Cambridge Assessment International Education

Teaching Pack

4.1 Forces and Equilibrium

Cambridge International AS & A Level Mathematics 9709

Version 1

Cambridge Pathway

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Introduction

This pack will help you to develop your learners' skills in mathematical thinking and mathematical communication, which are essential for success at AS & A Level and in further education.

Mathematical thinking and communication will be developed by focusing on:

1. Conceptual understanding – the 'why' behind the 'what'
2. Strategic competence – forming and solving problems
3. Adaptive reasoning – explanations, justifications and deductive reasoning

Throughout all activities, the learners will also develop:

- procedural fluency – know when, how and which rules to use
- positive disposition – believe maths can be learned, applied and is useful
- their skills in writing mathematically – writing working & proofs

These link to the course Assessment Objectives (AOs) which you can find in detail in the syllabus:

A01 Knowledge and understanding

A02 Application and communication

Each *Teaching Pack* contains one or more lesson plans and associated resources, complete with a section of preparation and reflection.

Each lesson is designed to be an hour long but you should adjust the timings to suit the lesson length available to you and the needs of your learners.

Important note
Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other topics.

This content is designed to give you and your learners the chance to explore a more active way of engaging with mathematics that encourages independent thinking and a deeper conceptual understanding. It is not intended as specific practice for the examination papers.

The *Teaching Packs* are designed to provide you with some example lessons of how you might deliver content. You should adapt them as appropriate for your learners and your centre. A single pack will only contain at most four lessons, it will **not** cover a whole topic. You should use the lesson plans and advice provided in this pack to help you plan the remaining lessons of the topic yourself.

Lesson preparation

This *Teaching Pack* will cover the following syllabus content:

Candidate should be able to:	Notes and examples
• identify the forces acting in a given situation	e.g. by drawing a force diagram .
• understand the vector nature of force, and find and use components and resultants	Calculations are always required, not approximate solutions by scale drawing
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero	Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions
• use Newton's third law	e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle

The remaining *three* bullet points for topic 4.1 Forces and equilibrium are not covered in this *Teaching Pack* (see the syllabus for detail). You will need to write your own lesson plans for these items.

Candidate should be able to:	Notes and examples
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leq \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.

Dependencies

For all lesson plans in this *Teaching Pack*, knowledge from the following 9709 syllabus content is required:

Candidate should be able to:	Notes and examples
• use the identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$	e.g. in proving identities, simplifying expressions and solving equations.

Prior knowledge and skills

For all lessons, it is assumed that learners have already completed Cambridge IGCSE™ Mathematics 0580, or a course at an equivalent level. See the syllabus for more details of the expected prior knowledge for taking Cambridge International AS & A Level Mathematics 9709.

When planning any lesson, make a habit of always asking yourself the following questions about your learners' prior knowledge and skills:

- Do I need to re-teach this or do learners just need some practice?
- Is there an interesting activity that will efficiently achieve this?

Learners must be confident in using trigonometry in right-angled triangles to find missing lengths or angles. In the first lesson this skill is assessed and any learners struggling with this must be given support to build their fluency.

Learners will also need to have mastered the algebraic methods from the content for Paper 1: Pure Mathematics in order to confidently solve equations. In particular they should be confident at solving simultaneous equations by substitution and rearranging simple formulae.

Key learning objectives

The following list represents the main underlying concepts that you should make sure your learners have understood by the end of this topic:

- All forces can be resolved into components to simplify problems.
- A resultant force can be calculated from the sum of the horizontal and vertical components.
- Systems in equilibrium have no resultant force.
- The frictional force F for a particular block and surface is not constant, but increases as the applied force P increases until the force F reaches a maximum F_{max}. At this point the system is said to be in limiting equilibrium.

Why this topic matters

The ability to resolve forces into components and use these to solve problems is significant in further topics as part of the Mechanics (for Paper 4) section of the 9709 syllabus. This will provide the basis for calculations in section 4.4 Newton's laws of motion.

Key terminology and notation

Your learners will need to be confident with the following terminology and notation:

coefficient of friction	the value that shows the relationship between the force of friction between two objects and the normal reaction between the objects that are involved
friction	the resistive force that one surface or object encounters when moving over another
limiting equilibrium	when a particle is in equilibrium but the friction force has reached its maximum or limiting value and the particle is on the point of moving, the particle is said to be in limiting equilibrium
limiting friction	the maximum friction that can be generated between two static surfaces in contact with each other
normal reaction	the force exerted by a surface on an object in contact with it, which prevents the object from passing through the surface; the force is

	perpendicular to the surface, and is the only force that the surface exerts on the object in the absence of frictional forces
resultant	the resultant force is the overall result of all forces acting on an object
scalar	just a number – it has no direction – e.g. mass, time, etc.
vector	a quantity which has both magnitude and direction

Insights video

There is an Insights video linked to this *Teaching Pack*:

- **4.1 Forces and equilibrium** – watch this video which will show some of the challenges learners face when analysing the forces that act on objects in different situations.

Teacher tutorials

There are *two* tutorials linked to this *Teaching Pack*:

- **Forces as vectors**
  Review this tutorial before teaching Lesson plan 1. This will demonstrate how to explain to learners how to resolve a force into its *x* and *y* components. It will then go through explaining finding resultant force in a system of forces.
- **Forces in equilibrium**
  Review this tutorial before teaching Lesson 2. This will show you how to set up an experiment to demonstrate forces in equilibrium around a point. It then demonstrates how to explain the process of finding missing forces and/or angles in a system in equilibrium.

Common misconceptions

Misconception	Problems this can cause for learners	An example way to resolve the misconception
That forces should always be resolved in the *x* and *y* direction.	Problems on an inclined plane become more complicated and difficult to solve.	Give an example of forces on an inclined plane. Ask learners which direction might be considered 'horizontal' in this case. Ask which directions for horizontal and vertical will require the least amount of calculation.
Every action has an equal and opposite reaction. Learners may take this to mean that because an unbalanced force is required for acceleration that no object can start moving as all forces are balanced.	Learners will confuse which objects forces are acting on and therefore will make assumptions about the resultant forces on a particular object.	Use the Horse and Cart example in Worksheet J to identify and talk through the implications of the assumption.

Lesson progression

Lesson 1 focuses on understanding the nature of forces as vectors and how to resolve them into their components. It then goes on to finding a resultant force. Lesson 2 follows on directly from the concept of resultant force to considering the idea of no resultant force and therefore systems in equilibrium. Lesson 3 then focuses on understanding Newton's third law and identifying forces acting on a specific object.

Going forward

This topic links to section 4.4 Newton's laws of motion

Lesson plan 1: Forces as vectors

Preparation	• Review the Teacher tutorial *Forces as vectors*.
Resources	• Worksheet A: *Missing sides and angles* • Worksheet B: *Components of forces* • Worksheet C: *Resultant force* • Worksheet D: *Exit ticket* • Lesson slides *Forces as vectors* • Paper, Mini whiteboards or other writing materials
Learning objectives	By the end of the lesson: • ***all*** learners should be able to resolve the horizontal and vertical components of a force when given the angle with the x-axis. • **most** learners should be able to find the sum of components in the x and y direction and parallel and perpendicular to an inclined plane in a system of forces • **some** learners should be able to calculate the resultant force using components in a system of forces

Dependencies

Learners need to know how to use trigonometry to find missing sides in right-angled triangles. Learners should understand the difference between scalar and vector quantities.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
That forces should always be resolved in the x and y direction.	Problems on an inclined plane become more complicated and difficult to solve.	Give an example of forces on an inclined plane. Ask learners which direction might be considered 'horizontal' in this case. Ask which directions for horizontal and vertical will require the least amount of calculation.

Timings	Activity
5 min	**Starter/Introduction** Worksheet A: *Missing sides and angles* Use mini whiteboards (or Worksheet A) to check that learners can remember how to calculate missing sides and angles from right-angled triangles. Learners should be able to show fluency with examples that need the use of sin, cos and tan.

Timings	Activity
	Main lesson
25 min	**Finding components** Have a discussion with learners about what a vector is and how they have represented them at IGCSE or equivalent. Lesson slides *Forces as vectors* slides 2–5 Show learners the examples of vectors on squares (Teacher tutorial slides – ‘Describing vectors’) and ask how they can describe them. The idea of this is that learners will recall using column vectors – ensure you ask them what the two parts of the column vector represent. Once learners are clearly able to give the *x* and *y* components on examples with squares then show a vector without the squares. Ask what other information we need, to be able to describe the *x* and *y* direction. Learners should make the link with the starter and realise that if we have an angle we can find the individual parts. Demonstrate that the *x* and *y* components can be considered as a right-angled triangle. Run through worked examples of how to find *x* and *y* components – show an example that includes an inclined plane. Make sure you discuss setting a positive and a negative direction on each axis and consider the direction of each component.
20 min	Worksheet B: *Components of forces* Learners then work through Worksheet B. This starts with single forces on a set of *x*-*y* axes, moves onto single forces on an inclined plane and then finding the sum of components in a system of forces, both on a set of axes and on an inclined plane. **Challenge:** Encourage learners to complete Questions 3 and 4 of Worksheet B – finding the sum of components in a system of forces. This will help them when they come to work on Worksheet C. **Support:** Encourage learners to draw the *x* and *y* components of each force as a triangle to help them visualise the use of trigonometry.
	Resultant force Lesson slides *Forces as vectors* slides 6–9 Worksheet C: *Resultant force* Show a system of forces diagram (Worksheet C). Ask how we could represent all of these forces as a single force. Think/pair/share – ask learners individually to think about what they could do for 1 minute. Then ask them to discuss with their partner for 2 minutes, then finally ask each pair to share ideas with the group on how to represent this as a single force. Prompts for discussion: - What information would we need to include to effectively describe the final force? - How could we represent the system of forces as just two separate parts? - Can we do the inverse of the process from our last activity?

Timings	Activity
	- What should we call this single final force? (resultant force – key terminology) Ask each pair to work through the problem and then choose a pair to show their method to the group. This could be done by asking a learner to write their method on the board. Ask learners to complete the other examples on Worksheet C.
10 min	**Plenary** Worksheet D: *Exit ticket* Collect Worksheet D in from learners as they leave and use it to assess how much of the lesson each learner has understood.

Reflection	Reflect on your lesson; use the **Lesson reflection** notes to help you.

Lesson plan 2: Forces in equilibrium

Preparation	• Review the Teacher tutorial *Forces in equilibrium*.
Resources	• Two smooth pulleys – string, connecting ring, masses (potentially multiple sets if running the activity in groups). The Physics/Science department may have suitable equipment. Set up before the lesson • Paper, Mini whiteboards or other writing materials • Worksheet E: *True/False resolving forces* • Worksheet F: *Forces around a point experiment* • Worksheet G: *Equilibrium match-up task* • Worksheet H: *Equilibrium exam question*
Learning objectives	By the end of the lesson: • ***all*** learners should recognise that if there is no resultant force then a system is in equilibrium • **most** learners should be able to set up equations for horizontal and vertical components of systems in equilibrium • **some** learners should be able to calculate the magnitude and direction of missing forces in systems of equilibrium

Dependencies

Learners need to know how to resolve forces into horizontal and vertical components.
Learners need to know how to solve simultaneous equations by substitution.
Learners need to be able to use the identity $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$

Timings	Activity
15 min	**Starter/Introduction** Worksheet E: *True/False resolving forces* Use mini whiteboards and the scenarios on Worksheet E to check that learners can find resultant force. You should remind them of the skills from Lesson 1. Learners need to be able to justify their decision
15 min	**Main lesson** Worksheet F: *Forces around a point experiment* Use the set-up instructions on worksheet F for an experiment/demonstration. Having set up the pulleys with the three masses hanging from the ends of the strings, ask learners the questions below while they investigate/while demonstrating the effect of changing the masses by small amounts: - Does the length of the string affect the angle? - What could we call this position of rest that is achieved? - What would happen if we changed one of the masses? - What is the relationship between the masses and the angle? Challenge: Depending on availability of equipment you could ask learners to investigate the relationship between the angle D and the mass B, initially keeping mass A and C equal.

Timings	Activity
	Learners should realise that the length of the string does not matter and that it is the tension in the string caused by the hanging masses that causes the change in equilibrium position. Try to use the conversation and questions to get learners to consider the components in each of the directions and that the resultant force must be 0.
20 min	Teacher tutorial *Forces in equilibrium* slides 3–9 Once learners have understood the concept of equilibrium and the sum of all x components and y components being 0, go through the first examples (Teacher tutorial slides 'Finding missing forces and angles in equilibrium' slides 3-5) of how to create the equations of x and y and then solve them simultaneously using substitution. Worksheet G: *Equilibrium match-up task* There are six scenarios where the system is in equilibrium. By doing calculations, learners should pair each scenario with the correct missing values. Depending on time available you may decide to have learners work together in pairs. **Support:** Less able learners should focus on completing the first three scenarios on Worksheet G.
10 min	**Plenary** Worksheet H: *Equilibrium exam question* Use the exam question on Worksheet H to summarise what has been learnt this lesson. Give learners 5 minutes individually to come up with an answer and then show the worked solution. Learners could mark each other's work and give each other feedback on the approach they have used and how clear the working and answers were.

Reflection Reflect on your lesson; use the **Lesson reflection** notes to help you.

Lesson plan 3: Newton’s third law

Preparation	• Read the explanation from Worksheet J: *Answers* in order to be clear on the concept of Newton’s third law.

Resources	• Lesson slides *Newton’s third law* • Worksheet I: *Newton’s third law – multiple-choice questions* • Worksheet J: *Horse and cart problem* • Balloon • Paper, Mini whiteboards or other writing materials

Learning objectives	By the end of the lesson: • ***all*** learners should be able to identify forces acting in given situations • **most** learners should be able to use Newton’s third law to identify reaction forces • **some** learners should be able to consider contact forces between two surfaces at an angle

Dependencies

Learners should recall that if forces are in equilibrium then an object is either stationary or moving at a constant velocity.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Every action has an equal and opposite reaction. Learners may take this to mean that because an unbalanced force is required for acceleration that no object can start moving as all forces are balanced.	Learners will confuse which objects forces are acting on and therefore will make assumptions about the resultant forces on a particular object.	Use the Horse and cart example in Worksheet J to identify and talk through the implications of the assumption.

Timings	Activity
15 min	**Starter/Introduction** **Identify forces in given situations** [Lesson slides: *Newton’s third law slides 2-10*] In this exercise learners draw clear diagrams of the forces acting on the objects named by you. They should draw diagrams on mini whiteboards to represent the forces. The clarity of the forces is what’s important here, not the realism of the drawings. In each of the cases learners should focus on the forces acting on the object in italics: - A *gymnast* hanging at rest on a bar

Timings	Activity
	- A *lamp* hanging from a ceiling - A *book* at rest on a table - A *horizontal plank* being used to bridge a stream - An *ice hockey puck* at the moment it is being hit by a stick, assuming the ice is smooth. - An *ice hockey puck* at the moment it is being hit by a stick, assuming the ice is rough. - A *block* at rest on a slope - A *plane* flying at a constant speed and maintaining height Learners should think carefully about all the forces acting in each situation. Try to elicit the idea that if the object is not moving or is at a constant speed then forces acting on the object are in equilibrium. Appropriate wording is important, e.g. weight, tension, reaction, friction etc. Try to encourage the use of 'reaction' forces.
10 min 20 min	**Main lesson** **Newton's third law – discussion** From the starter activity you should have introduced the idea that if an object exerts a force on another object we get a reaction force. Through discussion try to elicit a description of what Newton's third law is. You are likely to get the response: 'For every action there is an equal and opposite reaction'. Discuss what this really means: if two bodies A and B are in contact and exert forces on each other, then the force exerted on B by A is equal in magnitude and opposite in direction to the force exerted on A by B. Discuss examples – you could blow up a balloon and release it, demonstrating that the force of air being released causes a reaction force on the balloon which moves it forward. Guns or cannons firing show recoil; one person on ice skates pushing another away – both move, not just the one who pushed – and equal and opposite force. Worksheet I: *Newton's third law – multiple-choice questions* Get learners to work in pairs on the multiple-choice questions. For each question they need to make a choice and then explain why they have chosen this answer. They should use their understanding of Newton's third law to build their explanations. Once learners have worked through the questions, go through the answers as a group, paying close attention to where they have made mistakes. Ask each pair in turn and pass around the class for disagreeing with the answer / building a more in-depth reason for choosing an answer. **Challenge:** This exercise involves a need to truly understand what is meant by Newton's third law. More able learners are likely to give significantly more detailed descriptions and reasons for their answers. **Support:** A good strategy for this exercise would be to pair more able learners with less able learners.

Timings	Activity
15 min	**Plenary** Worksheet J: *Horse and cart problem* Lesson slides: *Newton's third law slides 11-15* This question is adapted from the book *Thinking Physics is Gedanken Physics* by Lewis Epstein. It is one of the most well-known physics puzzles. You may get some interesting responses to this as it is a problem that will highlight misconceptions learners have about Newton's third law. A complete answer to the problem can become complicated, but in order to give a clear explanation learners should recall that any object will accelerate if it has a resultant force acting on it. They must also realise and include the explanation that when considering the motion of an individual object you can only consider the forces acting on that object, and not those acting on other objects around it. Slide 11 provides an example you can project and draw on for explanation.

Reflection	Reflect on your lesson; use the **Lesson reflection** notes to help you.

Planning your own lessons

You now need to plan lessons to cover the following bullet points:

Candidate should be able to:	Notes and examples
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leq \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions

Follow the structure of the *Teaching Pack*, and use techniques from the 'How to' guides, to create your own engaging lessons to cover these bullet points. Consider what preparation you need for each lesson: what prior knowledge is needed, what are the key objectives, what are the dependencies, what common misconceptions are there, and so on.

Below, we have provided an outline of some activities and approaches you might like to try.

Lesson 4: Friction
Common misconceptions: The friction between two surfaces is a constant value.
Starter: You could try a discussion of the meaning of 'smooth' in a mathematical modelling context – bring out the ideas of the limitations of this model. Indicate it being useful in systems involving pulleys, but in other contexts it is not similar to real situations. Be clear that in most situations when there is contact between two surfaces you will have two components of the force between them: a) the normal and b) friction.
Main: You could have a class discussion about how friction will increase as the force trying to move an object increases until at some point you reach a maximum friction and beyond this the object will move. Ensure you use the keywords 'limiting friction' and 'limiting equilibrium'.
You could use resource https://study.com/academy/lesson/coefficient-of-friction-definition-formula-examples.html. Introduce the relationship $F = \mu R$ and demonstrate examples of questions involving it.
Plenary: You could try calculations involving the relationship $F = \mu R$

You will find some other activity suggestions in the Scheme of Work.

Lesson reflection

As soon as possible after the lesson you need to think about how well it went.

One of the key questions you should always ask yourself is:

Did all learners get to the point where they can access the next lesson? If not, what will I do?

Reflection is important so that you can plan your next lesson appropriately. If any misconceptions arose or any underlying concepts were missed, you might want to use this information to inform any adjustments you should make to the next lesson.

It is also helpful to reflect on your lesson for the next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time. There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.

To help you reflect on your lesson, answer the most relevant questions below.

Were the lesson objectives realistic?
What did the learners learn today? Or did they learn what was intended? Why not?
What proportion of the time did we spend on the most important topics?
Were there any common misconceptions?
What do I need to address next lesson?
What was the learning atmosphere like?
Did my planned differentiation work well?
How could I have helped the lowest achieving learners to do more?
How could I have stretched the highest achieving learners even more?
Did I stick to timings?
What changes did I make from my plan and why?

Summary evaluation

What two things went really well? (Consider both teaching and learning.)

What two things would have improved the lesson? (Consider both teaching and learning.)

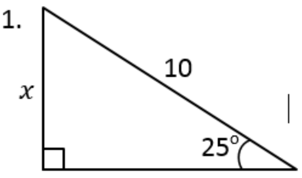
What have I learned from this lesson about the class or individuals that will inform my next lesson?

Worksheets and answers

Worksheet A: Missing sides and angles

For each triangle find the missing values correct to 1 decimal place.

1.



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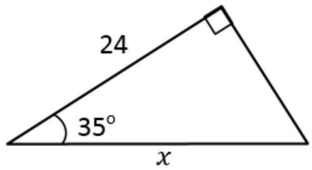

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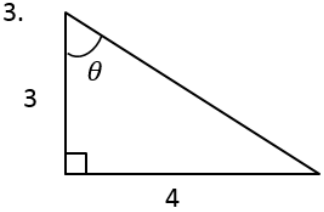

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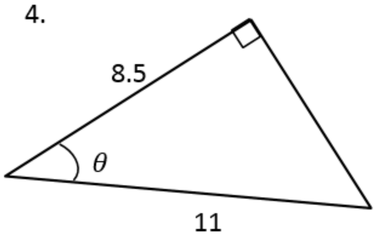

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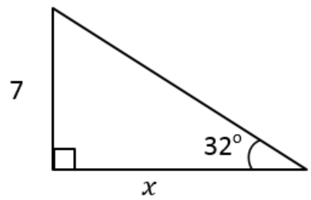

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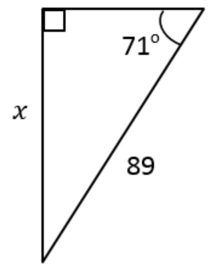

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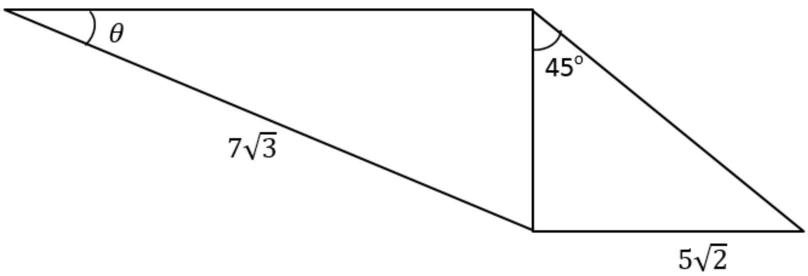
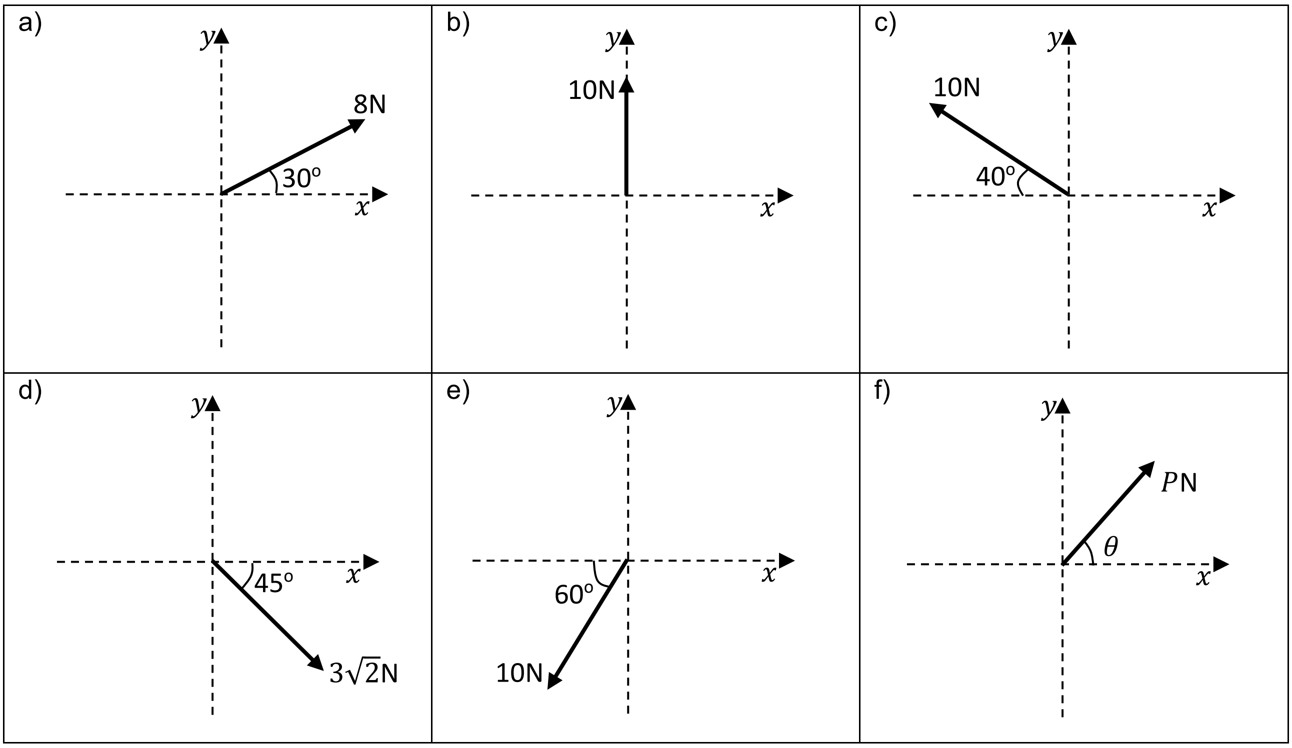


Worksheet B: Components of forces

The component of a force in any given direction is a measure of the effect of the force in that direction.

For each of the force diagrams below, find the components or sum of the components in the direction of (i) the *x*-axis and (ii) the *y*-axis.

1.



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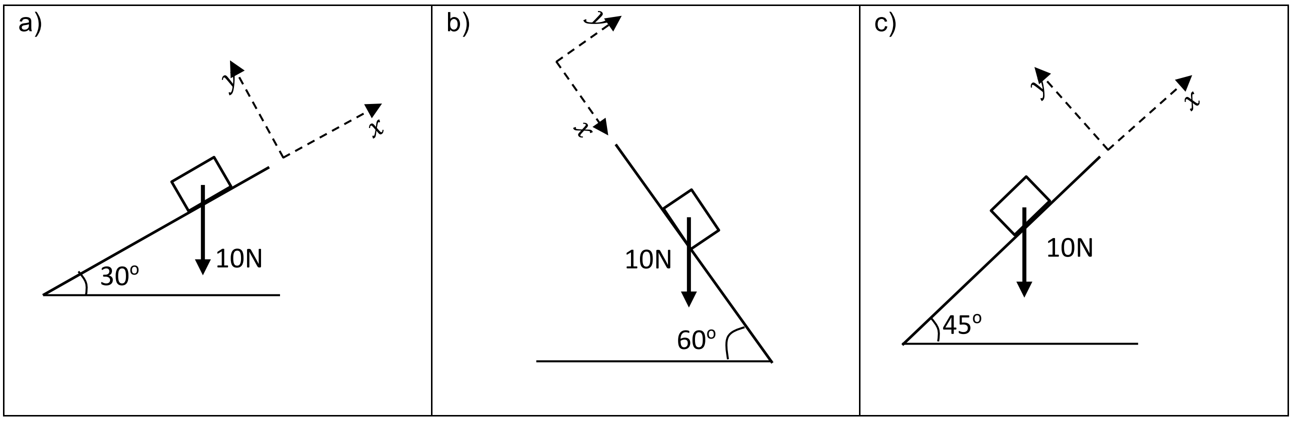
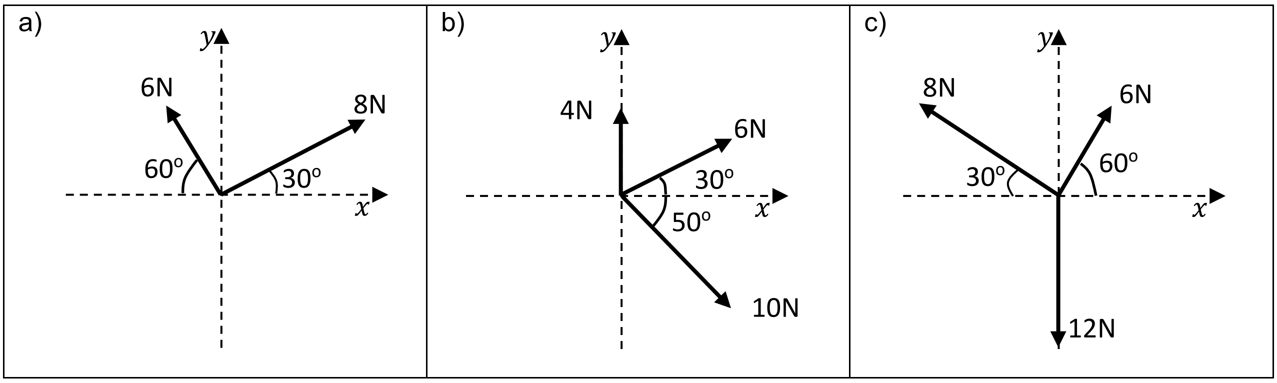


Worksheet B: Components of forces continued

3.



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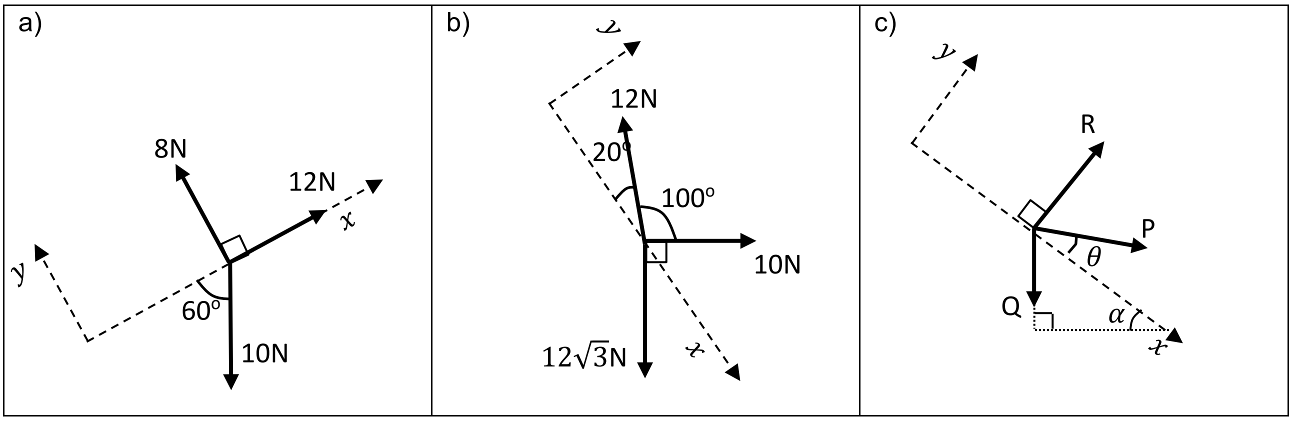
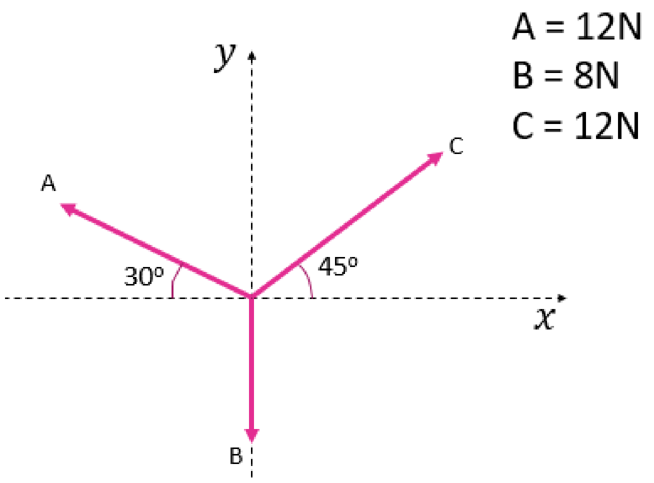


Worksheet C: Resultant force

Can you represent each of these force systems as a single force?

a)



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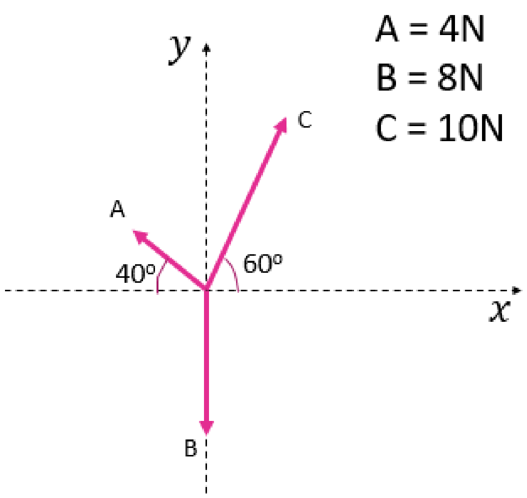
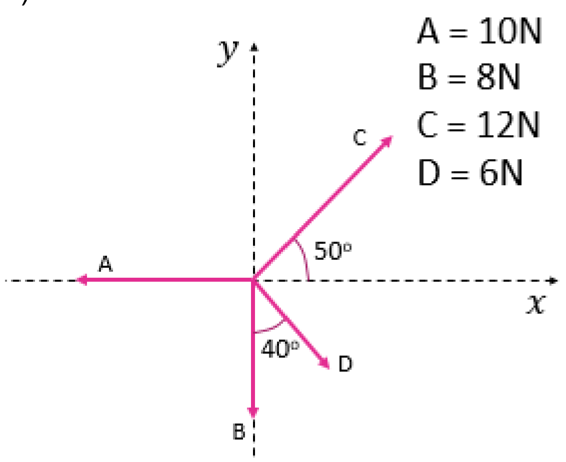
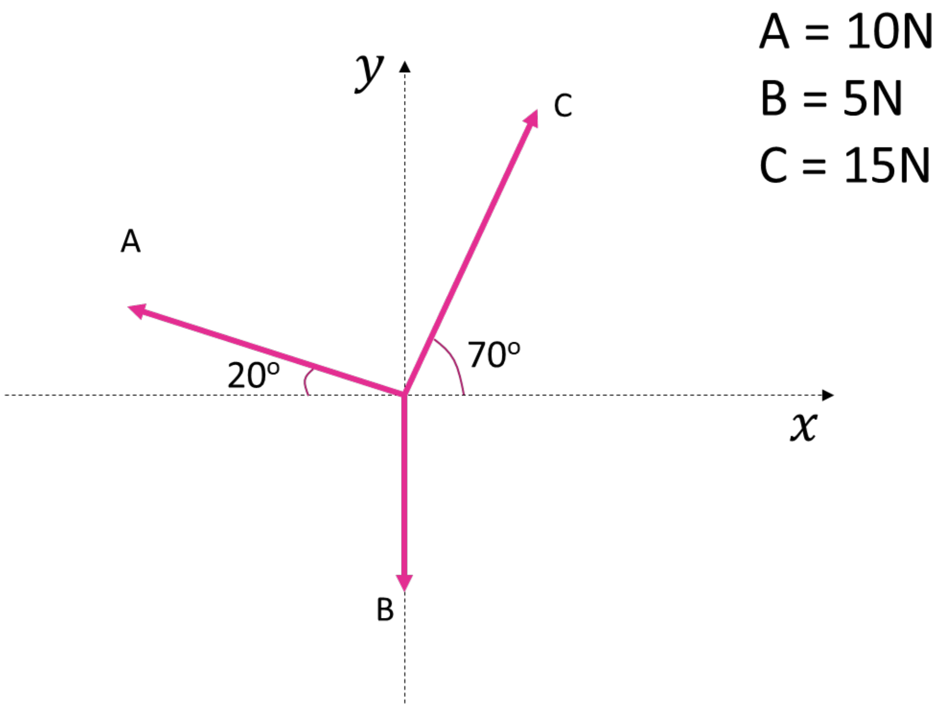


c)



Worksheet D: Exit ticket

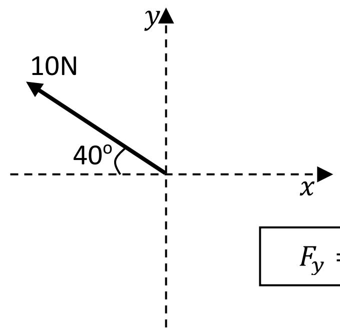


1. Find the *x* and *y* components of force A.
2. Find the sum of the components of all three forces.
3. Find the size and direction of the resultant force.

Worksheet E: True/False resolving forces

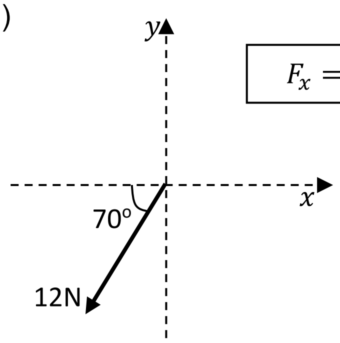
For each of the situations, decide whether the stated fact is True or False.

a)



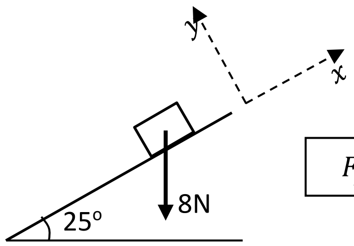
$F_y = 10\cos 40$

b)



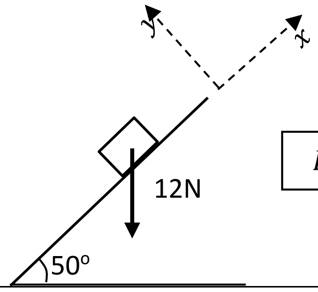
$F_x = 12\cos 70$

c)



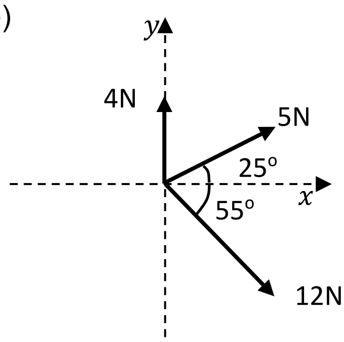
$F_y = -8\cos 25$

d)



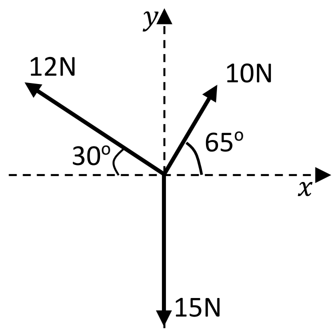
$F_x = -12\sin 50$

e)



$R_y = 5\sin 25 - 12\sin 55$

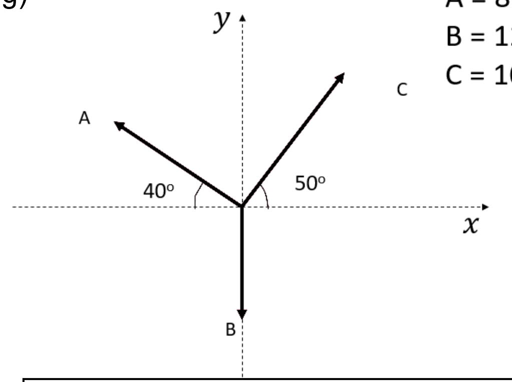
f)



$R_x = 10\cos 65 - 12\cos 30$

g)

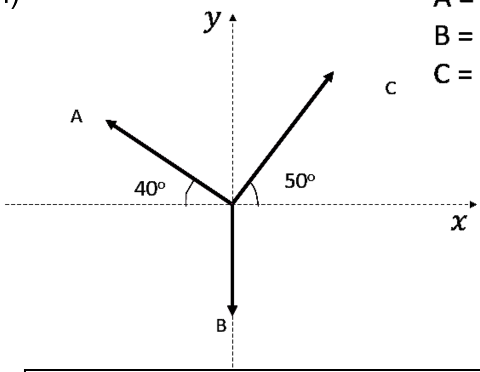
A = 8N
B = 12N
C = 10N



$R = 0.86N$

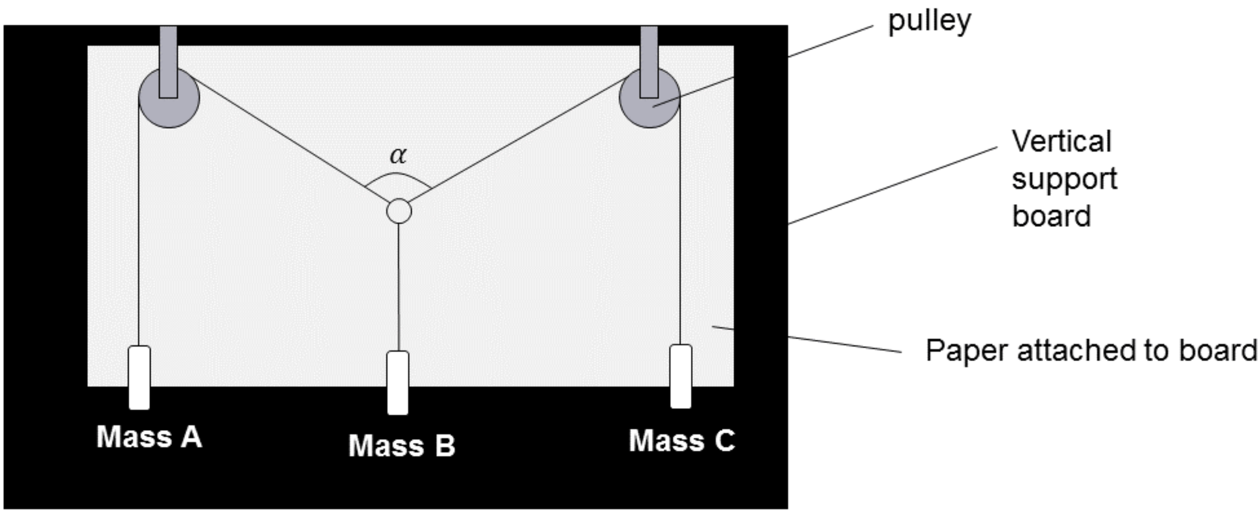
h)

A = 8N
B = 12N
C = 10N



$\theta = 20.5°$

Worksheet F: Forces around a point experiment



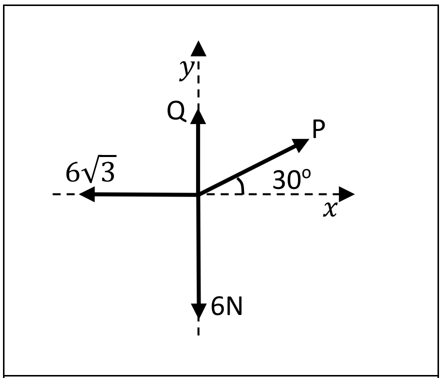
The diagram shows two strings tied to a light ring and passing over smooth pulleys.
Masses A and C are then suspended from the free end of these strings.
A third string is attached to the ring and mass B is hung directly below.
The whole system is mounted on a vertical board.

Good results can be achieved if the pulleys are smooth and the mass of the system is enough to counteract the effect of the friction.

You may want to start with 100g on B and 60g each on A and C.

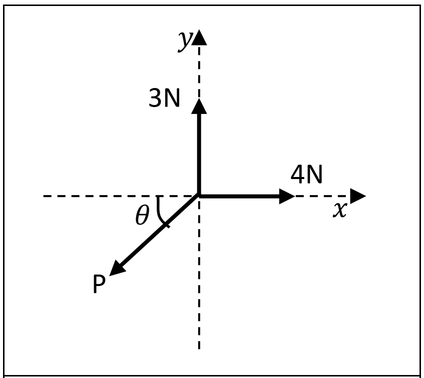
- What would happen to the angle if we reduced the mass of B by 10g?
- Can you come up with a rule for the angle and the relationship with the mass at A and B (keeping A and C equal)?
- What happens if we change the mass at A or C?

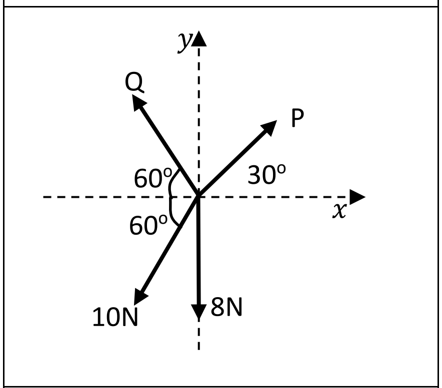
Worksheet G: Equilibrium match-up task



P = 12.7N
Q = 11.9N

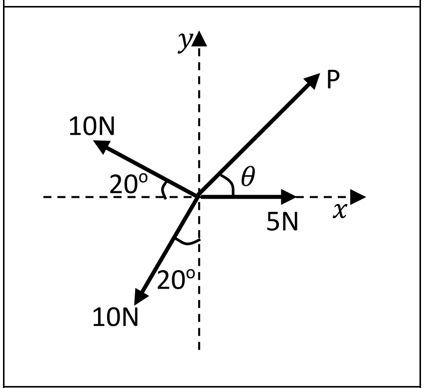
P = 8.9N
θ = 26.6°

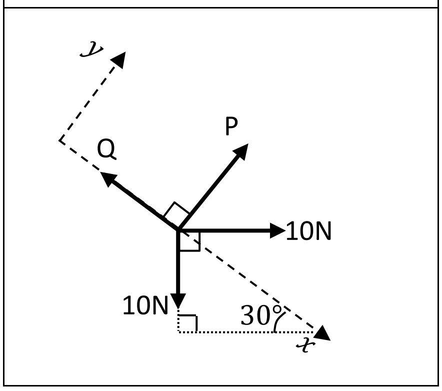




P = 3.7N
Q = 13.7N

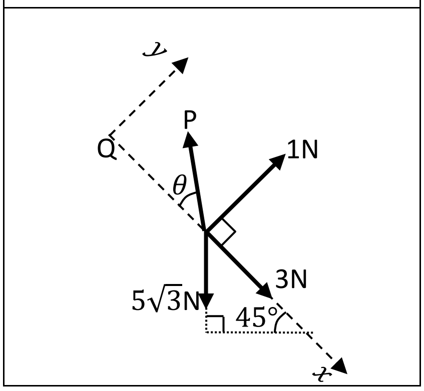
P = 9.8N
θ = 37.4°





P = 8N
Q = 2N

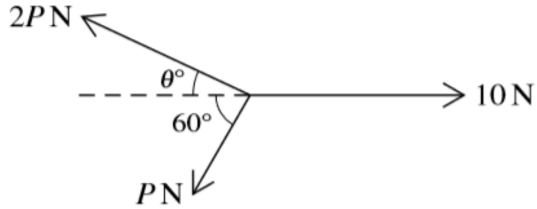
P = 5N
θ = 36.9°



Worksheet H: Equilibrium exam question

Exam question:

9709/42 March 2018 question 2



The three coplanar forces shown in the diagram are in equilibrium. Find the values of θ and P. [4]

Worksheet I: Newton's third law – multiple choice questions

For each question below, choose an answer and then give a clear reason why, making sure you use appropriate mathematical language.

1. When you step on a toy, the toy applies an equal and opposite force on your foot as your foot applies to the toy.

   ☐ True

   ☐ False

2. If you are an astronaut slowly drifting away from the space station, you might be able to drift back to the station by throwing a 5kg tool rapidly in the direction that you are travelling (away from the station).

   ☐ True

   ☐ False

3. Gravity is pulling on you downwards with a force which we call your weight. The reason why you aren't accelerating downwards is that there is an equal and opposite force of the floor (let's assume you are standing up) pushing you upwards that nets out against the force of gravity. This is the 'equal and opposite' force described by Newton's third law of motion.

   ☐ True

   ☐ False

4. Which best explains why we are able to accelerate forward when starting to run?

   ☐ The striking foot pushes backward against the ground. The friction with the ground provides an equal and opposite force forward.

   ☐ The foot not touching the ground propels the entire body as it swings forward.

   ☐ No acceleration takes place. Runners are always at a fixed velocity.

   ☐ As one leg moves backward, it provides an equal and opposite force for the other foot to move forward.

   ☐ The runner's upper body quickly leans forward, causing the entire body to begin accelerating forward.

5. You and a friend are pulling on a rope in opposite directions as hard as you can. What is the 'equal and opposite force' to the force of your hand pulling on the rope described by Newton's third law?

   ☐ The force of friction between the ground and your shoes

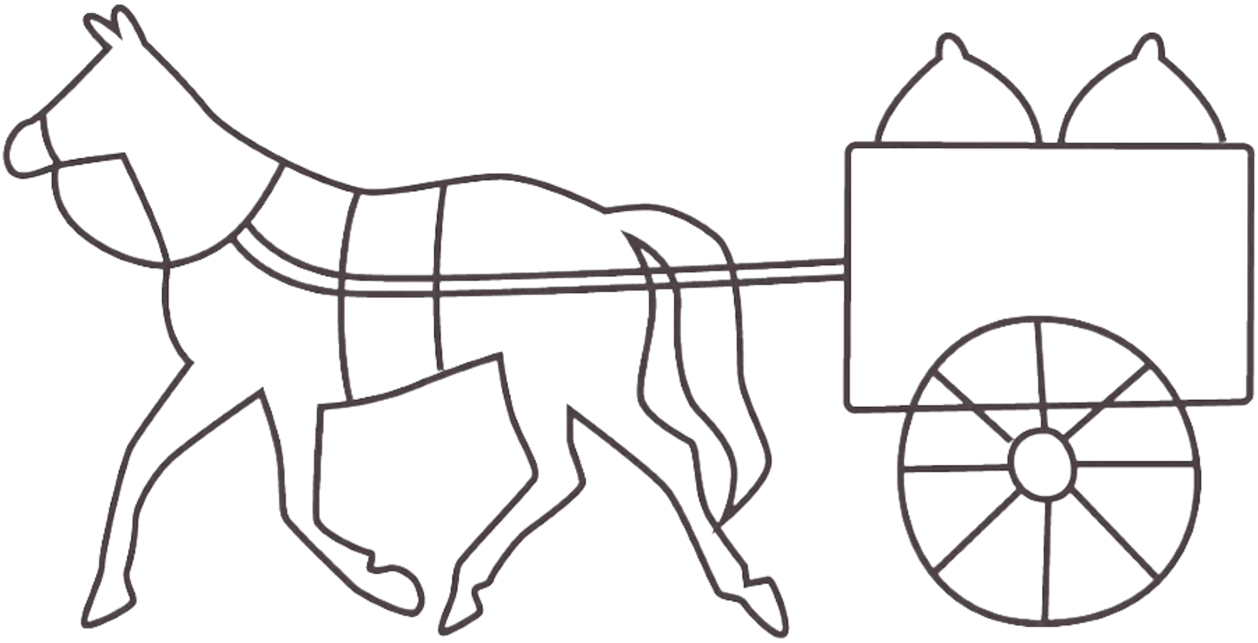
   ☐ The force of the rope pulling your friend's hand

   ☐ The force of your arm pulling back on your hand

   ☐ The force of your arm pulling back on your hand

   ☐ The force of the rope pulling on your hand in the opposite direction

Worksheet J: Horse and cart problem



A farmer attaches his horse to his cart one day, and climbs up on the cart with his son.

The farmer's son says, 'I learnt in school yesterday about Newton's laws of motion. Did you know that Newton's third law says that every action force has an equal and opposite reaction force?'.

'Yes, I do' says the farmer.

'Newton's third law says that if the horse pulls on the cart, the cart exerts an equal and opposite force on the horse. Don't you agree?', asks the son.

'Yes... but...'

'If these two forces are equal and opposite, they will cancel, so that the net force is zero, right?', argues the son.

'Well, I suppose so,' says the farmer.

'The net force is always the important thing. If the net force is zero, then Newton's second law (and Newton's first law, too) says that the acceleration of the wagon must be zero.'

'Yes.', says the farmer, but let's get going!'

'But that's the point!', says the son, 'If the wagon's pull is always equal and opposite to my pull, then the net force will always be zero, so the wagon can never move! Since it is at rest, it must always remain at rest!

*Newton's laws **are** correct, and horses **can** pull wagons. What is the error in the son's argument?*

Worksheet A: Answers

1. $\sin 25 = \frac{x}{10}$

   $x = 10 \sin 25 = 4.22618$

   $x = 4.2$

2. $\cos 35 = \frac{24}{x}$

   $x = \dfrac{24}{\cos 35}$

   $x = 29.3$

3. $\tan \theta = \frac{4}{3}$

   $\theta = \tan^{-1}\left(\dfrac{4}{3}\right)$

   $x = 53.1°$

4. $\cos \theta = \frac{8.5}{11}$

   $\theta = \cos^{-1}\left(\dfrac{8.5}{11}\right)$

   $\theta = 39.4°$

5. $\tan 32 = \frac{7}{x}$

   $x = \dfrac{7}{\tan 32}$

   $x = 11.2$

6. $\sin 71 = \frac{x}{89}$

   $x = 89 \sin 71$

   $x = 84.2$

7. $\tan 45 = \frac{5\sqrt{2}}{x}$

   $x = \dfrac{5\sqrt{2}}{\tan 45}$

   $x = 5\sqrt{2}$

   $\sin \theta = \dfrac{5\sqrt{2}}{7\sqrt{3}}$

   $\theta = \sin^{-1}\left(\dfrac{5\sqrt{2}}{7\sqrt{3}}\right)$

   $\theta = 35.7°$

Worksheet B: Answers

1. a) i) $F_x = 8\cos 30 = 6.93N$ ii) $F_y = 8\sin 30 = 4N$

   b) i) $F_x = 0N$ ii) $F_y = 10N$

   c) i) $F_x = -10\cos 40 = -7.66N$ ii) $F_y = 10\sin 40 = 6.43N$

   d) i) $F_x = 3\sqrt{2}\cos 45 = 3N$ ii) $F_y = -3\sqrt{2}\sin 45 = -3N$

   e) i) $F_x = -10\cos 60 = -5N$ ii) $F_y = -10\sin 60 = -5\sqrt{3}N$

   f) i) $F_x = P\cos\theta$ ii) $F_y = P\sin\theta$

2. a) i) $F_x = -10\sin 30 = -5N$ ii) $F_y = -10\cos 30 = -5\sqrt{3}N$

   b) i) $F_x = -10\sin 60 = -5\sqrt{3}N$ ii) $F_y = -10\cos 60 = -5N$

   c) i) $F_x = 10\sin 45 = 5\sqrt{2}N$ ii) $F_y = 10\cos 45 = 5\sqrt{2}N$

3. a) i) $F_x = 8\cos 30 - 6\cos 60 = 3.93N$

   ii) $F_y = 8\sin 30 + 6\sin 60 = 9.20N$

   b) i) $F_x = 6\cos 30 + 10\cos 50 = 11.6N$

   ii) $F_y = 4 + 6\sin 30 - 10\sin 50 = -0.660N$

   c) i) $F_x = 6\cos 60 - 8\cos 30 = -3.93N$

   ii) $F_y = -12 + 8\sin 30 + 6\sin 60 = -2.80N$

4. a) i) $F_x = 12 - 10\cos 60 = 7N$

   ii) $F_y = 8 - 10\sin 60 = -0.660N$

   b) i) $F_x = -12\cos 20 + 10\cos 60 + 12\sqrt{3}\cos 30 = 11.7N$

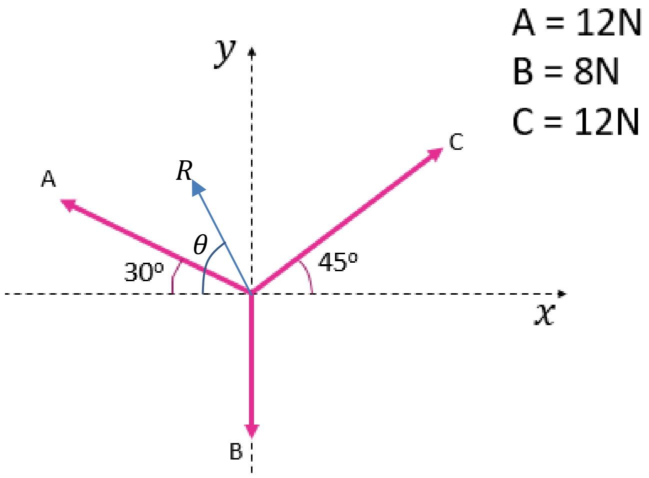
   ii) $F_y = 12\sin 20 + 10\sin 60 - 12\sqrt{3}\sin 30 = 2.37N$

   c) i) $F_x = P\cos\theta + Q\sin\alpha$

   ii) $F_y = R + P\sin\theta - Q\cos\alpha$

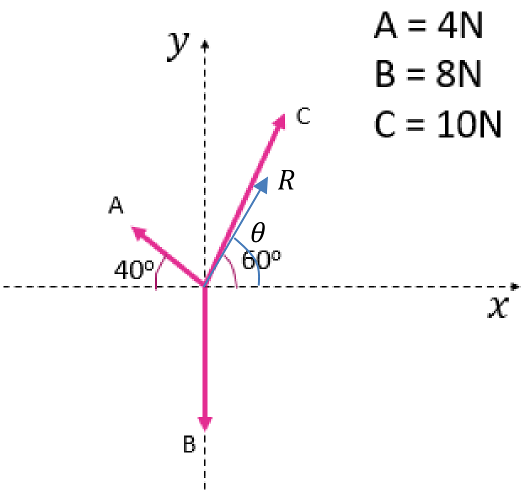
Worksheet C: Answers

a)



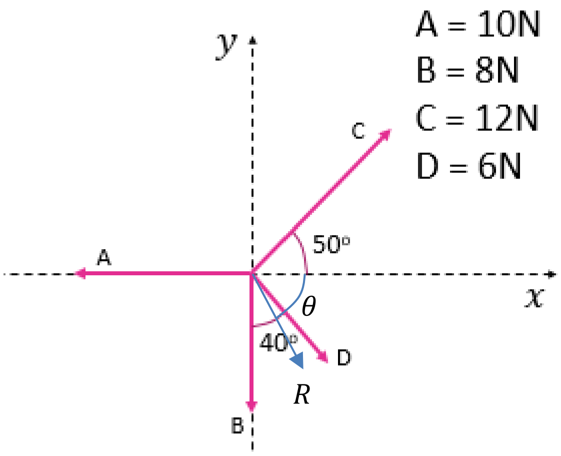
$$R_x = 12\cos 45 - 12\cos 30$$
$$= -6\sqrt{3} + 6\sqrt{2}$$
$$= -1.907N$$
$$R_y = 12\sin 45 + 12\sin 30 - 8$$
$$= -2 + 6\sqrt{2}$$
$$= 6.485N$$
$$R = \sqrt{(-1.9)^2 + 6.5^2}$$
$$R = 6.77N$$
$$\theta = \tan^{-1}\left(\frac{6.5}{1.9}\right) = 73.6°$$

b)



$$R_x = 10\cos 60 - 4\cos 40$$
$$= 1.935N$$
$$R_y = 4\sin 40 + 10\sin 60 - 8$$
$$= 3.231N$$
$$R = \sqrt{1.9^2 + 3.2^2}$$
$$R = 3.77N$$
$$\theta = \tan^{-1}\left(\frac{3.2}{1.9}\right) = 59.1°$$

c)



$$R_x = 12\cos 50 + 6\sin 40 - 10$$
$$= 1.570N$$
$$R_y = 12\sin 50 - 6\cos 40 - 8$$
$$= -3.404N$$
$$R = \sqrt{1.6^2 + (-3.4)^2}$$
$$R = 3.74N$$
$$\theta = \tan^{-1}\left(\frac{3.4}{1.6}\right) = 65.2°$$

Worksheet D: Answers

1. $F_x = -9.40\cos 20 = -9.4N$

   $F_y = 10\sin 20 = 3.42N$

2. $R_x = -10\cos 20 + 15\cos 70$

   $= -4.25N$

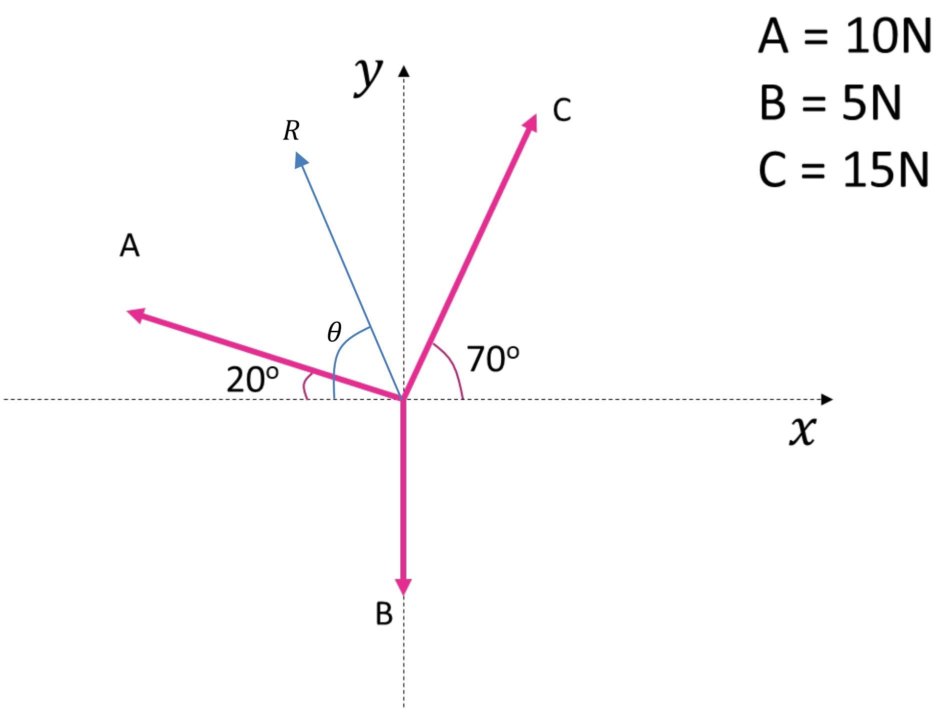
   $R_y = 10\sin 20 + 15\sin 70 - 5$

   $= 12.5N$

3. $R = \sqrt{(-4.3)^2 + (12.5)^2}$

   $R = 13.2N$

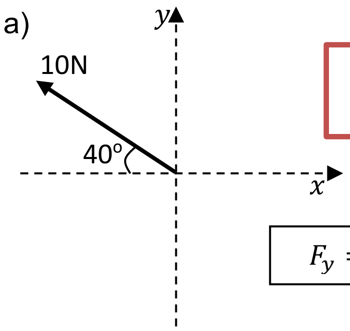
   $\theta = \tan^{-1}\left(\frac{12.5}{4.3}\right) = 71.2°$



Worksheet E: Answers

For each of the situations, decide whether the stated fact is True or False.

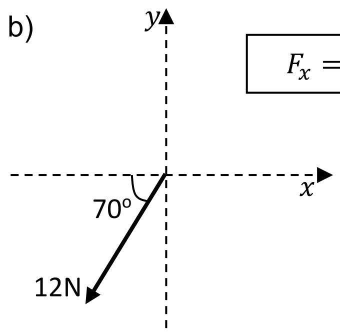
a)



$F_y = 10 \cos 40$

False

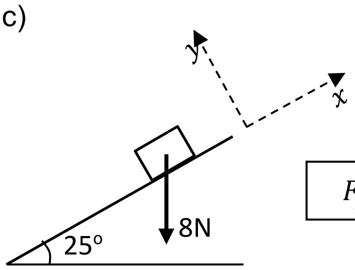
b)



$F_x = 12 \cos 70$

False

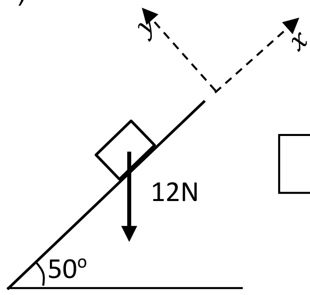
c)



$F_y = -8 \cos 25$

True

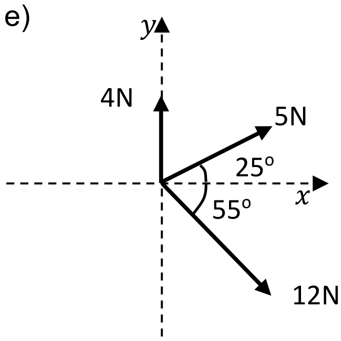
d)



$F_x = -12 \sin 50$

True

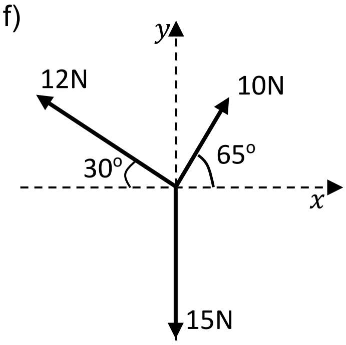
e)



$R_y = 5 \sin 25 - 12 \sin 55$

False

f)

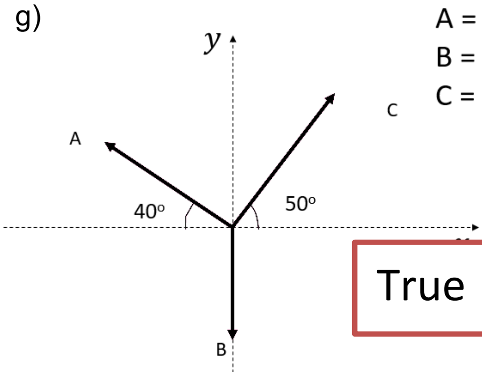


$R_x = 10 \cos 65 - 12 \cos 30$

True

g)

A = 8N
B = 12N
C = 10N

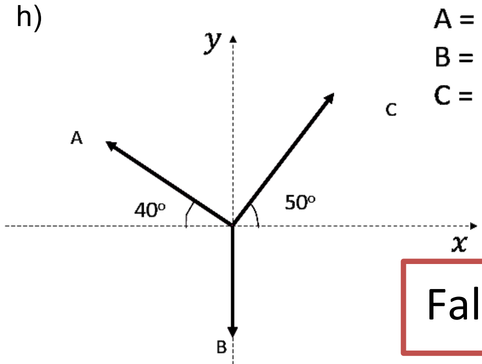


$R = 0.86N$

True

h)

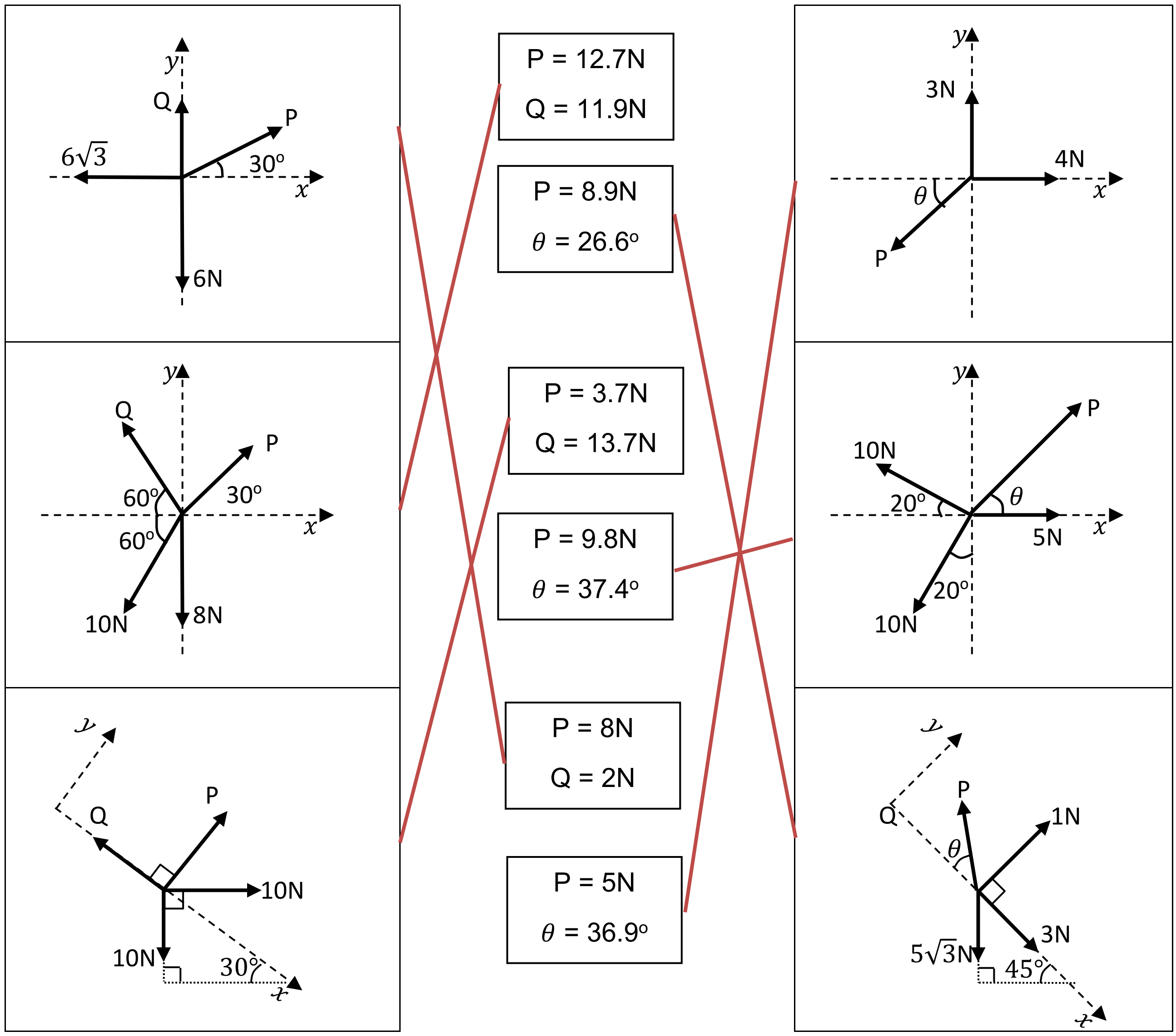
A = 8N
B = 12N
C = 10N



$\theta = 20.5°$

False

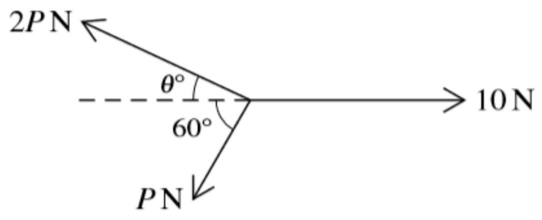
Worksheet G: Answers



Worksheet H: Answers

Exam question:

9709/42 March 2018 question 2



The three coplanar forces shown in the diagram are in equilibrium. Find the values of θ and P. [4]

Resolving vertically:

$$0 = 2P\sin\theta - P\sin 60$$
$$P\sin 60 = 2P\sin\theta$$
$$\sin 60 = 2\sin\theta$$
$$\frac{\sqrt{3}}{4} = \sin\theta$$
$$\theta = \sin^{-1}\frac{\sqrt{3}}{4} = 25.7°$$

Resolving horizontally:

$$0 = 10 - 2P\cos\theta - P\cos 60$$
$$2P\cos\theta + P\cos 60 = 10$$
$$2P\cos\theta + P\cos 60 = 10$$
$$P(2\cos\theta + \cos 60) = 10$$
$$P = \frac{10}{2\cos\theta + \cos 60}$$
$$P = \frac{10}{2\cos 25.66 + \cos 60}$$
$$P = 4.34N$$

Worksheet I: Answers

1. When you step on a toy, the toy applies an equal and opposite force on your foot as your foot applies to the toy.

   True: This comes directly from Newton's third law.

2. If you are an astronaut slowly drifting away from the space station, you might be able to drift back to the station by throwing a 5kg tool rapidly in the direction that you are travelling (away from the station).

   True: To throw the tool, you'll apply a force on the tool directed away from the station. By Newton's third law, the tool will apply an equal and opposite force on you. If this force is large enough and applied long enough, it may be able to change the direction of your velocity towards the station.

3. Gravity is pulling on you downwards with a force which we call your weight. The reason why you aren't accelerating downwards is that there is an equal and opposite force of the floor (let's assume you are standing up) pushing you upwards that nets out against the force of gravity. This is the ‘equal and opposite’ force described by Newton's third law of motion.

   False: It is true that the upwards force of the floor nets against the force of gravity, but this is not the ‘equal and opposite’ force described by Newton's third law.
   The ‘equal and opposite’ force described by the third law is the force of gravity pulling the Earth towards you (the equal and opposite forces in the third law act on the two bodies interacting, not just on one body).

4. Which best explains why we are able to accelerate forward when starting to run?

   The striking foot pushes backward against the ground. The friction with the ground provides an equal and opposite force forward.
   In order for the runner to accelerate forward, a net force has to be applied to the runner. The only object making contact with the runner is the ground. As the runner's striking foot attempts to rub backward against the ground, the force of friction keeps it ‘in place’ (relative to the ground) by providing an equal and opposite force forward.
   Think about what would happen if you tried to run barefoot on ice (other than getting frostbite on your feet). If there were no friction between the foot/shoe and the ground, it would be impossible to ‘run.’

5. You and a friend are pulling on a rope in opposite directions as hard as you can. What is the ‘equal and opposite force’ to the force of your hand pulling on the rope described by Newton's third law?

   The force of the rope pulling on your hand in the opposite direction.
   If object A applies a force to object B, then the ‘equal and opposite force’ is the force that B applies to A (same magnitude, but opposite direction). They don't net out with each other because they are acting on two different bodies (when we net forces, we are talking about forces on the same body).
   Newton's third law, therefore, is describing the force of the rope pulling on your hand in the opposite direction.

Worksheet J: Answers

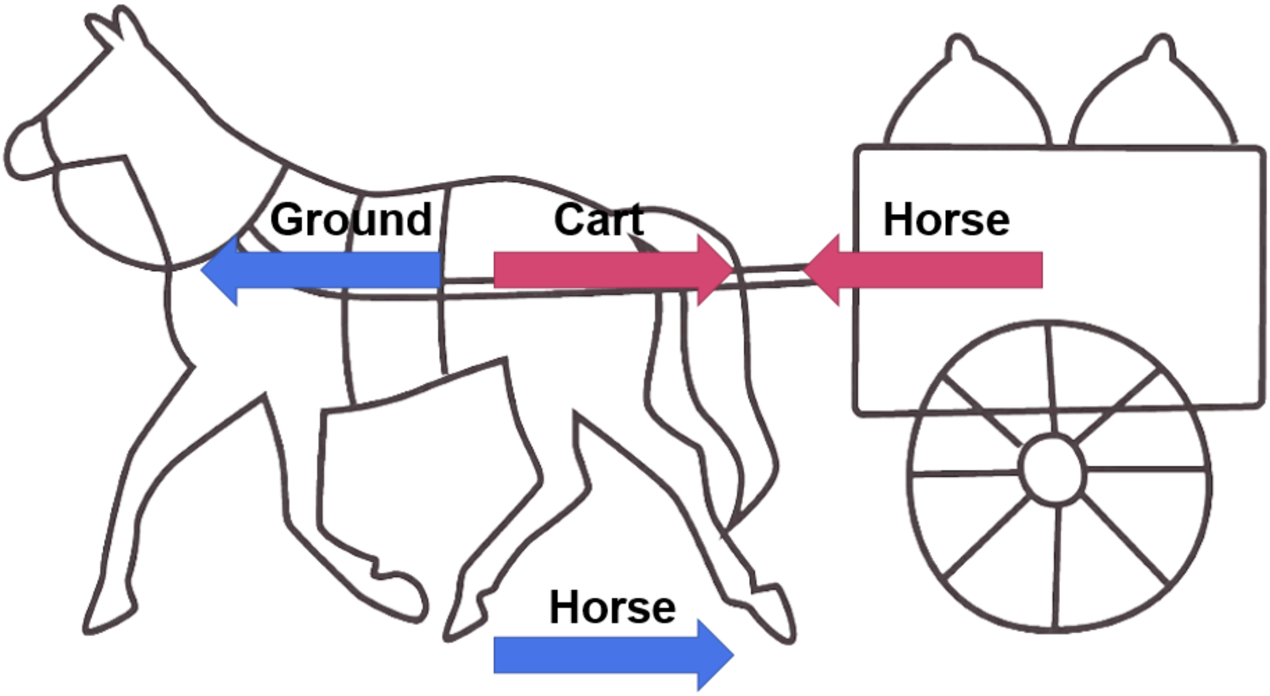
A complete answer to the Horse and cart problem can get rather involved, but a clear explanation only depends on a couple of simple points:

- An object accelerates (or not) because of the forces that push or pull on it. (Newton's second law)
- Only the forces that act on an object can cancel. Forces that act on different objects don't cancel - after all, they affect the motion of different objects.

The forces – no friction

The diagram shows the **horizontal** forces that act on the horse, the cart, and the earth. The convention for drawing the forces in the diagram is:

- The force is drawn as an arrow pointing in the direction of the force.
- The force is drawn on the object getting pushed or pulled.
- The force is labelled with the object doing the pushing or pulling.



For example: the red arrow labelled ‘cart’ is a force exerted **by** the cart **on** the horse; the blue arrow labelled ‘horse’ is a force exerted **by** the horse **on** the ground.

What are the Newton's third law force pairs?

The two forces coloured red in the diagram are a Newton's third law force pair – ‘horse pulls cart and ‘cart pulls horse’. They are equal in magnitude and opposite in direction.

The two forces coloured blue in the diagram are a Newton's third law force pair - ‘horse pushes ground’ and ‘ground pushes horse’. They are also equal in magnitude and opposite in direction.

Why does the cart accelerate?

Newton's second law says that an object accelerates if there is a net (unbalanced) force on it. Looking at the cart in the diagram above, you can see that there is just one force exerted on it – the force that the horse exerts on it. The cart accelerates because the horse pulls on it. The amount of acceleration equals the net force on the cart divided by its mass (Newton's second law).

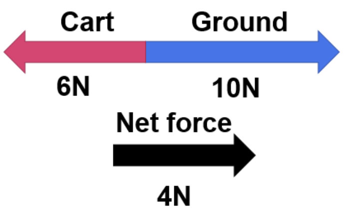
Worksheet J: Answers continued

Why does the horse accelerate?

There are two forces that push or pull on the horse in the diagram above. The cart pulls the horse backwards, and the ground pushes the horse forward. The net force is determined by the relative sizes of these two forces.

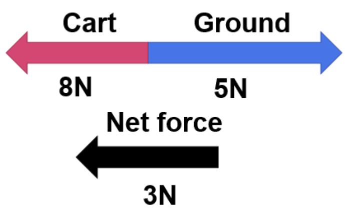
If the ground pushes harder on the horse than the cart pulls, there is a net force in the forward direction, and the horse accelerates forward.

Forward net force on the horse



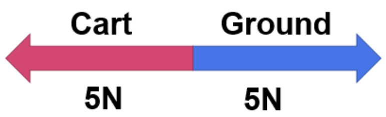
If the cart pulls harder on the horse than the ground pushes, there is a net force in the backward direction, and the horse accelerates backward. (This wouldn't happen on level ground, but it could happen on a hill...)

Backward net force on the horse



If the force that the cart exerts on the horse is the same size as the force that the ground exerts, the net force on the horse is zero, and the horse does not accelerate.

NO net force on the horse



In any case, the acceleration of the horse equals the net force on the horse divided by the horse's mass (Newton's second law).

Why does the ground push on the horse, anyway?

The force 'ground pushes horse' is the Newton's third law reaction force to 'horse pushes ground'. These two forces are exactly the same size. If the horse wants the ground to push him forward, he just needs to push backwards on the ground.

These two forces do not cancel because they act on different objects. The force 'ground pushes horse' tends to accelerate the horse, and the force 'horse pushes ground' tends to accelerate the ground.

Worksheet J: Answers continued

What about the ground?

Looking at the force diagram at the top of the worksheet, you see that there is one horizontal force pushing on the ground – the horse pushes on the ground. Therefore, there is a net force on the ground, so the ground should accelerate. Does it?

Of course it does. However the amount of acceleration equals the size of the net force divided by the mass of the Earth – and the mass of the earth is about 6×10^{24} kg. This means that the acceleration of the ground is much, much too small to notice.

Summary:

So, it is possible for horses to pull carts. It is true that the force that the horse exerts on the cart is the same size as the force that the cart exerts on the horse, but these forces do not combine to produce a zero net force. The force exerted on the cart (by the horse) affects the motion of the cart, and the force exerted on the horse affects the motion of the horse.

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