

Teaching Pack

Brownian Motion

Cambridge International AS & A Level
Physics 9702

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Icons used in this pack:



Briefing lesson



Lab lesson



Debriefing lesson

Introduction

This pack will help you to develop your learners' experimental skills as defined by assessment objective 3 (AO3 Experimental skills and investigations) in the course syllabus.

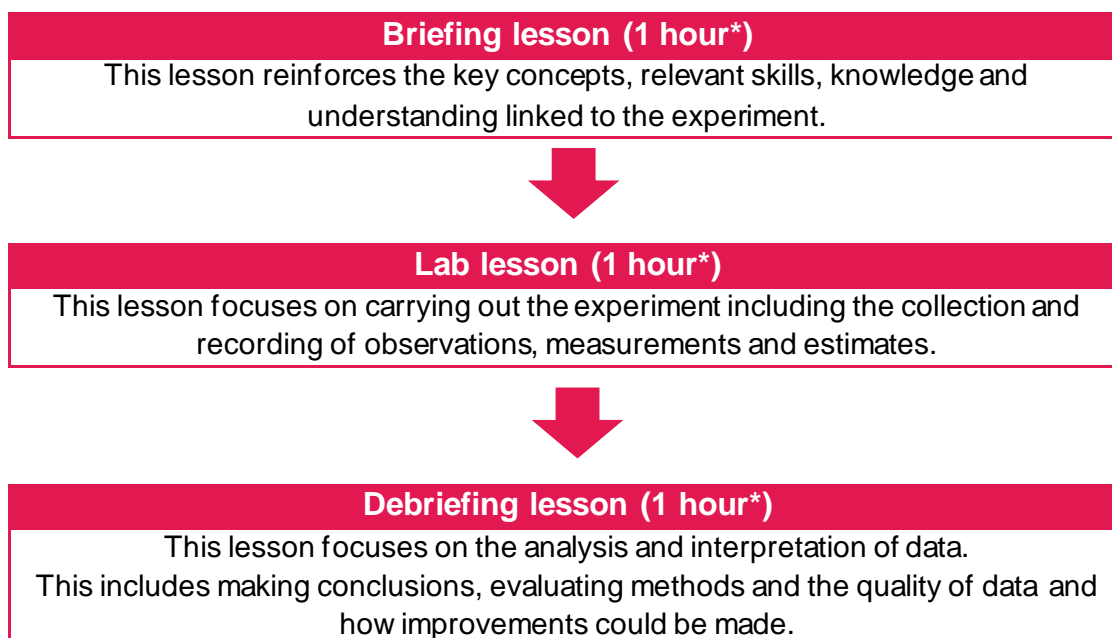
Important note

Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other experiments.

This content is designed to give you and your learners the chance to explore practical skills. It is not intended as specific practice for Paper 3 (Advanced Practical Skills) or Paper 5 (Planning, Analysis and Evaluation).

This is one of a range of *Teaching Packs* and each pack is based on one experiment. The packs can be used in any order to suit your teaching sequence.

The structure is as follows:



** the timings are a guide only; you may need to adapt the lessons to suit your circumstances.*

In this pack you will find lesson plans, worksheets and teacher resource sheets.

Experiment: Brownian motion

This *Teaching Pack* focuses on a Brownian motion.

The basic understanding that gases consist of molecules moving at speed that undergo collisions is called kinetic theory. Robert Brown visualised this movement in 1827. In this experiment you will use a smoke cell to replicate the type of observations he made.

This experiment has links to the following syllabus content (see syllabus for detail):

- 15.3 Kinetic theory of gases

The experiment covers the following experimental skills, as listed in **AO3: Experimental skills and investigations**:

- plan experiments and investigations
- collect, record and present observations, measurements and estimates
- analyse and interpret data to reach conclusions
- evaluate methods and quality of data and suggest improvements.

Prior knowledge

Knowledge from the following syllabus topics is useful for this experiment.

- 15.1 The Avogadro constant
- 3.1 Momentum and Newton's laws of motion
- 3.3 Linear momentum and its conservation
- 15.3 Kinetic theory of gases

Briefing lesson: Brownian motion







Resources

- Internet access

Learning objectives

By the end of the lesson:

- **all** learners should state that the Brownian motion demonstration provides evidence for the random movement of molecules.
- **most** learners should correctly use the term 'random walk'.
- **some** learners will explain the significance of the proof of atomic theory.

Timings	Activity
 5 min	Starter / Introduction Ask your learners to consider this question: 'Before the existence of powerful microscopes, how were scientists able to be sure of the existence of particles so small they are invisible to the naked eye?'
 25 min	Main lesson Using the internet, ask your learners to research the history of the development of atomic theory. This important branch of science was present in Ancient Greece as a philosophy, and gained greater attention in the nineteenth century. However, despite advances in chemistry that supported atomic theory, it remained disputed. The debate continued until Albert Einstein produced proof in 1905, later confirmed by Jean Baptiste Perrin who was awarded the Nobel Prize for Physics in 1926 for this work. The Brownian motion experiment is widely seen as the beginning of the confirmation of atomic theory – that all matter is created of atoms, which are discrete units. Ask learners to prepare and deliver a short talk to include a presentation or poster on this relatively recent history of science. Their presentations should cover these points: <ol style="list-style-type: none"> 1. starting from Robert Brown and Jan Ingenhousz in the early nineteenth century, why it was important to see the phenomenon in non-organic as well as organic particles, 2. the 'random walk', which should be illustrated 3. a description of the disagreement between the 'Atomists' and the 'Energetics' 4. how the theory was confirmed by Albert Einstein and Jean Perrin in the early twentieth century.
 25 min	Learners should deliver their presentations to the group. These can be assessed by you, or their peers, or just enjoyed, as time permits.
 5 min	Plenary Ask your learners to reflect on the lesson and write five sentences that sum up the lesson, then three words that sum up the ideas.

Lab lesson: Brownian motion



Resources	<ul style="list-style-type: none"> Equipment as outlined in the Teacher notes Teacher walkthrough video Worksheet A and Worksheet B
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Learning objectives	<p>By the end of the lesson:</p> <ul style="list-style-type: none"> all learners should state evidence for the movement of molecules. most learners should explain how molecular movement causes the pressure exerted by a gas. some learners will be able to deduce the relationship $PV = \frac{1}{3} Nm \langle c^2 \rangle$.
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Timings	Activity
	<p>Starter / Introduction</p> <p>Put the following question on the board: ‘What is the significance of the Brownian motion experiment?’</p> <p>Your learners should be able to discuss this on the basis of their work in the briefing lesson.</p>
	<p>Main lesson</p> <p>This set-up can be quite tricky to get right. You may need to demonstrate putting the smoke into the cell.</p> <p>Your learners should set up their own equipment (groups of 2–3 if there are sufficient resources). Encourage your learners to look for the spots of light which indicate smoke particles being buffeted by molecules making up the air.</p> <p>Once learners have seen the motion for themselves they should sketch their observations into their books.</p> <p>Safety</p> <p>Circulate the classroom at all times during the experiment, making that your learners are safe. Check that the observations they are making are correct, for example, they are seeing smoke particles, not specks on the microscope lens.</p> <p>Lit straws should be put out and left on heat proof mats until the clearing up session at the end of the lesson (no smouldering straws in bins).</p>
	<p>Explain to your learners that by using simple mathematics we can now deduce the relationship between pressure, volume, and molecular velocity. Using Worksheet A as your guide, elicit from your learners the equations they will need to start to derive the equation. Depending on their ability guide them to the final equation. They should keep a note of the derivation in their books, or you can provide them with a copy of Worksheet A.</p> <p>They should use Worksheet B to practise applying the equation.</p>
	<p>Plenary</p> <p>Minute Labs offer an interactive animation allowing learners to look at Brownian motion under different situations (change in energy, change in mass of particles). Learners may consolidate their ideas by spending a few minutes working with this.</p> <p>http://labs.minutelabs.io/Brownian-Motion/</p>



Teacher notes

Watch the *Teacher walkthrough* video and read these notes.

Each group will require:

- smoke cell (lamp and lens incorporated)
- power supply
- leads
- microscope
- cover slips
- wax coated paper straws
- dropping pipette
- heat proof mat.

Safety

- Wear a lab coat and eye protection.
- No eating or drinking in the lab.
- Make sure lit straws are fully extinguished and left to cool on heat proof mats.

It is your responsibility to carry out an appropriate risk assessment for this experiment.



Teacher method

This is your version of the method for this experiment that accompanies the *Teacher walkthrough* video.

Do not share this method with learners.

Before you begin

Plan how you will group your learners during the experiment session.

Think about:

- the number of groups you will need (group size 2–4 learners)
- the amount of equipment/chemicals required.

Experiment

Walk around the learners during the experiment in case they encounter any difficulties.

Steps

1. Learners should collect the equipment they require from the front of the class.

2. They should find a space in the classroom where the equipment can be assembled safely

3. Make sure your learners can use the straw to insert smoke into the cell. This is quite tricky.

Notes

Learners should light the straw at one end then put out the flame. The other end of the straw is put into the cell.

Learners need time to understand what they are seeing. Any dirt on the microscope lens is likely to become their main observation and will certainly confuse many. Because the smoke particles are moving in three dimensions they will go in and out of focus, and even vanish and reappear. Convection currents inside the smoke cell (even when it is sealed) cause particles to move in a certain way. Discuss these effects so that learners can see them, account for them, and then discount them when they make their observations.

Clean-up

After the experiment learners should:

- carefully return microscopes to the front of the class
- tidy up their work space
- ensure all straws are completely out before throwing away
- return all equipment to you.

Debriefing lesson: Density of air







Resources

- large polythene container with tap such as a collapsible water container for camping
- foot pump
- bucket
- water
- rubber connecting tube
- large measuring cylinder
- digital scales
- Worksheet C

Learning objectives

By the end of the lesson:

- **all** learners should explain an experimental method to find density of air.
- **most** learners will use their knowledge to answer examination questions to a good level.
- **some** learners will be able to apply their understanding to complete all questions to a high standard.

Timings	Activity
 5 min	Starter / Introduction Ask your learners to estimate the mass of air in the lab you are working in.
 20 min	Main lesson Now that we have established that the air is made up of particles, can we find the density of air under normal atmospheric pressure? This experiment can be used to find the density of air under normal atmospheric conditions: <ol style="list-style-type: none"> 1. Using the foot pump, blow up the water container as much as possible. 2. Find the mass of the container and the air inside it. Add the rubber tube to the nozzle of the container, keeping the tap closed. 3. Fill the measuring cylinder with water, turn it upside down in the bucket, making sure it is filled with water. Feed the rubber tube into the measuring cylinder. 4. Release air into the measuring cylinder, noting the volume of air released. Continue until most of the air has been removed. Immediately close the tap so that no more air can enter the container. 5. Now reweigh the container and remaining air. 6. Find density by calculation. Ask your learners to suggest why the air had to be bubbled out into the measuring cylinder rather than simply weighing the container empty and then fully pumped up?
 20 min	Using Worksheet C your learners can undertake some past paper practice.
 15 min	Plenary Using the answers for Worksheet C your learners should peer mark each others' work and give their feedback.

Worksheets and answers

	Worksheet	Answers
For use in <i>Lab lesson</i>:		
A: Deriving the equation $PV = \frac{1}{3} (Nm\bar{c}^2)$	12	
B: Using the equation $PV = \frac{1}{3} (Nm\bar{c}^2)$	13	16
For use in <i>Debrief lesson</i>:		
C: Past paper practice	14–15x	17–18



Worksheet A: Deriving the equation $PV = \frac{1}{3}(Nm\bar{c}^2)$

How to derive the formula for the pressure exerted by a gas

A gas is made up of a very *large number of molecules*, undergoing *elastic collisions* (momentum is conserved). Their motion is *random* and the molecules are of *negligible size* (because the container is very large by comparison).

Step 1 – for one molecule moving in one plane

Time between collisions on the end side: $t = 2l/u$

where l = length of the container, u = velocity of the particle

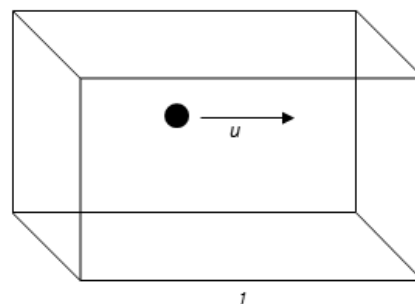
The force on the wall: $F = \Delta mu / \Delta t = 2mu / \Delta t$

where m = mass of the particle

Substituting in the value for t shows that: $F = mu^2/l$

Since $P = F/A$ we can substitute again and show that: $P = mu^2/lA$

But since the volume of the box, $V = lA$ we can now say that $P = mu^2/V$ which rearranges to give a more linear version: $PV = mu^2$



Step 2 – for one molecule moving in three planes

However, the molecules as we have seen are moving randomly in all directions, so the velocity should be treated as a vector sum problem. Since there are three directions the velocity can take, we see that

$$\bar{c}^2 = u_x^2 + u_y^2 + u_z^2$$

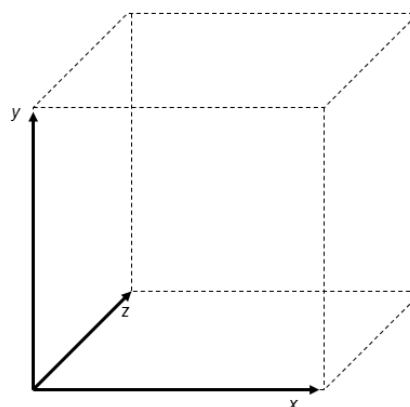
where \bar{c}^2 is the mean squared speed and u_x , u_y and u_z are the velocities in each plane.

We call \bar{c} the mean squared speed since it is giving an average value. Therefore $\sqrt{\bar{c}^2}$ is the root mean squared speed, or r.m.s. speed.

At any time, only a third of the molecules would be moving in any one direction (because the number of molecules is so large). We can therefore say that $u_x = u_y = u_z$ and so:

$$\bar{c}^2 = 3u_x^2, \text{ or; } u_x^2 = \bar{c}^2/3$$

If we take the equation at the end of Step 1 and replace the velocity called u with our value for the average vector velocity we find that; $PV = m\bar{c}^2/3$



Step 3. for N molecules moving in three planes

Finally we must account for the fact there are n molecules in the container. Our answer so far shows the pressure exerted by one molecule, so we simply multiply it by N :

$$PV = Nm\bar{c}^2/3$$

Worksheet B: Using the equation $PV = \frac{1}{3}(Nm\overline{c^2})$



Some physics learners are playing basketball and want to know the r.m.s. of the air inside it. They find through measurements that the air inside the ball has a mass of 2.1×10^{-2} kg.

- (a) The molar mass of air is $2.90 \times 10^{-2} \text{ kg mol}^{-1}$ and Avogadro's constant is $6.02 \times 10^{23} \text{ mol}^{-1}$. How many particles does the ball contain?
- (b) The pressure of the basketball is $5.5 \times 10^4 \text{ Pa}$ and volume is $7.1 \times 10^{-3} \text{ m}^3$. Calculate the mean square speed $\overline{c^2}$ of the particles.
- (c) Suggest a simple method the learners may have used to find the mass of the air inside the ball.





Worksheet C: Past paper practice

1. What is the internal energy of a system?
 - a. The amount of heat supplied to the system
 - b. The random energy of the atoms of the system
 - c. The total kinetic energy of the system
 - d. The total potential energy of the system

2. Below are four short paragraphs describing the molecules in a beaker of water at 50°C. Which paragraph correctly describes the molecules?
 - A The molecules all travel at the same speed. This speed is not large enough for any of the molecules to leave the surface of the water. There are attractive forces between the molecules.
 - B The molecules have a range of speeds. Some molecules travel sufficiently fast to leave the surface of the water. There are no forces between the molecules.
 - C The molecules have a range of speeds. The fastest molecules are unable to leave the surface of the water. There are attractive forces between the molecules.
 - D The molecules have a range of speeds. Some molecules travel sufficiently fast to leave the surface of the water. There are attractive forces between the molecules.

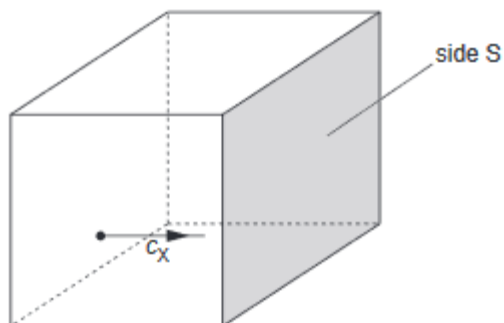
3. Why does an ideal gas exert pressure on its container?
 - a. The molecules of the gas collide continually with each other.
 - b. The molecules of the gas collide continually with the walls of the container.
 - c. The molecules of the gas collide inelastically with the walls of the container.
 - d. The weight of the molecules exerts a force on the walls of the container.

4. The kinetic theory of gases is based on some simplifying assumptions. The molecules of the gas are assumed to behave as hard elastic identical spheres. State the assumption about ideal gas molecules based on
 - a. The nature of their movement [1]
 - b. Their volume. [2]



Worksheet C: Past paper practice, continued

5. A cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_x of velocity normal to one side S of the cube, as shown in the figure below.



The pressure p of the gas due to the component c_x of velocity is given by the expression

$$pV = Nmc_x^2$$

where m is the mass of the molecule.

- a. Explain how the expression leads to the relation

$$pV = \frac{1}{3} Nm \langle c^2 \rangle$$

where $\langle c^2 \rangle$ is the mean square of the molecules. [3]

- b. The molecules of an ideal gas have a root-mean-square (r.m.s.) speed of 520 ms^{-1} at a temperature of 27°C .

Calculate the r.m.s. speed of the molecules at a temperature of 100°C . [3]



Worksheet B: Answers

Some physics learners are playing basketball and want to know the r.m.s. of the air inside it. They find through measurements that the air inside the ball has a mass of 2.1×10^{-2} kg.

- (a) The molar mass of air is 2.90×10^{-2} kg mol⁻¹ and Avogadro's constant is 6.02×10^{23} mol⁻¹. How many particles does the ball contain?

$$4.36 \times 10^{23} \text{ particles}$$

- (b) The pressure of the basketball is 5.5×10^4 Pa and volume is 7.1×10^{-3} m³. Calculate the mean square speed $\overline{c^2}$ of the particles.

$$5.58 \times 10^4 \text{ m}^2 \text{s}^{-2}$$

- (c) Suggest a simple method the learners may have used to find the mass of the air inside the ball.

Weighed the basketball fully deflated, then again fully inflated, and subtracted the first value from the second one.





Worksheet C: Answers

1. What is the internal energy of a system?
 - a. The amount of heat supplied to the system
 - b. The random energy of the atoms of the system
 - c. The total kinetic energy of the system
 - d. The total potential energy of the system

2. Below are four short paragraphs describing the molecules in a beaker of water at 50°C. Which paragraph correctly describes the molecules?
 - A The molecules all travel at the same speed. This speed is not large enough for any of the molecules to leave the surface of the water. There are attractive forces between the molecules.
 - B The molecules have a range of speeds. Some molecules travel sufficiently fast to leave the surface of the water. There are no forces between the molecules.
 - C The molecules have a range of speeds. The fastest molecules are unable to leave the surface of the water. There are attractive forces between the molecules.
 - D The molecules have a range of speeds. Some molecules travel sufficiently fast to leave the surface of the water. There are attractive forces between the molecules.

3. Why does an ideal gas exert pressure on its container?
 - a. The molecules of the gas collide continually with each other.
 - b. The molecules of the gas collide continually with the walls of the container.
 - c. The molecules of the gas collide inelastically with the walls of the container.
 - d. The weight of the molecules exerts a force on the walls of the container.

4. The kinetic theory of gases is based on some simplifying assumptions. The molecules of the gas are assumed to behave as hard elastic identical spheres. State the assumption about ideal gas molecules based on
 - a. The nature of their movement [1]

either random motion or constant velocity until hits wall / other molecule
 - b. Their volume [2]

(total) volume of molecules is negligible [1] compared to volume of containing vessel [1]

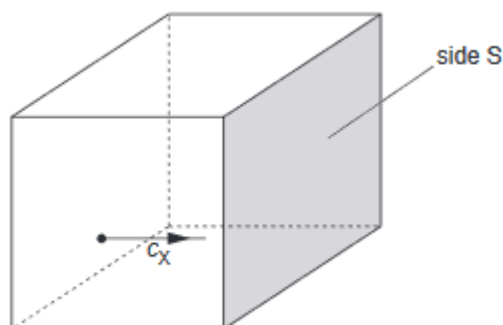
or

Radius / diameter of a molecule is negligible [1] compared to the average intermolecular distance [1]



Worksheet C: Answers, continued

5. A cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_x of velocity normal to one side S of the cube, as shown in the figure below.



The pressure p of the gas due to the component c_x of velocity is given by the expression

$$pV = Nmc_x^2$$

where m is the mass of the molecule.

- a. Explain how the expression leads to the relation

$$pV = \frac{1}{3} Nm \langle c^2 \rangle$$

where $\langle c^2 \rangle$ is the mean square of the molecules. [3]

either molecule has component of velocity in three directions

$$\text{or } c^2 = c_x^2 + c_y^2 + c_z^2 \quad [1]$$

random motion and averaging, so $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$ [1]

$$\langle c^2 \rangle = 3 \langle c_x^2 \rangle \quad [1]$$

$$\text{so, } pV = \frac{1}{3} Nm \langle c^2 \rangle$$

- b. The molecules of an ideal gas have a root-mean-square (r.m.s.) speed of 520 ms^{-1} at a temperature of 27°C .

Calculate the r.m.s. speed of the molecules at a temperature of 100°C . [3]

$$\langle c^2 \rangle \propto T \quad \text{or } c_{\text{rms}} \propto \sqrt{T} \quad [1]$$

Temperatures are 300K and 373K [1]

$$c_{\text{rms}} = 580 \text{ ms}^{-1} \quad [1]$$

Do not allow any marks for use of temperature in units of $^\circ\text{C}$ instead of K

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