

Teaching Pack Mensuration – The circle

Cambridge IGCSE[™] Mathematics 0580

This *Teaching Pack* can also be used with the following syllabuses:

- Cambridge IGCSE™ (9–1) Mathematics 0980
- Cambridge IGCSE[™] International Mathematics 0607
- Cambridge O Level Mathematics 4024





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Contents

Introduction	. 4
Skill: Mensuration – The circle	. 5
Lesson 1 – Circumference of a circle	. 6
Lesson 2 – Area of a circle	. 7
Lesson 3 – Arc length and sector areas	. 8
Teacher's notes	. 9
Worksheets and answers	10

Icons used in this pack:			
	Lesson plan		
i	Teacher's notes		
	Lesson resources		

Introduction

This *Teaching Materials Pack* focuses on supporting learners to develop confidence and fluency with the formulas associate with the circumference and area of a circle. Also derive and use the formulas for the arc length and area of a sector.

The lessons presented here are designed for learners that are already familiar with the concept of a circle and the parts of a circle so they can apply this understanding to the circumference, arc length and sectors of circles.

It is expected that learners should already understand the concept of perimeter as a distance around a shape and area as the amount of space inside the boundary of a flat (2-dimensional) object such as a circle. They should also understand the concept of units in terms of measurements of distance and area (unit squared).

It would be useful if students could also multiply a whole number by a fraction both using a calculator and using written methods. It would also be useful if they could simplify fractions.

Important note

Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other skills.

This content is designed to give you and your learners the chance to explore mathematical skills. It is not intended as specific practice for exam papers.

This is one of a range of Teaching Packs. Each pack is based on one mathematical topic with a focus on specific mathematical techniques. The packs can be used in any order to suit your teaching sequence.

In this pack you will find the lesson plans and worksheets you will need to successfully complete the teaching of this topic.

Skill: Mensuration – The circle

This *Teaching Pack* links to the following syllabus content (see syllabus for detail):

- C5.2 Carry out calculations involving the circumference and area of a circle Solve simple problems involving the arc length and sector area as fractions of the circumference and area of a circle.
- E5.2 Carry out calculations involving the circumference and area of a circle. Solve problem involving the arc length and sector area as fractions of the circumference and area of a circle.

For assessments from 2025

• C5.3	Carry out calculations involving the circumference and area of a circle.
	Carry out calculations involving arc length and sector area as fractions of the
	circumference and area of a circle, where the sector angle is a factor of 360°.
• E5.3	Carry out calculations involving the circumference and area of a circle.
	Carry out calculations involving arc length and sector area as fractions of the
	circumference and area of a circle.

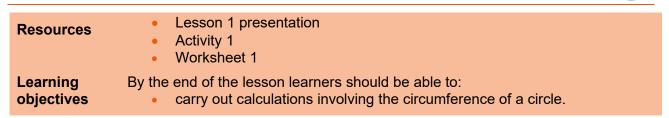
The pack covers mathematical skills, adapted from AO1: Demonstrate knowledge and understanding of mathematical techniques / Knowledge and understanding of mathematical techniques and AO2: Reason, interpret and communicate mathematically when solving problems / Analyse, interpret and communicate mathematically.

Before you begin

This *Teaching Pack* includes a **Teacher Introduction** video to which you should refer before using the resources in this pack. The video is available to watch in Resource Plus within the topic section relevant to this **Teaching Pack**.

The video introduces the resources available for teaching this topic, and explains how they can be used to successfully deliver the topic to your learners. In particular, the video highlights typical learner misconceptions and common errors this *Teaching Pack* will help you to overcome.

Lesson 1 – Circumference of a circle



Timin	gs Activity
	Starter / Introduction
min •••••	Slide 2 – 3 Use this activity to check learners' knowledge of the different parts of the circle. This activity can also be done in groups as a reading images activity by printing the slide without the boxes/answers and asking learners to reproduce the diagram. Using this approach, learners work in small groups and one person at a time comes up and looks at the image for a short period (approximately 30 seconds). They then go back to the group and try to explain to the group how to reproduce the image. They cannot touch or draw on the image themselves. It might be useful to provide a blank circle template for learners to use if approaching this activity in this way.
	Explain to learners what is meant by a minor arc or segment. Tell them we will be looking at arcs and sectors later in this series of lessons but for the first two lessons we will be focusing on the perimeter and the area of a circle.
	Main lesson
5 min	Slide 4 This fill in the gap activity is designed to test learners understanding of pi number. Complete this as a whole class activity, clicking to show the answers.
20 min	Slide 5 – <u>Activity 1</u> - you can use this activity to investigate the formulae for the circumference of a circle or you can go straight to presenting the formula using slides 6.
20 min	Work through the examples Slides 7 to 9. Tell learners it is generally useful to draw a diagram. Pause to give learners the opportunity to work out the answers independently before moving on to answers. The example on slide 8 introduces the use of $\frac{22}{7}$ as an approximation for π .
5 min	Plenary Summarise the learning in the lesson and introduce: Homework – <u>Worksheet 1</u>

Lesson 2 – Area of a circle

Timings

Resources	 Lesson 2 presentation Scissors Worksheet 2 Area of a circle
Learning objectives	By the end of the lesson learners should be able to:carry out calculations involving the area of a circle.

Activity

20 min	Slides 2 - 6 Area of a circle This practical activity gives learners the opportunity to see how the formula for the area of a circle can be approximated.				
10 min	Slide 7 demonstrates how the formula can be rearranged depending on the information that you are given in the question. Give learners the opportunity to do the rearrangements before animating the slides by clicking on the boxes. The final rearrangement may need some further explanation depending on the group you are working with.				
20 min	Slide 8 to 11 work through a series of examples that demonstrate how to use the different rearrangements depending on the information in the question.				
	Plenary				
10 min	What is the minimum information you need to be able to find the circumference and area of a circle? <i>The minimum information you need is either the length of the radius or the diameter.</i>				
	How would you go about finding the area of a circle if you know the circumference? The formula for area of a circle is πr^2 . The formula for the circumference is $2\pi r$. You can rearrange this second formula to make r the subject $r = \frac{c}{2\pi}$. You can then substitute this into the formula for the area. Area = $\pi (\frac{C^2}{2\pi}) = \frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}$.				
	Why is it easy to distinguish between the formulae for the circumference and area of a circle? The formula for the circumference is $2\pi r$ or πd . In either case it only uses r once, so it is a measure of length. The formula for area of a circle is πr^2 . In this case the length (the radius) is squared so this is a measure of area.				
	All circles are similar. Use what you know about the ratio of the circumference to the diameter of a circle to help you explain this. The ratio of the circumference to the diameter for all circles will always approximate to π . Therefore, all circles are similar.				
	Here we are the Alford as the				

Homework – Worksheet 2

Lesson 3 – Arc length and sector areas

Resources	 Lesson 3 presentation Worksheet 3 Activity 2 Constructing Polygons
Learning objectives	 By the end of the lesson should be able to: solve simple problems involving the arc length and sector area as fractions of the circumference and area of a circle

Timin	gs Activity
5 min	Slide 2 Remind learners what the sector, arc and radius of a circle are.
20 min	Slide 3 Use example to explain how you calculate the fraction of the circle enclosed in a sector when you know the angle at the centre of that sector.
	Slides 4 Use to show how you can calculate the length of the arc once you know the fraction of the whole circle enclosed in the arc.
10	Slide 5 extends this work to demonstrate how you work out the area of the sector again using a specific example.
min	Slide 6 presents the general formula. Give learners the opportunity to fill in the boxes before you animate the slide.
20 min	Slide 7 and 8 Work through a series of examples that demonstrate how to calculate arc length and area of a sector.
	Activity 2 Constructing Polygons (Extension activity) This activity will also prepare students for the first activity in the circle theorem series lessons.
•••••	Plenary
5 min	Summarise the learning in the lesson and introduce:
` O •⊕•O`	Homework – <u>Worksheet 3</u>

Teacher's notes



Key words / concepts you could highlight during the lesson, or have pre-taught before the lesson:

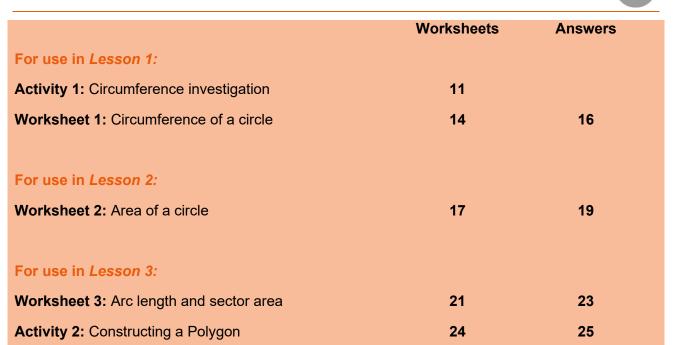
Students need a basic understanding of perimeter before tackling lesson 1.

Key words

- Perimeter
- Circumference
- Area
- Radius
- Diameter
- Arc
- Sector
- Chord

The time required for Lesson 1 will depend on whether the activity is used and how quickly the students become competent and fluent with the formula for the circumference. Depending on these two points you may want to combine lessons1 and 2 as a single lesson.

Worksheets and answers



Activity 1

<u>Task</u>

- 1. Work out the diameters for each of the circles on the next two pages.
- 2. Using a piece of string measure the circumferences of each circle.
- 3. Fill in the table
- 4. Complete the table by calculating $\frac{c}{d}$ for each circle

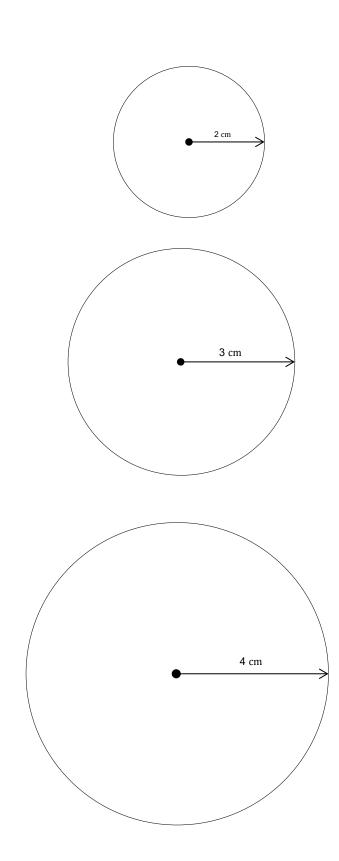
Radius r (cm)	Diameter d (cm)	Circumference C (cm)	$\frac{C}{d}$
2			
3			
4			
5			
6			

Conclusion

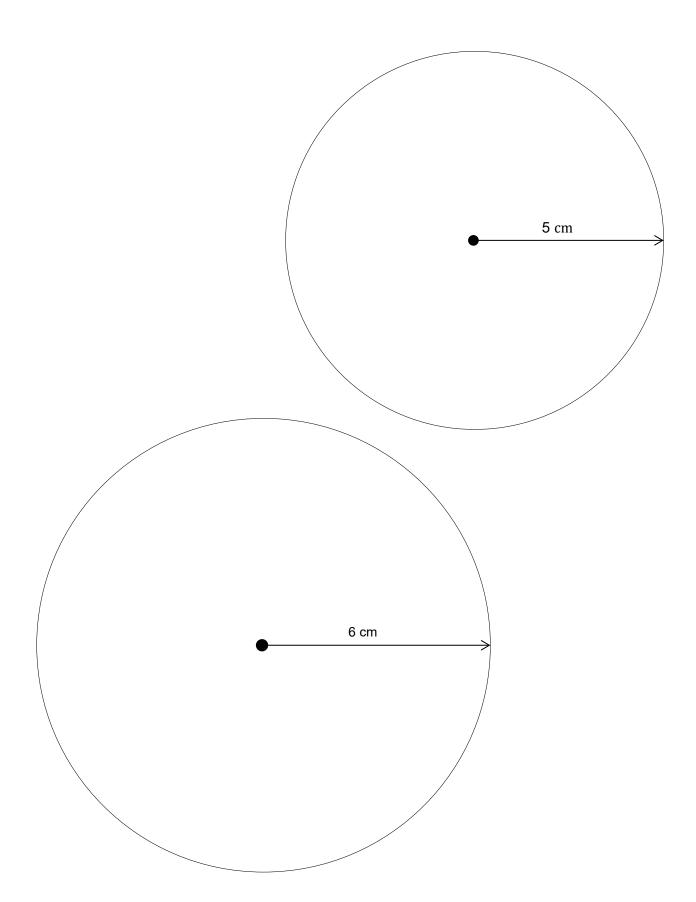
 $\frac{c}{d} =$

C =

Activity 1 – continued



Activity 1 – continued



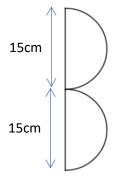
Worksheet 1

Use this sheet to become confident and fluent with using the formula for the circumference of a circle to answer a range of problems.

- 1. Fill in the gaps
 - A circle is a set of points that are at a given distance from its.....
 - The..... is the distance round the circle
 - The..... is the distance from the centre to the.....
- 2. A circular pool has radius 8 m. Calculate the circumference of the pool.

3. A circle has a radius of 6.4 cm. Work out the circumference of the circle.

A letter B is made out of a piece of wire.
 It has a straight side and to equal semicircles.
 How much wire is required to make this letter?

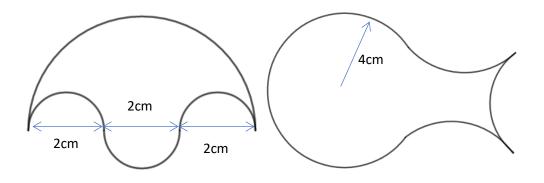


Worksheet 1 - continued

Extension questions

- 5. Abjit's bicycle wheel has a radius of 29 cm.
 - a) Calculate the circumference of the wheel. Give your answer correct to 1 decimal place.
 - b) Calculate the number of complete turns the wheel makes when travelling 500 m.

6. All curves in the left-hand figure semicircles. All curves in the right-hand figure are quarter circles or three-quarter circles. Calculate the perimeter of each shape.



Worksheet 1 – Answers

- 1. Fill in the gaps
 - A circle is a set of points that are at a given distance from its centre.
 - The circumference is the distance round the circle
 - The radius is the distance from the centre to the circumference.
- 2. $2 \times 8 \times \pi = 16 \times \pi = 50.24 \, cm$
- 3. $2 \times 6.4 \times \pi = 12.8 \times \pi = 40.192 \, cm$
- 4. Circumference of one semicircle = $\left(\frac{2 \times 7.5 \times \pi}{2}\right) + 15 = 38.55 \ cm$

Circumference of letter B = $2 \times 38.55 = 77.1 \, cm$

77.1 cm wire is required to make this letter.

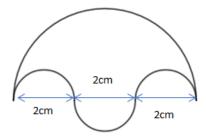
5. (a) $2 \times 29 \times \pi = 58 \times \pi = 182.1 \, cm$

(b) $500 \div 1.821 = 274.5$ The wheel will make 274 complete turns.

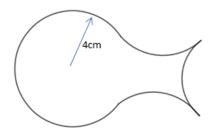
6. $\frac{6 \times \pi}{2} = 9.42 \ cm$ Circumference of the big semicircle

 $\frac{2 \times \pi}{2} = 3.14 \, cm$ Circumference of one small semicircle

Circumference of the shape = $9.42 + 3 \times 3.14 = 18.84$ cm



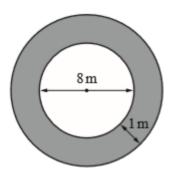
$$\frac{3}{4} \times 2 \times 4 \times \pi = 18.84 \, cm$$
 Circumference of big circle
$$\frac{1}{4} \times 2 \times 4 \times \pi = 6.28 \, cm$$
 Circumference of small circle
Circumference of the shape = 37.68 cm





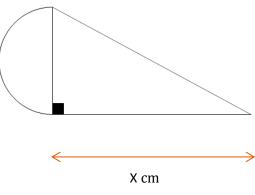
Worksheet 2

1. Work out the shaded area in this diagram. Give your answer in terms of π .



- 2. The diagram shows a shape made from a semicircle, radius 8 cm and a right-angled triangle with the hypotenuse of 25 cm.
 - a) Find x.
 - b) Calculate the area of the shape.

Give your answers correct to one decimal place.



3.

- a) A small Pizza has a diameter of 25 cm. What is the surface area of the top of the pizza?
- b) A large pizza has twice the surface area of the small one. What is the diameter of the large pizza?

Give your answers correct to 2 decimal places.

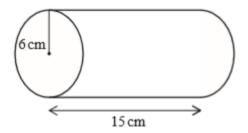
4. A door is in the shape of a rectangle with a semi-circular arch on top. The rectangular part is 2m high and the door is 90cm wide. What is the area of the door? Give your answers correct to 2 decimal places.

Worksheet 2 - continued

Extension questions

1. A solid cylinder has length 15 cm and radius 6 cm.

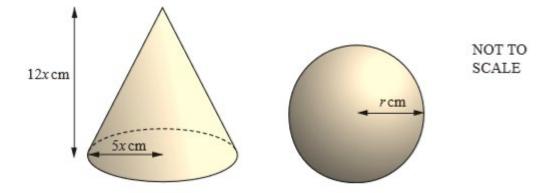
Calculate the total surface area of the cylinder. Give your answer in terms of $\boldsymbol{\pi}.$



2. Running track. The inside perimeter of a running track has the shape below. Both straights are 80m. Both ends are identical semicircles. The track is 8 m wide. What is the area of the smallest rectangular field needed to contain it?



3. The diagram below shows a solid circular cone and a solid sphere.

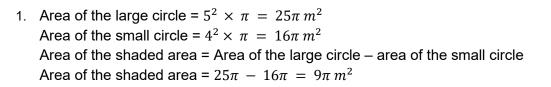


The cone has radius 5x cm and height 12x cm. The sphere has radius r cm. The cone has the same **total** surface area as the sphere.

Show that $r^2 = \frac{45}{2}x^2$.

[The curved surface area, *A*, of a cone with radius r and slant height I is $A = \pi r l$.] [The surface area, *A*, of a sphere with radius r is $A = 4\pi r^2$.]

Worksheet 2 – Answers



$$162 + x2 = 252$$

$$x2 = 625 - 256 = 369$$

$$x = \sqrt{369} = 19.2 cm$$

(b)

Area of the triangle = $\frac{16 \times 19.2}{2}$ = 153.6 cm²

Area of the semicircle = $\frac{\pi \times 8^2}{2} = 100.5 \ cm^2$

Area of the shape = $153.6 + 100.5 = 254.1 \, cm^2$

3. (a)

 $r = 25 \div 2 = 12.5 cm$ Area of the small pizza = $\pi \times 12.5^2 = 490.63 cm^2$

(b) Area of the large pizza = $2 \times 490.63 = 981.26 \ cm^2$ Radius of the large pizza = $\sqrt{\frac{a}{\pi}} = \sqrt{\frac{981.26}{\pi}} = 17.68 \ cm$ Diameter of the large pizza = $35.36 \ cm$

4. Area of the rectangle = $2 \times 0.9 = 1.8 m^2$

Area of the semicircle = $\frac{\pi \times 0.45^2}{2} = 0.32 m^2$



Worksheet 2 – Answers

Extension questions

- 1. Area of the cylinder = $2\pi r^2 + 2\pi rh$ Area of the cylinder = $2 \times \pi \times 36 + 2 \times \pi \times 6 \times 15 = 72 \times \pi + 180 \times \pi = 252\pi cm^2$
- 2. Area of the smallest field is = $88 \times 8 = 704 m^2$
- 3. Slant height = $\sqrt{(5x)^2 + (12x)^2} = 13x$

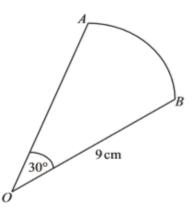
$$\pi \times (5x)^2 + \pi \times (5x) \times (13x) = 4 \times \pi \times r^2$$

$$25x^2 + 65x^2 = 4r^2$$

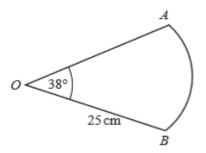
$$r^2 = \frac{90x^2}{4} = \frac{45x^2}{2}$$

Worksheet 3

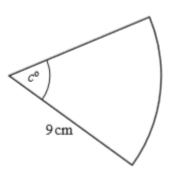
1. OAB is a sector of a circle with radius 9 cm and centre O. Calculate the area of this sector. Give your answers in terms of π .



2. The diagram shows a sector of a circle, centre *O*, radius 25 cm. Calculate the length of the arc AB. Give your answer correct to 4 significant figures.

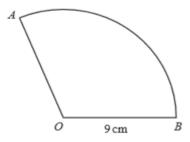


 The perimeter of this sector of a circle is 28.2 cm. Calculate the value of c.



Worksheet 3 - continued

4. AB is an arc of a circle, centre O, radius 9 cm. The length of the arc AB is 6π cm. The area of sector AOB is $k\pi$ cm². Find the value of k.



Worksheet 3 – Answers

1.
$$A = \frac{30}{360} \times \pi \times 81 = \frac{27}{40} \pi \, cm^2$$

- 2. Arc length = $\frac{38}{360} \times 2 \times \pi \times 25 = 16.58 \, cm$
- 3. Perimeter of the sector = 9 + 9 + $\frac{c}{360}$ × 2 × π × 9 = 64.95°
- 4. Length of $AB = \frac{Angle \ AOB}{360} \times 2 \times \pi \times 9 = 6 \pi$

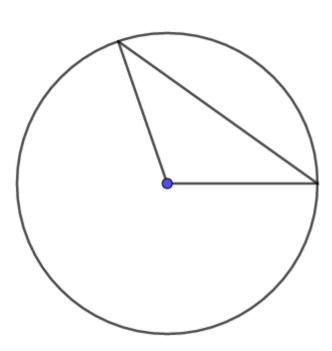
Angle AOB = 120°

Area of Sector $AOB = \frac{120}{360} \times \pi \times 9^2 = k\pi$

$$k = 27 cm$$

Activity 2

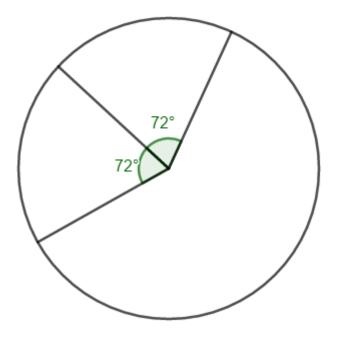




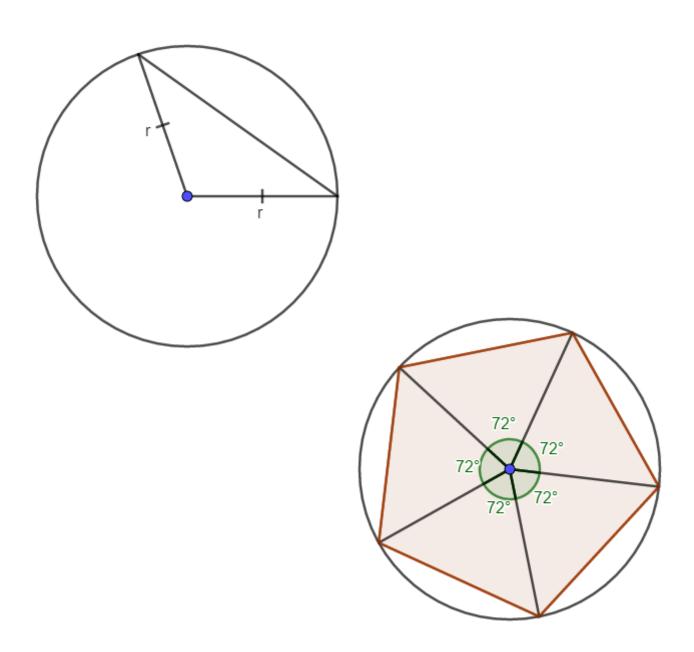
Look at this diagram. It is constructed by using two radii together with a chord of the circle which links the points were two radii touch the circumference of the circle. What type of triangle is formed how can you justify this?

Use this diagram to demonstrate how if you continue to form equal sectors around the circle this can help you to draw a regular polygon. The resulting polygon is called an inscribed polygon because it is drawn inside the circle.

Explain why if you draw a chord the same length as the radius of the circle the inscribed polygon you will get will be a hexagon.



Activity 2 – Constructing Polygons Exemplar Solution



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