Cambridge Assessment International Education

Teaching Pack

Number - Surds

Cambridge IGCSE™

Mathematics 0580

Version 1.0

Cambridge Pathway

Contents

Icons used in this pack:	
	**Lesson plan**
	**Teacher's notes**
	**Lesson resources**

Introduction: Surds

This *Teaching Materials Pack* focuses on supporting learners to develop a confidence and fluency in working with surds in a wide range of different calculations and contexts. It starts with relatively simple simplifying problems and goes through all of the standard mathematical operators, how to deal with brackets and finishes with rationalising the denominators of fractions.

The lesson presented here is designed for learners that have little to no experience of surds up until now. They will often be aware of how the square root symbol works in relation to square numbers, and will have used square roots in a calculator in calculations such as Pythagoras' theorem.

It is expected that learners should already understand how to multiply out both single and double brackets, as well as cancel down fractions by dividing by a common factor.

Important note
Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other skills.

This content is designed to give you and your learners the chance to explore mathematical skills. It is not intended as specific practice for exam papers.

This is one of a range of Teaching Packs. Each pack is based on one mathematical topic with a focus on specific mathematical techniques. The packs can be used in any order to suit your teaching sequence.

In this pack you will find the lesson plans and worksheets you will need to successfully complete the teaching of this topic.

Syllabus links

This *Teaching Pack* links to the following syllabus content (see syllabus for detail):

- E1.18 Understand and use surds, including simplifying expressions
  Rationalise the denominator

The pack covers mathematical skills, adapted from **AO1: Demonstrate knowledge and understanding of mathematical techniques** and **AO2: Reason, interpret and communicate mathematically when solving problems**.

Previous Knowledge

Before you begin

This *Teaching Pack* includes a **Teacher Introduction** video to which you should refer before using the resources in this pack. The video is available to watch in Resource Plus within the topic section relevant to this **Teaching Pack**.

The video introduces the resources available for teaching this topic, and explains how they can be used to successfully deliver the topic to your learners. In particular, the video highlights typical learner misconceptions and common errors this *Teaching Pack* will help you to overcome.

Lesson 1 – Multiplying Surds

Resources	• Lesson 1 PowerPoint presentation • Worksheet 1
Learning objectives	By the end of the lesson learners should be able to: • Understand what a surd is, and why we use them • Multiply simple surds together • Multiply surds with coefficients together

Timings	Activity
5 mins	**Starter / Introduction** **Slides 2 and 3 –** Introduce the idea of a surd, and reinforce why it is important to be able to work with them non-calculator.
	Main lesson
10 mins	**Slides 4 and 5** – Show how the basics of multiplying surds together works. There are 3 worked examples for the teacher to go through, and then 3 for the learners to work on afterwards to check for understanding.
10 mins	**Slides 6, 7 and 8** – Extend the rules to understand how to multiply surds with numbers, and also how to multiply two surds together when both have a coefficient. (Students typically find this difficult at first). Then 3 for the learners to work on afterwards to check for understanding. (Don't worry about simplifying the last one, that is covered next lesson)
Rest of the lesson	Work on worksheet #1

Lesson 2 – Simplifying Surds

Resources	• Lesson 2 PowerPoint • Worksheet 2
Learning objectives	By the end of the lesson learners should be able to: • Simplify surds • Multiply surds, then simplify the result afterwards.

Timings	Activity
10 mins	**Starter / Introduction** Recap of surds questions from the previous lesson to get the students back up to speed.
	Main lesson
5 mins	**Slide 3** – Quick introduction to the idea that in order to simplify a surd you need the number inside the surd to be as small as possible.
5 mins	**Slide 4** – Practising the key skill required – finding the biggest square factor of a number.
10 mins	**Slides 5 and 6** – Two teacher led examples of how to simplify a surd, with step by step instructions modelled. The next slide has 3 questions for the students to practice the skill themselves, with answers provided.
10 mins	**Slides 7 and 8** – This links the simplifying from this lesson with the multiplication from the previous one. Sometimes after multiplying surds together you can then simplify the result. There are 2 examples of this, the second one being quite difficult for students as both surds contain coefficients. There are then a further 3 for the students to practice on.
Rest of the lesson	Work through worksheet #2

Lesson 3 – Brackets and Rationalising Denominators

Resources	• Lesson 3 PowerPoint • Worksheet 3
Learning objectives	By the end of the lesson: • **all learners** will be able to expand a single bracket • **most learners** will be able to rationalise denominators where the denominator is a single term and expand double brackets • **some learners** will multiply double brackets containing multiple surds, with coefficients

Timings	Activity
	Starter / Introduction
1 min	**Slide 2 –** Quick explanation that you will be looking to incorporate surds in to lots of other mathematical techniques
	Main lesson
5 mins	**Slides 3 and 4** – Work on adding and subtracting surds. Students often incorrectly assume that you can 'just add the numbers beneath the surds together'. Keep reminding them that you must treat them like algebra. There are 4 questions for the students to practise on slide 4.
10/15 mins	**Slides 5,6 and 7 –** Work on how to multiply both single and double brackets that contain surds. The added complexity of multiplying the surds will slow the students down, so be prepared to allow more time. There are 3 questions for the students to practise on slide 7.
10/15 mins	**Slides 8,9, 10 and 11** – Introduce the concept of rationalising a denominator with one term in the denominator only. This is essentially dividing with surds. Students initially struggle to understand *why* you do the multiplication with the fraction. They are generally quite good at actually carrying them out. On slide 11 there are 3 questions for the students to practise the skill.

Lesson 4 – Harder Rationalising Denominators

Resources	• Lesson 4 PowerPoint presentation • Worksheet 4
Learning objectives	By the end of the lesson: • **all learners** will recognise what to multiply the top and bottom of the fraction by • **most learners** will rationalise a two-term denominator when the numerator is a '1' • **some learners** will rationalise a two-term denominator when the numerator isn't a '1'

Timings	Activity
	Starter / Introduction
10 mins	Recap of surds questions from the previous 3 lessons to get the students back up to speed.
	Main lesson
5 mins	**Slide 3** – Quick investigation into multiplying out two brackets containing at least one surd, where the second bracket is the same as the first except the sign is reversed (+ to - , or – to +). Get the students to work through these questions. The main point they should spot is that the result always ends up without a surd. This will be important for the next technique.
10 mins	**Slides 4,5 and 6** – Show the students the two fully worked examples on rationalising the denominator when there are two terms in the denominator. There is a quiz in the middle allowing the students to practice picking the correct fraction to multiply by.
5/10 mins	**Slides 7 and 8** – There are two examples for the students to work through themselves, one at a time. The answers are revealed step-by-step to help students pinpoint any inaccuracies they make, as this is a longer calculation with many places to make errors along the way.

Teacher's notes

Key words / concepts you could highlight during the lesson, or have pre-taught before the lesson:

Key words

- Surd
- Rational numbers
- Irrational numbers
- Expand
- Coefficient

There is a fair amount of content to cover here. You might find that some parts require you to take longer than the allotted time. This would be particularly true for lesson 3, where you could easily use a lesson and a half on practicing those techniques.

Lesson resources

Worksheet 1: Multiplying Surds
Worksheet 1: Multiplying Surds Answers

Worksheet 2: Simplifying Surds
Worksheet 2: Simplifying Surds Answers

Worksheet 3: Brackets and Rationalising Denominators
Worksheet 3: Brackets and Rationalising Denominators Answers

Worksheet 4: Harder Rationalising Denominators
Worksheet 4: Harder Rationalising Denominators Answers

Worksheet 1: Multiplying Surds

1. Calculate the following. Leave your answers in **surd** form.

   a) $\sqrt{3} \times \sqrt{2}$

   b) $\sqrt{3} \times \sqrt{3}$

   c) $\sqrt{3} \times \sqrt{4}$

   d) $\sqrt{3} \times \sqrt{5}$

   e) $\sqrt{5} \times \sqrt{3}$

   f) $\sqrt{5} \times \sqrt{5}$

   g) $\sqrt{7} \times \sqrt{2}$

   h) $\sqrt{7} \times \sqrt{20}$

   i) $\sqrt{7} \times \sqrt{200}$

   j) $\sqrt{6} \times \sqrt{1}$

   k) $\sqrt{5} \times \sqrt{1}$

   l) $\sqrt{3} \times \sqrt{1}$

   m) $\sqrt{3} \times \sqrt{2} \times \sqrt{5}$

   n) $\sqrt{3} \times \sqrt{2} \times \sqrt{10}$

   What do you notice about your answers to: b and f? e and d?

2. Find the area of this rectangle. Give your answer as a **surd**.

$\sqrt{6}\ cm$

$\sqrt{13}\ cm$

3. Calculate the following. Leave your answers in **surd** form.

   a) $2\sqrt{3} \times \sqrt{2}$

   b) $4\sqrt{3} \times \sqrt{8}$

   c) $7\sqrt{3} \times 7\sqrt{11}$

   d) $\sqrt{10} \times 7\sqrt{5}$

   e) $8\sqrt{7} \times 2$

   f) $3\sqrt{30} \times 2\sqrt{20}$

   g) $(11\sqrt{5})^2$

   h) $2\sqrt{5} \times 2\sqrt{3} \times 2\sqrt{8}$

4. Extension

   a) Work out $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$

   b) Use this to help you to work out $(\sqrt{2})^6$

   c) Now try $(3\sqrt{2})^6$

Worksheet 1: Multiplying Surds ANSWERS

1. Calculate the following. Leave your answers in **surd** form.

a) $\sqrt{3} \times \sqrt{2}$ $\sqrt{6}$

b) $\sqrt{3} \times \sqrt{3}$ 3

c) $\sqrt{3} \times \sqrt{4}$ $\sqrt{12}$

d) $\sqrt{3} \times \sqrt{5}$ $\sqrt{15}$

e) $\sqrt{5} \times \sqrt{3}$ $\sqrt{18}$

f) $\sqrt{5} \times \sqrt{5}$ 5

g) $\sqrt{7} \times \sqrt{2}$ $\sqrt{14}$

h) $\sqrt{7} \times \sqrt{20}$ $\sqrt{140}$

i) $\sqrt{7} \times \sqrt{200}$ $\sqrt{1400}$

j) $\sqrt{6} \times \sqrt{1}$ $\sqrt{6}$

k) $\sqrt{5} \times \sqrt{1}$ $\sqrt{5}$

l) $\sqrt{3} \times \sqrt{1}$ $\sqrt{3}$

m) $\sqrt{3} \times \sqrt{2} \times \sqrt{5}$ $\sqrt{30}$

n) $\sqrt{3} \times \sqrt{2} \times \sqrt{10}$ $\sqrt{60}$

What do you notice about your answers to: b and f? $\sqrt{a} \times \sqrt{a} = a$

d and e? Order doesn't matter, same outcome

2. Find the area of this rectangle. Give your answer as a **surd**.

$\sqrt{6}\ cm$

$\sqrt{13}\ cm$

$\sqrt{68}$

3. Calculate the following. Leave your answers in **surd** form.

a) $2\sqrt{3} \times \sqrt{2}$ $2\sqrt{6}$

b) $4\sqrt{3} \times \sqrt{8}$ $4\sqrt{24}$

c) $7\sqrt{3} \times 7\sqrt{11}$ $49\sqrt{33}$

d) $\sqrt{10} \times 7\sqrt{5}$ $7\sqrt{50}$

e) $8\sqrt{7} \times 2$ $16\sqrt{7}$

f) $3\sqrt{30} \times 2\sqrt{20}$ $6\sqrt{600}$

g) $(11\sqrt{5})^2$ 605

h) $2\sqrt{5} \times 2\sqrt{3} \times 2\sqrt{8}$ $8\sqrt{120}$

4. Extension

a) Work out $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$ 4

b) Use this to help you to work out $(\sqrt{2})^6$ 8

c) Now try $(3\sqrt{2})^6$ $3^6 \times 8 = 5832$

Worksheet 2: Simplifying Surds

1. Simplify the following.

   a) $\sqrt{48}$
   b) $\sqrt{63}$
   c) $\sqrt{12}$
   d) $\sqrt{40}$
   e) $\sqrt{125}$
   f) $\sqrt{300}$
   g) $\sqrt{8}$
   h) $\sqrt{80}$
   i) $\sqrt{108}$
   j) $\sqrt{1000}$

2. Which of the following has the largest value:

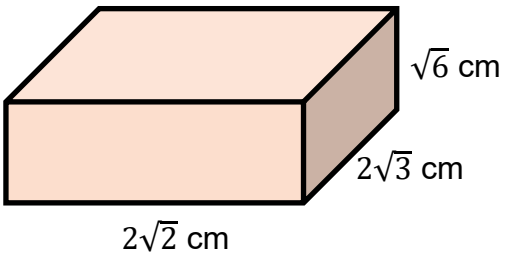
   $16\sqrt{3}$ $3\sqrt{108}$ $2\sqrt{147}$

3. Calculate the following. Leave your answers in **simplified surd** form.

   a) $\sqrt{5} \times \sqrt{10}$
   b) $\sqrt{8} \times \sqrt{6}$
   c) $\sqrt{10} \times \sqrt{15}$
   d) $2\sqrt{12} \times \sqrt{3}$
   e) $3\sqrt{6} \times \sqrt{18}$
   f) $4\sqrt{7} \times 3\sqrt{14}$
   g) $2\sqrt{3} \times 3\sqrt{6} \times 4\sqrt{5}$
   h) $(4\sqrt{5})^2$

4. Extension: Using the cuboid, calculate the following. Leave your answers as simplified surds.

   a) Volume b) Surface area c) Combined length of all edges



Worksheet 2: Simplifying Surds ANSWERS

1. Simplify the following.

a)	$\sqrt{48}$	$4\sqrt{3}$	f)	$\sqrt{300}$	$10\sqrt{3}$
b)	$\sqrt{63}$	$3\sqrt{7}$	g)	$\sqrt{8}$	$2\sqrt{2}$
c)	$\sqrt{12}$	$2\sqrt{3}$	h)	$\sqrt{80}$	$4\sqrt{5}$
d)	$\sqrt{40}$	$2\sqrt{10}$	i)	$\sqrt{108}$	$6\sqrt{3}$
e)	$\sqrt{125}$	$5\sqrt{5}$	j)	$\sqrt{1000}$	$10\sqrt{10}$

2. Which of the following has the largest value:

$16\sqrt{3}$ $3\sqrt{108}$ $2\sqrt{147}$

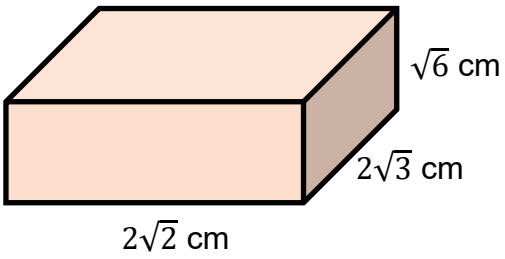
$3\sqrt{108}$

3. Calculate the following. Leave your answers in **simplified surd** form.

a)	$\sqrt{5} \times \sqrt{10}$	$5\sqrt{2}$	e)	$3\sqrt{6} \times \sqrt{18}$	$18\sqrt{3}$
b)	$\sqrt{8} \times \sqrt{6}$	$4\sqrt{3}$	f)	$4\sqrt{7} \times 3\sqrt{14}$	$84\sqrt{2}$
c)	$\sqrt{10} \times \sqrt{15}$	$5\sqrt{6}$	g)	$2\sqrt{3} \times 3\sqrt{6} \times 4\sqrt{5}$	$72\sqrt{10}$
d)	$2\sqrt{12} \times \sqrt{3}$	12	h)	$(4\sqrt{5})^2$	80

4. Extension: Using the cuboid, calculate the following. Leave your answers as simplified surds.

a) Volume b) Surface area c) Combined length of all edges



Volume = 24 cm^3

Surface Area = $12\sqrt{2} + 8\sqrt{3} + 8\sqrt{6}$ cm^2

Edges = $8\sqrt{2} + 8\sqrt{3} + 4\sqrt{6}$ cm

Worksheet 3: Brackets and Rationalising Denominators

1. Calculate the following. Leave your answers in simplified **surd** form.

   a) $\sqrt{75} + \sqrt{27}$

   b) $\sqrt{45} + \sqrt{80}$

   c) $3\sqrt{3} + \sqrt{48} - \sqrt{75}$

   d) $\sqrt{12} + \sqrt{27} - \sqrt{5}$

   e) $\sqrt{20} + \sqrt{80}$

   f) $\sqrt{28} + 3\sqrt{63} - \sqrt{7}$

2. Find the area **and** perimeter of this rectangle. Give your answer as a simplified **surd**.

$\sqrt{45}\ cm$

$\sqrt{80}\ cm$

3. Calculate the following. Leave your answers in **surd** form.

   a) $\sqrt{3}(2 + \sqrt{2})$

   b) $\sqrt{5}(3 - \sqrt{5})$

   c) $\sqrt{2}(4 - \sqrt{11})$

   d) $(4 + \sqrt{2})(3 - \sqrt{5})$

   e) $(4 + \sqrt{5})(2 + \sqrt{5})$

   f) $(4 + \sqrt{3})(2 - \sqrt{3})$

   g) $(11 + \sqrt{5})^2$

   h) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{5})$

4. Rationalise the denominators of the following. Simplify your answers where possible.

   a) $\frac{1}{\sqrt{17}}$

   b) $\frac{2}{\sqrt{2}}$

   c) $\frac{3}{\sqrt{3}}$

   d) $\frac{10}{\sqrt{5}}$

   e) $\frac{8}{\sqrt{2}}$

   f) $\frac{4}{\sqrt{12}}$

Worksheet 3: Brackets and Rationalising Denominators ANSWERS

1. Calculate the following. Leave your answers in simplified **surd** form.

a) $\sqrt{75} + \sqrt{27}$ $8\sqrt{3}$

b) $\sqrt{45} + \sqrt{80}$ $7\sqrt{5}$

c) $3\sqrt{3} + \sqrt{48} - \sqrt{75}$ $2\sqrt{3}$

d) $\sqrt{12} + \sqrt{27} - \sqrt{5}$ $5\sqrt{3} - \sqrt{5}$

e) $\sqrt{20} + \sqrt{80}$ $6\sqrt{5}$

f) $\sqrt{28} + 3\sqrt{63} - \sqrt{7}$ $10\sqrt{7}$

2. Find the area **and** perimeter of this rectangle. Give your answer as a simplified **surd**.

$\sqrt{45}\ cm$

$\sqrt{80}\ cm$

$60cm^2$ $14\sqrt{5}\ cm$

3. Calculate the following. Leave your answers in **surd** form.

a) $\sqrt{3}(2 + \sqrt{2})$ $2\sqrt{3} + \sqrt{6}$

b) $\sqrt{5}(3 - \sqrt{5})$ $3\sqrt{5} - 5$

c) $\sqrt{2}(4 - \sqrt{11})$ $4\sqrt{2} - \sqrt{22}$

d) $(4 + \sqrt{2})(3 - \sqrt{5})$ $12 - 4\sqrt{5} + 3\sqrt{2} - \sqrt{10}$

e) $(4 + \sqrt{5})(2 + \sqrt{5})$ $13 + 6\sqrt{5}$

f) $(4 + \sqrt{3})(2 - \sqrt{3})$ $5 - 2\sqrt{3}$

g) $(11 + \sqrt{5})^2$ $126 + 22\sqrt{5}$

h) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{5})$ $5\sqrt{5}$

4. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{1}{\sqrt{17}}$ $\frac{\sqrt{17}}{17}$

b) $\frac{2}{\sqrt{2}}$ $\sqrt{2}$

c) $\frac{3}{\sqrt{3}}$ $\sqrt{3}$

d) $\frac{10}{\sqrt{5}}$ $2\sqrt{5}$

e) $\frac{8}{\sqrt{2}}$ $4\sqrt{2}$

f) $\frac{4}{\sqrt{12}}$ $\frac{2\sqrt{3}}{3}$

Worksheet 4: Harder Rationalising Denominators

1. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{1}{\sqrt{2}}$

b) $\frac{1}{\sqrt{19}}$

c) $\frac{\sqrt{3}}{\sqrt{15}}$

d) $\frac{\sqrt{6}}{\sqrt{48}}$

e) $\frac{7}{\sqrt{63}}$

f) $\frac{\sqrt{12}}{\sqrt{156}}$

2. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{1}{1+\sqrt{2}}$

b) $\frac{1}{1-\sqrt{2}}$

c) $\frac{4}{3+\sqrt{5}}$

d) $\frac{10}{2+\sqrt{5}}$

e) $\frac{1}{1+\sqrt{3}}$

f) $\frac{1}{3-\sqrt{7}}$

2. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{6\sqrt{3}}{\sqrt{8}-\sqrt{7}}$

b) $\frac{\sqrt{7}}{\sqrt{7}-\sqrt{3}}$

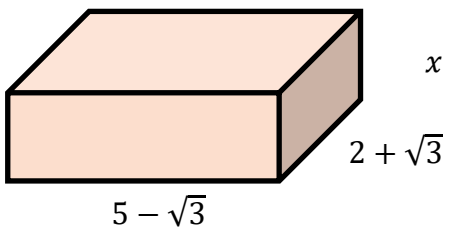
c) $\frac{7}{3+\sqrt{7}}$

d) $\frac{3-\sqrt{2}}{6-\sqrt{5}}$

e) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}$

f) $\frac{2}{(3-\sqrt{2})^2}$

4. Extension: The volume of the cuboid is 2 m^3. Work out the value of the missing side, giving your answer as a fully simplified surd with a rationalised denominator.



Worksheet 4: Harder Rationalising Denominators ANSWERS

1. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{2}}{2}$

b) $\frac{1}{\sqrt{19}}$ $\frac{\sqrt{19}}{19}$

c) $\frac{\sqrt{3}}{\sqrt{15}}$ $\frac{\sqrt{5}}{5}$

d) $\frac{\sqrt{6}}{\sqrt{48}}$ $\frac{\sqrt{2}}{4}$

e) $\frac{7}{\sqrt{63}}$ $\frac{\sqrt{7}}{3}$

f) $\frac{\sqrt{12}}{\sqrt{156}}$ $\frac{\sqrt{13}}{13}$

2. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{1}{1+\sqrt{2}}$ $-1+\sqrt{2}$

b) $\frac{1}{1-\sqrt{2}}$ $-1-\sqrt{2}$

c) $\frac{4}{3+\sqrt{5}}$ $3-\sqrt{5}$

d) $\frac{10}{2+\sqrt{5}}$ $-20+10\sqrt{5}$

e) $\frac{1}{1+\sqrt{3}}$ $\frac{-1+\sqrt{3}}{2}$

f) $\frac{1}{3-\sqrt{7}}$ $\frac{3+\sqrt{7}}{2}$

2. Rationalise the denominators of the following. Simplify your answers where possible.

a) $\frac{6\sqrt{3}}{\sqrt{8}-\sqrt{7}}$ $6\sqrt{21}+12\sqrt{6}$

b) $\frac{\sqrt{7}}{\sqrt{7}-\sqrt{3}}$ $\frac{7+\sqrt{21}}{4}$

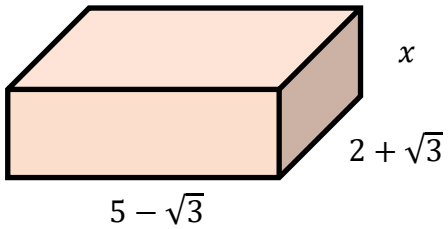
c) $\frac{7}{3+\sqrt{7}}$ $\frac{21-7\sqrt{7}}{2}$

d) $\frac{3-\sqrt{2}}{6-\sqrt{5}}$ $\frac{3+\sqrt{7}}{2}$

e) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}$ $\frac{9-\sqrt{77}}{2}$

f) $\frac{2}{(3-\sqrt{2})^2}$ $\frac{22+12\sqrt{2}}{49}$

4. Extension: The volume of the cuboid is 2 m^3. Work out the value of the missing side, giving your answer as a fully simplified surd with a rationalised denominator.



$\frac{7-3\sqrt{3}}{11}$

Cambridge Assessment International Education
The Triangle Building, Shaftsbury Road, Cambridge, CB2 8EA, United Kingdom
t: +44 1223 553554
e: **info@cambridgeinternational.org** **www.cambridgeinternational.org**