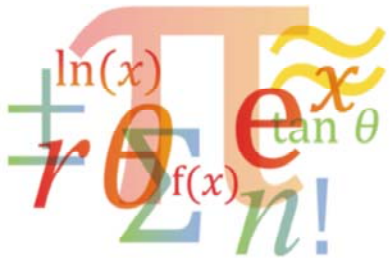
Cambridge Assessment International Education

Teacher Guide

How to … solve problems



Version 1

Cambridge Pathway

Problem-solving is one of three Key Concepts outlined in the new 9709 syllabus and it plays an important role in all topics of AS & A Level Mathematics. Learners need to be confident with all the methods they learn on the course and also to be able to apply the methods to solving problems in various different contexts. This guide will help you to support your learners in developing their problem-solving skills.

Approaching a problem

- Read the question carefully
- Identify the key facts in the question
- Do these suggest which mathematical topics you need to consider?

Some learners find it helpful to highlight or underline key terms or pieces of information in the question to help them focus on what it is about.

Analysing the problem

- Would a diagram or graph be helpful?
- If the question already has a diagram or graph, is there anything you can add to it?
- What do you notice?
- What would you like to know?
- Can you see any methods you could use?

Stopping to think is always a good idea before writing anything. Some learners are eager to start doing something mathematical but, if they jump in too quickly, the methods they use are not always relevant to the question.

Solving the problem (initial exploration)

- Can it be simplified?
- Can part of it be solved straight away?
- If it's an algebraic problem, can it be done with numbers first?

Any of these tactics can help learners to understand the mathematical structure of the problem more fully.
Being systematic and writing clearly are important throughout the process: learners need to see what they've done in order to decide what to do next.

Solving the problem (strategy)

- Pause and think before trying to solve the actual problem
- Work out a strategy for solving it
- Implement the strategy
- Would another diagram be helpful?

Perhaps the original diagram is too small to annotate the important part, so a larger version would be helpful. Or perhaps new information has come out of their initial exploration of the problem and so they can draw a different diagram or graph.

Collaborative problem-solving

Learners will benefit from solving problems in pairs or groups from time to time. This experience will:

- help them to communicate mathematically
- help them to justify their strategy and decisions
- encourage them to persevere
- allow them to learn from their peers
- help them to develop flexibility in solving problems

Sometimes introducing a competitive element within the classroom can be exciting for learners e.g. solving problems against the clock, team competitions, relay races (solving problems then submitting the solution to you in order to obtain the next problem).

Presenting the solution

- Learners need to decide how to present their solution logically in order to address the question fully
- The solution must relate to the context of the original problem
- It must be written clearly and fully, addressing all aspects of the problem
- It's a good idea to check that the solution has taken into account all the information given in the problem

Some problems do not have any structure so learners will need to decide how to create structure.
When solving a problem, learners need to make it clear how they have applied their mathematical methods in this particular context, not just using them in the abstract.
They may find it helpful to practise writing suitable sentences in English, complementing their use of mathematical notation (language), whenever they solve a problem.
It is important to check the solution against the original problem, making sure nothing has been ignored e.g. a restricted domain
There are more detailed suggestions in the guide 'How to … write mathematically'

Incorporating problem-solving into your course

- Introduce a problem then ask learners what they need to know to solve it. This creates a need for methods that you can teach them, returning to the problem later to apply the methods
- Teach relevant methods then introduce a problem to consolidate or assess your learners' understanding of the methods
- Emphasise to learners that problems in this course will make use of the methods they know. They should practise thinking about which methods might be appropriate
- If learners are stuck, you can ask a question to prompt them

There is another guide in this series 'How to … get unstuck' which gives more detailed help on what learners can do to help themselves when they are stuck.

Helping learners to get better at solving problems

You can model to learners the thought processes they will need to solve problems on their own

- Think out loud while you're solving problems on the board.
- In this way you demonstrate how you select methods to use
- You can also model getting stuck and thinking what to do next
- Explain how you make connections between topics
- Include a problem for the whole class to discuss

Learners need to internalise the questions they can ask themselves when faced with a problem. With practice and experience, they will learn to develop a 'feeling' about a problem they meet so that they can make good decisions about which methods to apply to it.
Solving problems can provide motivation for learners to practise the methods and to be confident with them.
Learners may need encouragement to see that topics are interconnected, not isolated parts of the mathematics syllabus. This will help them to draw on disparate areas of knowledge when solving problems later.
When you manage a whole class discussion, encourage learners to contribute ideas and others to respond to them. The aim is for learners to progress as a group towards the solution rather than expecting you to give them the definitive answer. It is better if the learners do the talking!

Further reading

- Mason, Burton & Stacey 'Thinking Mathematically' (2nd edition 2010), published by Prentice-Hall. Talks through the thinking process that is part of problem-solving. Contains some useful questions and ideas
- Polya 'How to solve it' (2nd edition 2015), published by Princeton University Press. New edition of a classic book on problem-solving.
- Problem solving guide (MEI) – http://mei.org.uk/problem-solving-guide. Contains many useful ideas and mathematical examples. Written for UK mathematics syllabuses but applicable to the international AS & A Level 9709 too.