used once only

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$$

- (b) each character can be repeated?

$$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^8 = 16777216$$

Q3 You do not have to evaluate your expressions in this question.

Halima is arranging 50 different characters to make a 50-character pattern.

- (a) Write an expression for the number of ways Halima can do this when each character can be repeated. Simplify your answer.

$$50 \times 50 \times 50 \times 50 \times \dots \times 50 \times 50 \times 50 = 50^{50}$$

You do not have to write all the missing terms of the expression, just the first 3 and the last 3.

- (b) Write an expression for the number of ways Halima can do this when each character can be used once only.

$$50 \times 49 \times 48 \times 47 \times \dots \times 3 \times 2 \times 1 = 50!$$

Worksheet B: Answers



Q1 In this question, each letter can be used once only.

(a) Complete the table to show the number of arrangements of the letters given.

Letters	Possible Arrangements	Number of letters	Number of arrangements
A	A	1	1
AB	AB, BA	2	2
ABC	ABC, ACB BAC, BCA CAB, CBA	3	6

(b) By listing only the arrangements that start with A and using the results for 3 letters, find the number of possible arrangements of these 4 letters.

ABCD	ABCD, ABDC ACBD, ACDB ADBC, ADCB	4	24
------	---	---	----

(c) Show how you found your answer.

$$4 \times 6 = 24$$

(d) Complete each of these statements with numbers.

For example

The number of arrangements with 4 letters is⁴..... times the number of arrangements with³..... letters.

For example

The number of arrangements with 5 letters is⁵..... times the number of arrangements with⁴..... letters

(e) Complete this statement.

There are³⁶²⁸⁸⁰⁰..... arrangements of the 10 letters ABCDEFGHIJ when each letter is used exactly once.

Worksheet B: Answers continued



(f) Show how you found your answer.

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

Q2 In how many different ways can the eight characters

6 7 8 9) (* &

be arranged to make an 8-character pattern when

(a) each character can be used once only

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$$

(b) each character can be repeated?

$$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^8 = 16777216$$

Q3 Complete the following expressions for the number of different ways n different characters can be arranged to make a pattern that is n characters long when

(a) each character can be repeated

$$n \times n \times n \times n \times \dots \times n \times n \times n = n^n$$

You cannot write all the missing terms of the expression, so just write the first 3 and the last 3.

(b) each character can be used once only.

$$n \times (n-1) \times (n-2) \times (n-3) \times \dots \times 3 \times 2 \times 1 = n!$$



Worksheet D: Answers

Independent study Lesson 1 Arrangements, Model answers

1 (i) $8! = 40320$

(ii) Even digits are 2, 4, 8

$$\underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{3}$$

7 digits remaining when one even digit is used for the last space

3 even digits so 3 ways to fill this space

Answer: $7! \times 3 = 15120$

2 (i) $\underline{A} \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \underline{E}$

Answer: $4! = 24$

The first and last spaces are fixed and so only 4 spaces remain to be filled

(ii) $\boxed{\underline{L} \underline{A} \underline{N}} \times \underline{3} \times \underline{2} \times \underline{1}$

Answer: $3! \times 4! = 144$

3 Even digits are 2 and 4.

(i) $\boxed{\underline{2} \underline{4}} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad}$

$$\underline{2}! \times 6 \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

Answer: $2! \times 6! = 1440$

When you are using digits to make numbers take care not to confuse the digits from the question with the number of ways of filling each space!

(ii) $\underline{2} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{4}$ or

$$\underline{4} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{2}$$

$$\underline{2} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{1}$$

Answer: $2 \times 5! = 240$

2 ways to fill the first space and one way to fill the last space (as there is only one even number remaining).

Worksheet D: Answers continued



4 (i) $10! = 3\,628\,800$

(ii) T _____ R _____ 2 times or
 _____ T _____ R _____ 2 times or
 _____ T _____ R _____ 2 times

R and T can
change
spaces.

8 spaces left
to fill with the
remaining 8
letters

Answer: $8! \times 2 \times 3 = 241\,920$

5 (i) $20 \times 19 \times 18 \times 17 \times 16 = 1\,860\,480$

(ii) Back row only:

B K T _____ or
 _____ B K T _____ or
 _____ B K T _____

So $3! \times 3$ ways of B, K and T sitting in the back row.

One other row:

A L _____ or
 _____ A L _____ or
 _____ A L _____ or
 _____ A L _____

So $2! \times 4$ ways of A and L sitting in one of the other 3 rows.

Answer: $3! \times 3 \times 2! \times 4 \times 3 = 432$

BKT in
back
row

AL in
one row

3 rows where
A and L can sit



Worksheet F: Answers

Practice questions 1 – Answers

1 AVOCADOS

(i) Repeated $2 \times A$ and $2 \times O$; $\frac{8!}{2! \times 2!} = 10080$

(ii) O _ _ _ _ _ O with repeated $2 \times A$; $\frac{6!}{2!} = 360$

(iii) O O _ _ _ _ _ with repeated $2 \times A$; $\frac{7!}{2!} = 2520$

2 BREADBOARD

(i) Repeated $2 \times B$, $2 \times R$, $2 \times A$ and $2 \times D$; $\frac{10!}{2! \times 2! \times 2! \times 2!} = 226800$

(ii) Vowels: E, A, O, A

E A O A _ _ _ _ _

$$\frac{4!}{2!} \times 7 \times \frac{6!}{2! \times 2! \times 2!} = 7560$$

3 MISSISSIPPI

(i) Repeated $4 \times I$, $4 \times S$ and $2 \times P$;

I I I I _ _ _ _ _

$$\frac{8!}{4! \times 2!} = 840$$

(ii) M _ _ _ _ _ P or

P _ _ _ _ _ M

Only repeats are now $4 \times I$, $4 \times S$; $2 \times \frac{9!}{4! \times 4!} = 1260$

4 AMARANTHS

(i) Repeated $3 \times A$

N T H _ _ _ _ _

$$3! \times 7 \times \frac{6!}{3!} = 5040$$

(ii) _ _ _ _ _ M _ _ _ _ ; other consonants are R, N, T, H, S

5 ways to fill first space and 4 ways to fill last space; repeated $3 \times A$;

$$5 \times \frac{6!}{3!} \times 4 = 2400$$

Worksheet F: Answers continued



- 5
- | | | | | | |
|---|---|---|---|------|--|
| 1 | — | — | — | 3 or | Each of the middle 3 values could be 1, 2, 3 or 4;
$- 4^3 \times 4 = 256$ |
| 3 | — | — | — | 1 or | |
| 4 | — | — | — | 2 or | |
| 2 | — | — | — | 4 | |

Practice Questions 2 – Answers

- 1 (i) B₁ B₂ B₃ B₄ B₅ B₆ — — — —

$$6! \times 5! = 86400$$

- (ii) $\uparrow B_1 \uparrow B_2 \uparrow B_3 \uparrow B_4 \uparrow B_5 \uparrow B_6 \uparrow$ $6! \times 7 \times 6 \times 5 \times 4 = 604800$

- 2 $B \uparrow B \uparrow B \uparrow B \uparrow B$ $5! \times 4 = 480$

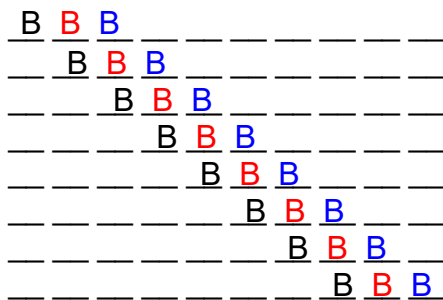
3 ALLOWABLE

- (i) Repeated $2 \times A$ and $3 \times L$; $\frac{9!}{2! \times 3!} = 30240$

- (ii) 5 consonants with repeated $3 \times L$; 4 vowels with repeated $2 \times A$

$$\uparrow C \uparrow C \uparrow C \uparrow C \uparrow C \uparrow \quad \frac{5!}{3!} \times \frac{6 \times 5 \times 4 \times 3}{2!} = 3600$$

- 4 (i) $10 \times 9 \times 8 = 720$
- (ii) 720 – the number that **are** next to each other



3! ways of arranging the 3 cycles
8 possible positions for the 3 cycles

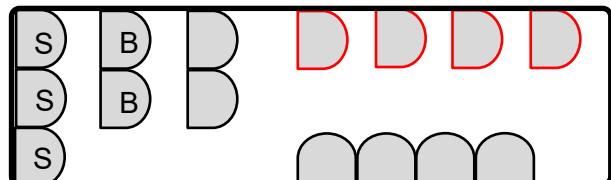
Answer: $720 - 3! \times 8 = 672$

Extension question

- 5 (i) $15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 = 259459200$

- (ii) $3!$ for the students
 $\times 4$ places for the policeman
 $\times 2!$ for Mr and Mrs Brown
 $\times 4 \times 2!$ for Mr and Mrs Green

$$3! \times 4 \times 2! \times 4 \times 2! = 384$$





Worksheet G: Answers

1 FOOTSTOOL

(i) $\frac{9!}{4! \times 2!} = 7560$

4 × O; 1 × F
2 × T; 1 × S
1 × L

(ii) T _ _ _ _ _ S

$$\underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

Answer: $\frac{7!}{4!} = 210$

Only the Os are repeated
as there is now only one
T to place.

(iii) O O O O _ _ _ _

$$\underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

Answer: $\frac{6!}{2!} = 360$

The Ts are repeated

2 ACCELERATED

(i) $\frac{11!}{2! \times 2! \times 3!} = 1663200$

2 × A; 1 × L
2 × C; 1 × R
3 × E; 1 × T
1 × D

(ii) C _ _ _ _ _ C

$$\underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

Answer: $\frac{9!}{2! \times 3!} = 30240$

The As and Es are
repeated

(iii) Number of different arrangements – Number **with** all 5 vowels together

A A E E E _ _ _ _

The As and
Es are
repeated

$$\frac{5!}{2! \times 3!} \times 7 \times \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2!}$$

The Cs are
repeated

Number **with** all 5 vowels together = 25200

Number that **do not** have all 5 vowels together

$$= 1663200 - 25200 \\ = 1638000$$

Worksheet G: Answers continued



3



Each arrow represents a place where a woman can stand

- 5! ways for the men to stand
- × 6 spaces for the 1st woman
- × 5 spaces for the 2nd woman
- × 4 spaces for the 3rd woman

One space is taken by the 1st woman and so the 2nd woman can only stand in one of 5 spaces and so on

Answer: $5! \times 6 \times 5 \times 4 = 14400$

4 1 2 3 4

- (i) If even, ends with 2 or 4.

____ 2 or ____ 4

$3 \times 2 = 6$ numbers end with 2

$3 \times 2 = 6$ numbers end with 4

So $6 + 6 = 12$ three-digit numbers are even

- (ii) 1-digit and even = 2 numbers

2-digit and even:

____ 2 or ____ 4

3 numbers end with 2

3 numbers end with 4

So $3 + 3 = 6$ two-digit numbers are even

3-digit and even = 12 numbers (from (i))

4-digit and even:

____ 2 or ____ 4

$3!$ numbers end with 2

$3!$ numbers end with 4

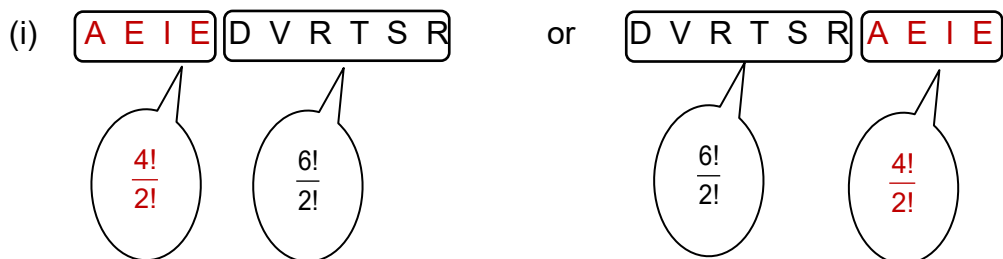
So $2 \times 3! = 12$ four-digit numbers are even

Number of digits and even	Number of numbers
1	2
2	6
3	12
4	12
Total	32



Worksheet G: Answers

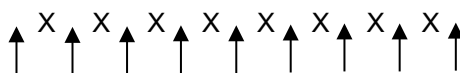
5 Vowels: A E I E
Consonants: D V R T S R



Answer: $2 \times \frac{4!}{2!} \times \frac{6!}{2!} = 8640$

(ii) Let X represent the letters that are 'not E'. There are 8 of them.

The Rs are repeated



$\frac{8!}{2!}$

different ways to arrange the letters that are not E

The Es are repeated

$\times \frac{9 \times 8}{2!}$

ways to place the 1st E and the 2nd E

Answer: $\frac{8!}{2!} \times \frac{9 \times 8}{2!} = 725760$

Worksheet H: Answers



Connect each question to its answer.

How many different 4-digit even numbers can be formed using 4 of the 6 digits

1, 2, 3, 4, 5, 6

if no digit is used more than once?

$$5 \times 4 \times 3 = 60$$

How many different 4-digit numbers can be formed using 4 of the 6 digits

3, 4, 5, 6, 7, 9

if no digit is used more than once?

$$8 \times 7 \times 6 \times 5 \times 4 = 6720$$

Five empty chairs are arranged in a row.

3 boys are to be seated in this row.

How many possible seating arrangements are there?

$$6 \times 5 \times 4 \times 3 = 360$$

A 1st prize, 2nd prize, 3rd prize and 4th prize are to be awarded to 4 different members of a group of 8 people. In how many ways can this be done?

$$5 \times 4 \times 3 \times 3 = 180$$

A woman has 8 different trees. 5 of these trees are to be planted in a line. How many possible arrangements are there?

$$8 \times 7 \times 6 \times 5 = 1680$$



Worksheet I: Answers

The number of 5-character passwords that can be made from a list of 10 different characters.

Permutation

The number of ways a captain and vice-captain can be chosen from a squad of 12 sportsmen.

Permutation

The number of hands of 7 randomly-chosen cards that can be dealt to one player in a card game.

Combination

The number of different **selections** of a committee of 3 senior and 3 junior students from a group of 10 senior and 8 junior students.

Combination

The number of **selections** of a team of 3 people from a group of 12 people.

Combination

The number of **arrangements** of 5 out of 7 paintings in a line on a wall.

Permutation

The number of ways 2 boys and 1 girl can be chosen from a group of 5 boys and 6 girls.

Combination

5 comedians are chosen from a list of 12 to star in a show. The names of the comedians are written on a list. How many different possible lists are there?

Permutation

The number of ways 3 of the 8 runners in a race can finish 1st, 2nd and 3rd.

Permutation

A selection of 3 letters from the word PEARL is made. How many different **selections** are possible?

Combination

Worksheet J: Answers



Permutations and combinations practice questions

- 1 Selections so nC_r .

$${}^{10}C_5 = 252$$

- 2 Arrangements so nP_r .

$${}^{12}P_8 = 19\,958\,400$$

- 3 Selections so nC_r .

1 boy 1 girl or 2 boys

$${}^{17}C_1 \times {}^8C_1 + {}^{17}C_2 = 272$$

- 4 Arrangements so nP_r .

$${}^5P_2 \times {}^6P_2 = 600$$

- 5 Selections so nC_r .

A	B
3	5
4	4

$${}^5C_3 \times {}^7C_5 = 210$$

$${}^5C_4 \times {}^7C_4 = \frac{175+}{385}$$

- 6 Arrangements so nP_r .

$${}^5P_3 \times {}^5P_4 = 7200$$

- 7 (a) Must start with 3, but the other digits could be any of the other 6

3 _ _ _ _

$$1 \times 6^3 = 216$$

- (b)

3 _ _ _ _

$$1 \times {}^5P_3 = 60 \text{ or } 1 \times 5 \times 4 \times 3 = 60$$

- 8 (a) Selections so nC_r .

$${}^{18}C_4 \times {}^9C_4 \times {}^6C_2 = 5783400$$

- (b) Arrangements so nP_r .

$${}^{18}P_8 = 1\,764\,322\,560$$



Worksheet J: Answers continued

- 9 (a) Selections so nC_r .

M	W
0	5
1	4
2	3

$$\begin{aligned}
 {}^8C_5 &= 56 \\
 {}^8C_4 \times {}^6C_1 &= 420 \\
 {}^8C_3 \times {}^6C_2 &= \underline{840} +
 \end{aligned}$$

- (b) Selections so nC_r . 1316

Case1: 3 brothers in team;
2 players left to pick; 11 people to choose from
 ${}^{11}C_2 = 55$

Case 2: 3 brothers not in team;
5 players to pick; 11 people to choose from
 ${}^{11}C_5 = 462$

$$\text{Total} = 55 + 462 = 517$$

- 10 Selections so nC_r .

(a) ${}^{10}C_3 = 120$

- (b) 120 – the number that do include the book of poetry

P, __, __

2 books left to choose; 9 books left to choose from

$$120 - {}^9C_2 = 120 - 36 = 84$$

There are alternative approaches:

not taking the book of poetry so 9 books left

$${}^9C_3 = 84$$

- 11 Arrangements so nP_r .

(a) ${}^{30}P_3 = 24360$

- (b) 24360 – the number of committees with A and N

$$24360 - {}^3P_2 \times {}^{28}P_1 = 24360 - 168 = 24192$$

There are alternative approaches:

Number of ways with neither = ${}^{28}P_3 = 19656$, Number of ways with A = ${}^{28}P_2 \times 3 = 2268$, Number of ways with N = ${}^{28}P_2 \times 3 = 2268$

- 12 (a) $12! = 479\,001\,600$

- (b) Selections so nC_r .

$${}^2C_1 \times {}^3C_1 \times {}^7C_2 = 126$$

$$2 \times (A)$$

$$2 \times (B)$$

$$3 \times (C)$$

$$4 \times (D)$$

$$1 \times (E)$$

Worksheet J: Answers continued



13 Selections so nC_r .

(a) T, ____, ____

$${}^3C_2 = 3 \text{ selections}$$

(b) Case 1: No Os & No Ts

$$F, S, L = 1 \text{ selection}$$

$$\text{Case 2: No Os \& 1 T} = 3 \text{ selections (part(a))}$$

Case 3: No Os & 2Ts

T, T, ____

$${}^3C_1 = 3 \text{ selections}$$

$$\text{Total} = 3 + 3 + 1 = 7 \text{ selections}$$

4 × O
2 × T
1 × F
1 × S
1 × L

14 Selections so nC_r .

At least 2 Es:

Case 1: 2Es

E, E, ____, ____

$${}^4C_2 = 6 \text{ selections}$$

Case 2: 3Es

E, E, E, ____

$${}^4C_1 = 4 \text{ selections}$$

$$\text{Total} = 6 + 4 = 10 \text{ selections}$$

2 × A
2 × C
3 × E
1 × L
1 × R
1 × T
1 × D

15 Selections so nC_r .

Case 1: 1E and 1R

E, R, ____, ____

$${}^6C_2 = 15 \text{ selections}$$

Case 2: 2Es and 2Rs

$$E, E, R, R = 1 \text{ selection}$$

$$\text{Total} = 15 + 1 = 16 \text{ selections}$$

1 × A
1 × D
1 × V
2 × E
2 × R
1 × T
1 × I
1 × S

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