

Teaching Pack

Probability of combined events

Cambridge IGCSE™

Mathematics 0580

This *Teaching Pack* can also be used with the following syllabuses:

- Cambridge IGCSE™ (9–1) Mathematics **0980**
- Cambridge IGCSE™ International Mathematics **0607**
- Cambridge O Level Mathematics **4024**



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Icons used in this pack:



Lesson



Video



Assessment opportunity

Introduction

This pack will help you to develop your learners' mathematical skills as defined by assessment objective 1 (AO1 Knowledge and understanding of mathematical techniques) in the course syllabus.

Important note

Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other skills.

This content is designed to give you and your learners the chance to explore mathematical skills. It is not intended as specific practice for exam papers.

This is one of a range of *Teaching Packs*. Each pack is based on one unit of work related to a mathematical theme. The packs can be used in any order to suit your teaching sequence.

In this pack you will find the lesson plans and worksheets for learners you will need to successfully complete the teaching of this unit.

Skill: Probability of combined events

This *Teaching Pack* links to the following syllabus content (see syllabus for detail):

- C/E8.5 Calculate the probability of simple combined events, using possibility diagrams, tree diagrams and Venn diagrams.
- E8.6 Calculate conditional probability using Venn diagrams, tree diagrams and tables.

For assessments from 2025	
• C/E8.3	Calculate the probability of combined events using where appropriate: • sample space diagrams • Venn diagrams • tree diagrams.
• E8.4	Calculate conditional probability using Venn diagrams, tree diagrams and tables.

The pack covers the following mathematical skills, adapted from **AO1: Demonstrate knowledge and understanding of mathematical techniques** (see syllabus for assessment objectives):

- performing calculations and procedures by suitable methods, including using a calculator

For assessments from 2025	
AO1: Knowledge and understanding of mathematical techniques	
•	understand and use mathematical notation and terminology
•	perform calculations with and without a calculator

Prior knowledge

Knowledge from the following syllabus topics is useful for the development of skills in this unit.

- C/E1.2 Understand notation of Venn diagrams.
- C/E1.8 Use the four rules for calculations with whole numbers, decimals and fractions (including mixed numbers and improper fractions), including correct ordering of operations and use of brackets.
- C/E8.1 Calculate the probability of a single event as either a fraction, decimal or percentage.
- C/E8.2 Understand and use the probability scale from 0 to 1.
- C/E8.3 Understand that the probability of an event occurring = $1 -$ the probability of the event not occurring.

For assessments from 2025	
• C/E1.2	understand and use set language, notation and Venn diagrams to describe sets
E1.2	represent relationships between sets.
• C/E1.6	use the four operations for calculations with integers, fractions and decimals, including correct ordering of operations and use of brackets.
• C/E8.1	Understand and use the probability scale from 0 to 1. Calculate the probability of a single event. Understand that the probability of an even not occurring = $1 -$ the probability of

E8.1	the even occurring. Understand and use probability notation.
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Going forward

The knowledge and skills gained from this *Teaching Pack* can be used for when you teach learners:

- Cambridge AS&A Level Mathematics 9709 – Probability and statistics



Before you begin

This *Teaching Pack* includes a **Teacher Introduction** video to which you should refer before using the resources in this pack. The video is available to watch in Resource Plus within the unit relevant to this **Teaching Pack**.

The video introduces the resources available for teaching this unit and explains how they can be used to successfully deliver these skills to your learners. In particular, the video highlights typical learner misconceptions and common errors that this *Teaching Pack* will help you to overcome.

Common misconceptions: Probability of combined events

Possibility space diagrams – When rolling two dice counting the event 1,3 and 3,1 as the same. The best way of dealing with this is to make the dice distinct (different colours). Some learners then believe that 5,5 can be then be the other way round as well, but this can be reasoned away more easily.

Tree diagrams – Learners often find these confusing. By tackling the problems using possibility space diagrams and fraction arithmetic first this should enable learners to better understand what a tree diagram represents. It is important to approach probability questions from a variety of methods so that learners can understand them and their limitations. The lessons are designed to encourage this approach and often show how problems can be solved by at least two different methods.

Conditional Probability – Failing to recognise that they have more information than they thought. Not using this information properly to reduce down the size of the possibility space to just those outcomes that are known to have happened. The best way of tackling this is to get the learners to go on a journey with you to see how the condition is applied. e.g. Roll a die, ask the learners to guess what number you have, then give them a hint, how has this reduced their chances?

Advanced probability without a tree diagram – Not including all of the cases e.g. I toss 3 coins, what is the probability of getting 2 heads – only multiplying 2 probabilities together and failing to notice that three things happened so 3 probabilities need to be multiplied together. You can get round this by ensuring that the learners always write down a number for each element.



Lesson 1: Sample (Possibility) space diagrams

Resources

- Whiteboard
- Lesson 1 Sample (Possibility) space diagrams presentation
- Worksheets 1a, 1b.

Learning objectives

By the end of the lesson:

- **all** learners should be able to calculate the probability of two combined events using a sample (possibility) space diagram.

Timings

Activity



Starter/Introduction

Teach this lesson using [Lesson 1 Sample \(Possibility\) space diagrams presentation](#).

Begin by introducing learners to an example of probability (slide 1) that will feed into the following activity (the example is game involving picking 6 numbered balls from 59 number balls).

Follow with discussion around matching games and playing the class game leading to the creation a of sample (possibility) space diagram to solve the problem (slides 4-7)

OR

Have learners completed the activity on [Worksheet 1a Two player game](#).

Learners play the Two player game and discuss winning strategies, leading to the creation of a sample (possibility) space diagram to solve the problem.



Main lesson

Using slides 8-10 with learners, complete the on-screen activity.

This is an example of how to draw up a sample (possibility) space diagram when probabilities are unequal. Focus here is to ensure that the learners understand the importance of the outcomes all being equally likely. This example deals with the situation for a simple case.

Follow up with [Worksheet 1b Sample \(Possibility\) space diagrams](#) with learners.

OR

Use the activity on slides 11 and 12 – Is rock, paper, scissors a fair game?

Investigate this by playing the game in the classroom and recording the numbers of wins, loses and draws to calculate an experimental probability. Does this match to theory via a possibility space diagram? Discuss reasons why they may not match.

**Plenary**

Using slide 13 with learners, complete the final lesson activity. This provides an opportunity to assess via contextual example.



Lesson 2: Finding probabilities using fraction multiplication

Resources

- Whiteboard
- Lesson 2 Finding probabilities using fraction multiplication.

Learning objectives

By the end of the lesson:

- **all** learners should be able to find probabilities of combined events using fraction arithmetic.

Timings

Activity



Starter/Introduction

Teach this lesson using [Lesson 2 Finding probabilities using fraction multiplication](#).

Start by using the test on slide 3, Fraction arithmetic reminder. Fraction arithmetic (multiplication and addition) is vital in using probability tree diagrams. The starter addresses this. Learners should attempt as many of the questions as they can. The teacher can then ask learners to give their answers and explain their method, before clicking on the slide to reveal the correct answer. This could be done as a competitive game between left and right halves of the classroom etc. Make certain Learners can multiply fractions and add with a common denominator before attempting the lesson.



Main lesson

This lesson should focus on non-calculator skills. Learners should tackle the problem on slide 4 first and then the teacher can show how the result can be obtained both using a possibility (sample) space diagram and using fraction arithmetic (slide 5). This leads to finding probabilities by multiplying fractions together. Slide 6 can be used as an example to practise this idea.

Using slides 7 to 11, and mini whiteboards (or paper) check learners' understanding of the multiply rule. It is important that they understand that they need to multiply 3 fractions together even if it says 'late on the first day, but not on the next two' – learners often miss this.

Extension: For a more able class you could provide some conditional probability style questions as an extension.

Give learners [Worksheet 2a Which competition?](#) to complete.

Can they find the probability of winning each competition? Which has the best chance of winning?



Plenary

Using slides 12 and 13, go back to the original question and introduce the concept of multiplying probabilities together and then adding up the ones you need. The key ideas from this lesson feed into the next lesson on tree diagrams to help make sense of them.

Lesson 3: Draw and interpret tree diagrams



Resources

- Whiteboard
- Lesson 3 Draw and interpret tree diagrams presentation
- Worksheet 3a.

Learning objectives

By the end of the lesson:

- **all** learners should be able to interpret and draw probability tree diagrams
- **all** learners should be able to use probability tree diagrams to solve probability questions involving independent events.

Timings

Activity

	Starter/Introduction Teach this lesson using Lesson 3 Draw and interpret tree diagrams presentation. Introduce the recap question on slide 3 to learners. Let the learners tackle this question using any method – fraction multiplication or possibility space diagrams. Slides 4 – 7 have the answers and show both methods to help understanding. This lesson should be taught as a non-calculator lesson.
	Main lesson Slides 8-15 show how the same question could be tackled using a tree diagram. Learners often get confused by tree diagrams, many failing to understand what the pathways are for and seeing it as a random assortment of fractions and lines. It is important to explain the structure by describing the journey and the way the tree diagram is a map of the 4 possible outcomes: WW, WL, LW, LL. The slides build the diagram in a way that helps this to be understood, but there is no substitute for drawing it yourself as you explain. Give learners Worksheet 3a Probability tree diagrams. This provides practice questions, scaffolded with the easiest at the start of the sheet. All are “with replacement”, although some mirror use of conditional probability.
	Plenary Complete the lesson by introducing learners to slides 16 and 17. These get the learners thinking about how the tree diagram would be altered by conditional events and to begin thinking about “without replacement” problems. Learners should hopefully see that tree diagrams can be much easier than sample (possibility) space diagrams for solving these kinds of problems.



Lesson 4: Conditional probability (extended)

Resources

- Whiteboard
- Lesson 4 Conditional probability
- Worksheets 4a, 4b.

Learning objectives

By the end of the lesson:

- **some** learners should be able to understand and be able to calculate conditional probabilities.

Timings

Activity



Starter/Introduction

Teach this lesson using [Lesson 4 Conditional probability presentation](#).

Introduce learners to the problem on slide 3, and ask them to suggest the answer and reason why. The puzzle is designed to generate debate about issues surrounding 'conditional probability'. It is a good starting point. This lesson is ideal to not use calculators.



Main lesson

Define conditional probability (slide 5) and demonstrate how we should think about conditional probability. Learners need to see how the extra information they have been given is reducing the size of the possibility space and that they calculate the probability for conditional cases using the reduced possibility space. Practical examples can help with this. E.g. roll a die, ask learners to guess what number you have, then give them a hint, how has this reduced their odds?

Next, give learners [Worksheet 4a Conditional probability](#) to complete. This provides practice questions – scaffolded with the easiest at the start of the sheet.

OR

Extension: [Worksheet 4b Hard conditional probability puzzle](#). This really tests learners' understanding of conditional probability and is quite challenging.

Follow up by playing the game on slide 9 or use the random number generator tool in Resource Plus.

The game allows learners to practise conditional probability and to help Learners grasp how conditional probability works.



Plenary

Slides 10 and 11 provide questions to check understanding and provide assessment opportunities – use mini whiteboards or paper.

Lesson 5: Tree diagrams & more complex probabilities



Resources

- Whiteboard
- Worksheets 5a.

Learning objectives

By the end of the lesson:

- **some** learners should be able to use tree diagrams to find the probability of more complex combined events involving conditional probability.

Timings

Activity

 10 min	Starter/Introduction Teach this lesson using Lesson 5 Tree diagrams and more complex probabilities presentation. This lesson is focused on developing more complex techniques to solve probability problems. The starter puzzle on slide 3 is deceptively easy to solve but provides a good starting point for learners.
 40 min	Main lesson Introduce the problem on slides 4 and 5 to learners, and work through it in class. This looks at how we can use tree diagrams to solve problems involving conditional probability not using calculators. Most of the questions in the lesson are “without replacement” questions. It is important for learners to understand that when two things are taken at the same time, this signifies “without replacement” and that they can draw tree diagrams to assist them. The first problem can be used to illustrate how to draw a tree diagram with conditional probabilities. The second problem, on slide 6, can also be solved in this way, but can instead be used as a platform to manage without a tree diagram. In fact drawing the tree diagram is cumbersome. Calculators can be used here. Give learners Worksheet 5a Probability tree diagrams, which contains practice questions that are scaffolded, with the easiest at the start of the sheet.
 10 min	Plenary Finish by presenting the problem on slide 7 to learners. Encourage learners to recognise that it is often easier to do $1 -$ the probability they need. This question can also be used to encourage learners to move away from using tree diagrams

Links to websites: Probability

The following links to websites provide further opportunities to create activities related to this topic are:

<https://nrich.maths.org/9646>

An article on conditional probability in real life.

<https://nrich.maths.org/506>

A hard puzzle that can be solved with a tree diagram.

<https://nrich.maths.org/512>

Coin tossing games.

Worksheets and answers

	Worksheets	Answers
For use in Lesson 1:		
1a: Two player game	18	
1b: Sample (Probability) space diagrams	21	34
For use in Lesson 2:		
2a: Which competition?	23	35
For use in Lesson 3:		
3a: Probability tree diagrams	24	36
For use in Lesson 4:		
4a: Conditional probability	28	39
4b: Hard conditional probability puzzle	31	40
For use in Lesson 5:		
5a: Probability tree diagrams	32	41

Worksheet 1a: Two player game

Equipment:

Playing Board, two dice, 10 blue tokens and 10 red tokens to represent crates.

Back story:

You both work for an interstellar packaging company and you need to pack all of your crates of goods onto the space craft as quickly as possible. The first to pack all their crates wins the game.

Rules:

Take it in turn to place your crates into the loading bays numbered 0 to 5.

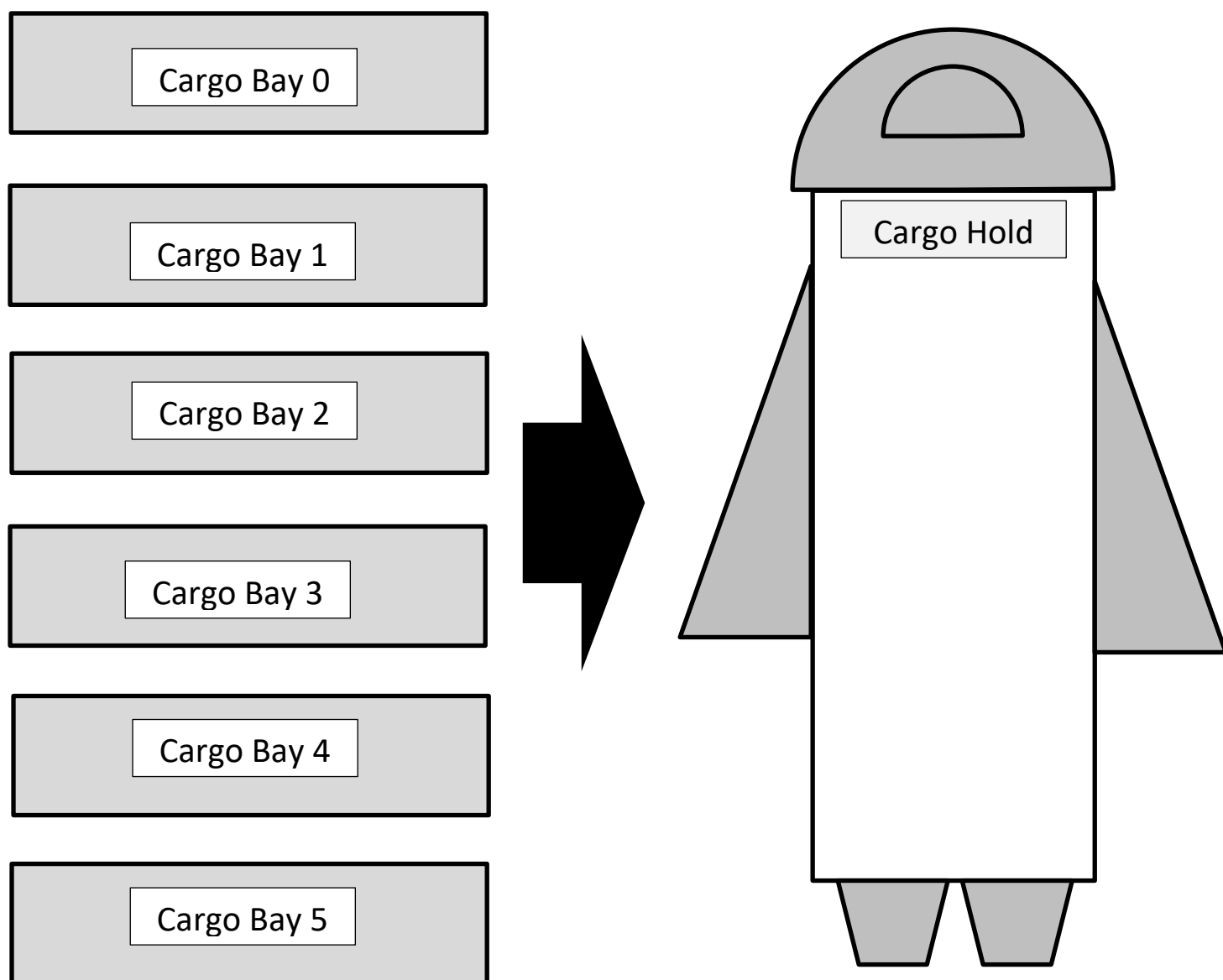
Only 4 crates can occupy each loading bay.

When all the crates have been allocated loading can begin.

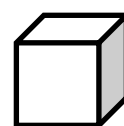
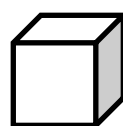
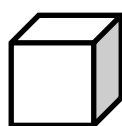
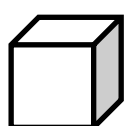
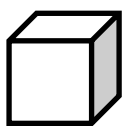
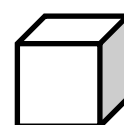
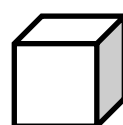
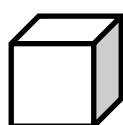
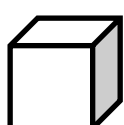
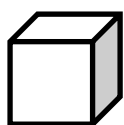
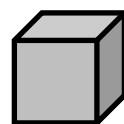
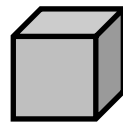
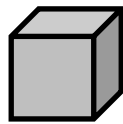
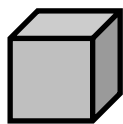
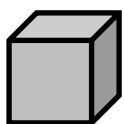
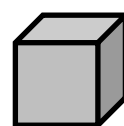
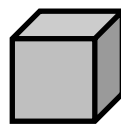
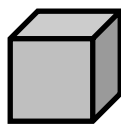
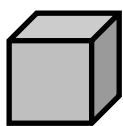
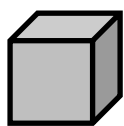
To load an item take it in turns to roll two fair six sided dice and find the difference in their scores. The difference represents the loading bay that you can pack a crate from. If you have a crate in that bay move it into the space craft's hold. If you do not have a crate on the bay, your turn ends.

The first person to clear all their crates from the loading bay into the hold is the winner.

Worksheet 1a: Two player game continued



Worksheet 1a: Two player game continued



Worksheet 1b: Sample (Probability) space diagrams

- 1) Draw a **sample (possibility) space diagram** to show all the possible outcomes when you spin the spinner shown opposite twice and **multiply the scores together**.



- a) What is the probability of the score being even?
- b) What is the probability of the score being below 10?

- 2) In a game the player must roll a tetrahedral die (numbered 1, 2, 3, 4) and a normal six sided die (numbered 1, 2, 3, 4, 5, 6).



- a) Draw a **sample (possibility) space diagram** to show the outcomes from rolling the dice.

- b) What is the probability that you get the same score on each die?

Worksheet 1b: Sample (Probability) space diagrams continued

- 3) Draw a **sample (possibility) space diagram** to show all the outcomes for a game of rock/scissors/paper.

Does your diagram indicate that the game is fair?

- 4) A bag contains a red sweet, a blue sweet and a yellow sweet. To win a sweet a player flips a coin, if they get a head then they select a sweet from the bag. Draw a sample (possibility) space to show all the possible outcomes.

What is the probability that the player wins a blue sweet?

- 5) How can you draw a **sample (possibility) space diagram** to show the outcomes from tossing a fair coin 3 times? Draw one. Use it to calculate the probability of getting exactly one head.

Worksheet 2a: Which competition?

By calculating the probability of winning each competition, decide which of these competitions gives you the best chance of winning:

Toss a fair coin 6 times. You win if all 6 tosses are heads

You roll a fair 4 sided dice 12 times. The dice is numbered:

2, 3, 4 and 5

You win if all of your throws show a prime number

You roll a fair 6 sided dice 3 times. The dice is numbered:

1, 2, 3, 4, 5 and 6

You win if all of your throws are a 6

You shuffle a standard deck of 52 playing cards – no jokers.

You draw a card and replace it before drawing another card.

You win if both cards are an ace

You are given 5 great features of a holiday resort and have to list them from best to worst.

To win you have to match the exact order of the expert panel

You are given a bag with 2 red tickets and a black ticket.

You have to draw 10 tickets from the bag without getting the black ticket.

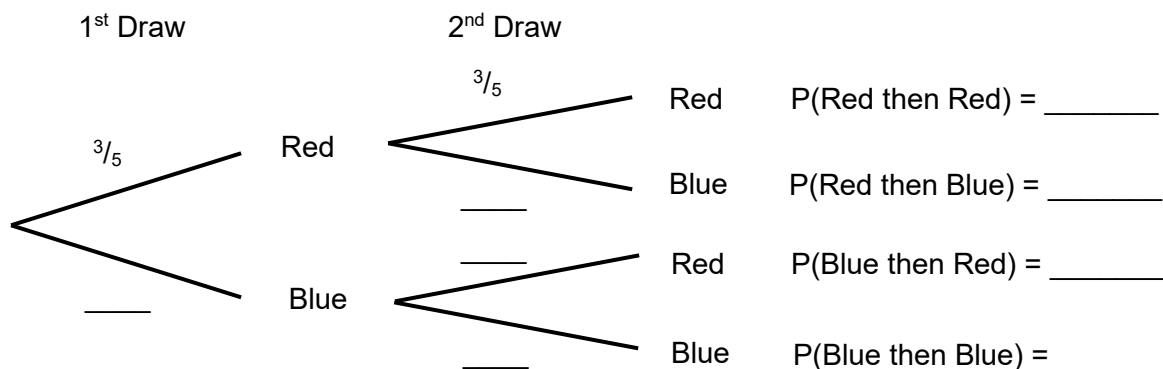
Every time you draw a ticket you replace it in the bag

Worksheet 3a: Probability tree diagrams



- 1) A bag contains 5 balls, 3 are red and 2 are blue. An experiment is conducted by taking a ball from the bag at random, noting its colour and then returning it to the bag. A second ball is then drawn randomly from the bag.

Copy and complete the tree diagram below to show the outcomes from the experiment.

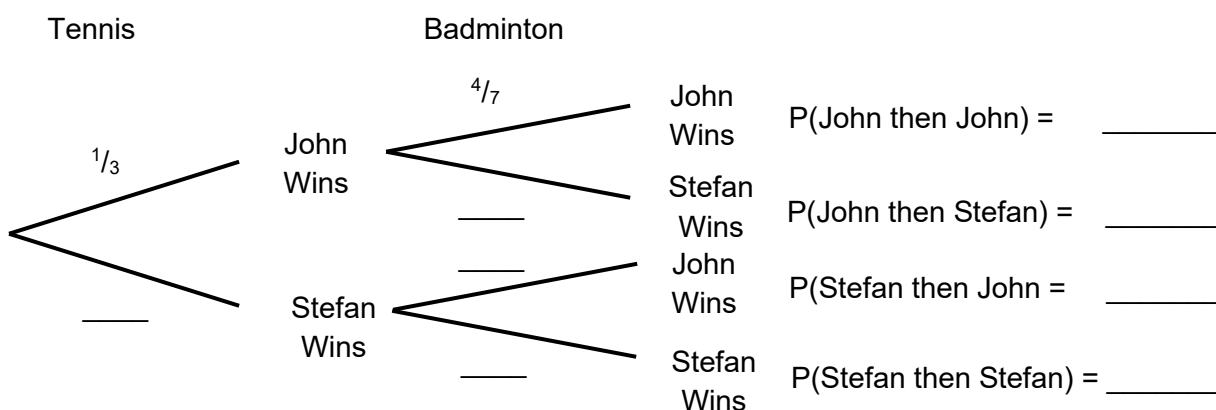


Use your tree diagram to calculate the probability of:

a) Two red balls being drawn from the bag.

b) Exactly one blue ball being drawn from the bag.

- 2) John and Stefan are playing a tournament in which they will play each other at tennis and then at badminton. They always play to win and no draws are allowed. The probability that John wins the tennis game is $\frac{1}{3}$. The probability that John wins the badminton game is $\frac{4}{7}$. Copy and complete the tree diagram.



Use your tree diagram to calculate the probability that:

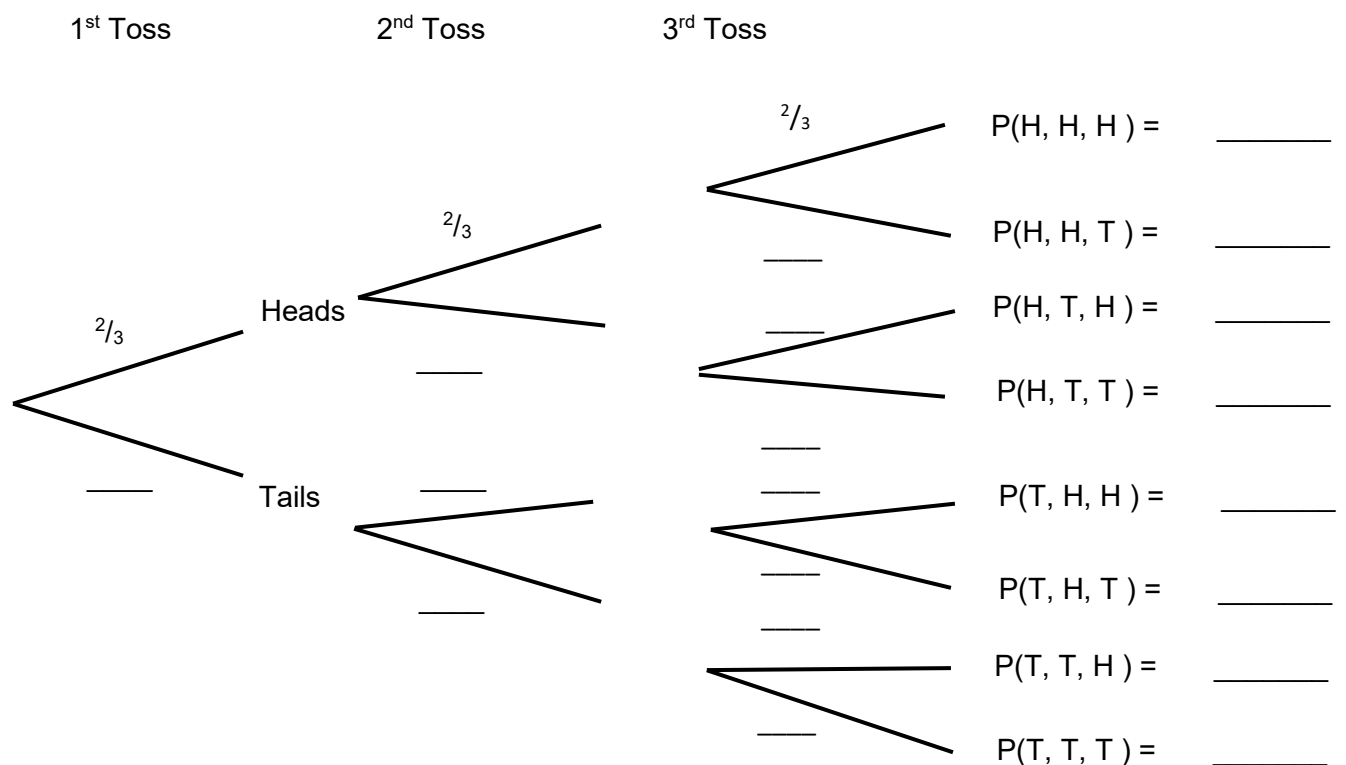
a) Stefan wins at both tennis and badminton.

b) John wins at least one of the games played.

Worksheet 3a: Probability tree diagrams continued

- 3) Experience shows that a bus is late on week day with probability $\frac{1}{5}$. Draw a tree diagram to show the likelihood of the bus being late on two consecutive week days. Use your tree diagram to calculate the probability that the bus is late on exactly one of the two days.

- 4) A biased coin has a probability of $\frac{2}{3}$ of landing heads up. An experiment involves tossing the coin three times. Copy and complete the tree diagram below showing the outcomes from the experiment. Use your tree diagram to calculate the probability that from the three tosses exactly 2 heads were obtained.



Worksheet 3a: Probability tree diagrams continued

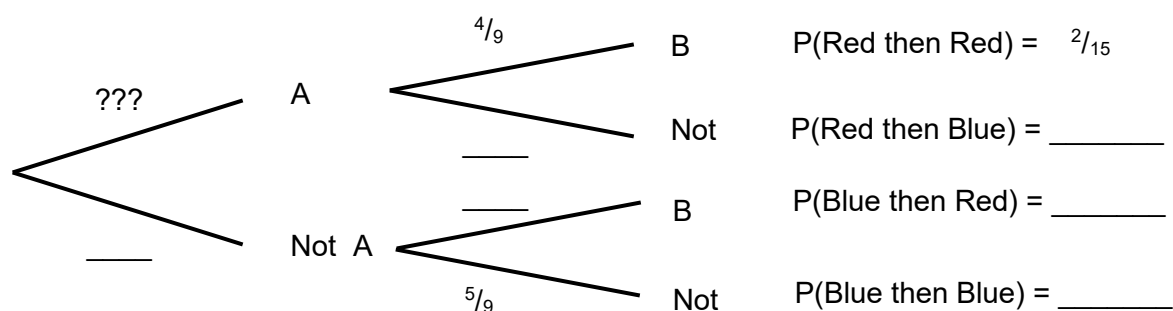
5) Asher is going to school tomorrow. If it is raining the probability that she walks to school is 0.3. If it is not raining the probability that she walks to school is 0.7. The probability that it rains is 0.1.

a) Draw a tree diagram to describe this situation.

b) Calculate the probability that Asher walks to school tomorrow.

6) The probability that a learner driver passes their driving test is set to be 0.4. If they fail the first test the probability that they pass on the second test is 0.5. If they fail the second test the probability that they pass on the third attempt is 0.3. Draw a tree diagram to show this information. Hence find the probability that the driver passes their test before their fourth attempt.

7) The tree diagram below shows the probability of event B happening after event A.



- Use the tree diagram to find the probability the probability of A.
- Find all the missing values in the tree diagram.

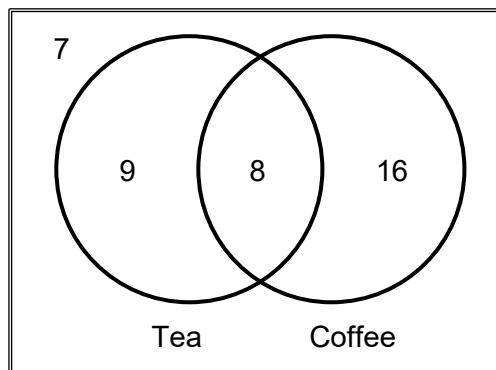
Worksheet 4a: Conditional probability



- 1) Forty people were asked if they had drunk tea or coffee in the last 24 hours. The results were put into the Venn diagram opposite.

Select a person at random from the survey, find:

- a) The probability that they drank tea

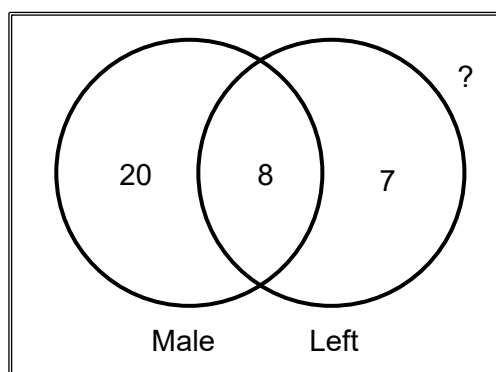


- b) The probability that they drank both tea and coffee

- c) If I choose a tea drinker what is the probability that they also drank coffee?

- 2) Fifty learners were asked if they were right or left handed. The results were put into a Venn diagram:

- a) What number is missing from the Venn diagram?



- b) If I pick a random male what is the probability that they are right handed.

- c) If I pick a learner at random what is the probability that they are left handed

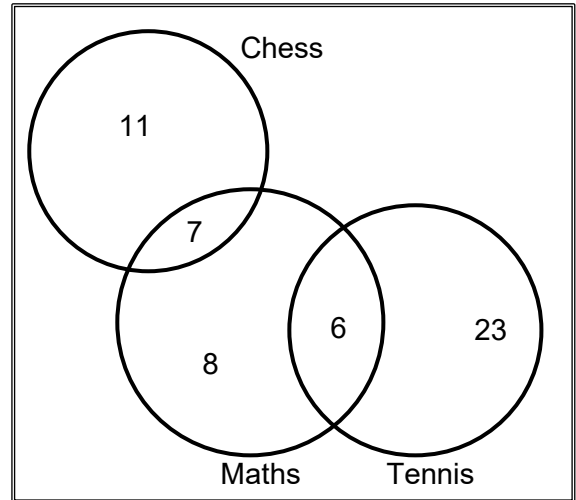
- d) If I pick a right handed learner at random what is the probability that they are a girl?

Worksheet 4a: Conditional probability continued

- 3) The Venn diagram shows the results when fifty learners at a school were asked which clubs they attended.

a) If I pick one of the fifty learners at random what is the probability that they attend maths club?

b) If I pick a maths club member what is the probability that they attend another club?



c) I pick a chess club member what is the probability that they attend maths club?

d) If I pick a member of the maths club what is the probability that they attend tennis club?

- 4) Let $\mathcal{S} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

From the set of numbers above:

a) What is the probability that I pick a prime number

b) What is the probability that I pick a square number that is a factor of 12

c) If I pick a factor of 12, what is the probability that it is a square number?

d) If I pick a square number what is the probability that it is a factor of 12?

e) If I pick a prime number what is the probability that it is a factor of 12?

Worksheet 4a: Conditional probability continued

- 5) I roll two fair dice, one is red and one is blue. What is the probability:
- a) That I have a score of more than three on the red die if my total score is nine?
 - b) The score on the red die is double that on the blue die if my total score is more than 8?
- 6) In a school of 90 learners, 47 study Maths and 57 study English. Twelve learners study neither subject. I pick an English learner at random, what is the probability that they also study Maths?

Worksheet 4b: Hard conditional probability puzzle

A red die and a blue die are rolled. The outcomes can be illustrated in the sample (possibility) space diagram below

		Red Die Roll					
		1	2	3	4	5	6
Blue Die Roll	1						
	2						
	3						
	4						
	5						
	6						

Let S = Total score of both dice is eight

E = Total score from both dice is even

B = The score on the blue die is prime

R = The score on the red die is more than 4

Calculate the following probabilities

a) $P(S)$

b) $P(B)$

c) $P(R)$

d) $P(E)$

e) $P(S \text{ given that } R)$

f) $P(R \text{ given that } S)$

g) $P(R \text{ given that } E)$

h) $P(E \text{ given that } R)$

i) $P(R \text{ given that } B)$

j) $P(B \text{ given that } R)$

k) $P(S \text{ given that } E)$

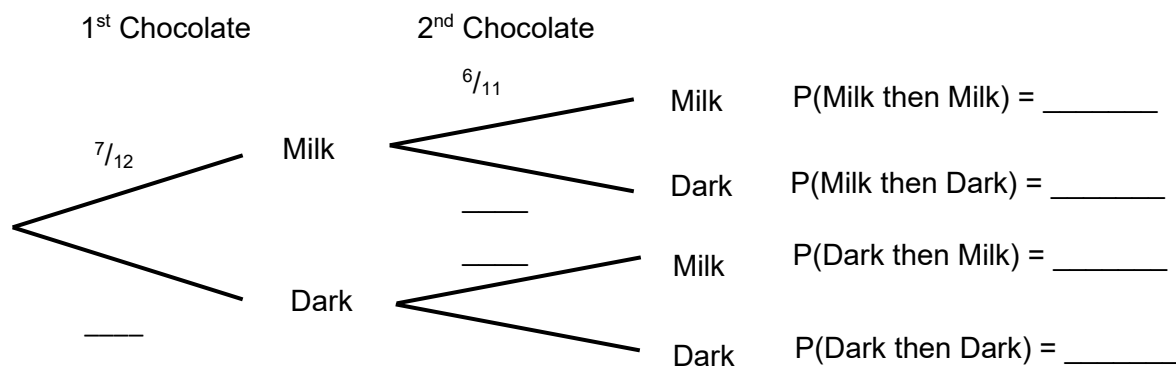
l) $P(E \text{ given that } S)$

m) $P(B \text{ given that } S)$

n) $P(S \text{ given that } B)$

Worksheet 5a: Probability tree diagrams

- 1) A box contains 12 chocolates, 7 are milk chocolate and 5 are dark chocolate. Two chocolates are taken from the box. Copy and complete the tree diagram below to show the types of chocolates taken from the box.



Use your tree diagram to calculate the probability of getting two different types of chocolate.

- 2) A bag contains ten tickets, numbered 1 to 10 inclusively. Two tickets are drawn from the bag at the same time. Draw a tree diagram to show the outcomes from. Find the probability that both tickets are even numbers.
- 3) The probability that it rains on a particular day is $\frac{1}{5}$. If it rains the probability that it rains the next day is $\frac{2}{3}$, if it does not rain the probability that it rains remains at $\frac{1}{5}$. Find the probability that it rains exactly once over a three-day period.

Worksheet 5a: Probability tree diagrams continued

- 4) Bag A contains 4 red discs and 3 green discs

Bag B contains 3 red discs and 3 green discs.

A disc is drawn at random from Bag A and placed in Bag B. A second disc is then drawn from Bag B.

What is the probability that the disc from Bag B is green?

- 5) The following numbered tiles are placed in a bag:



Claudia takes one of the tiles at random. She keeps it and then takes a second tile.

- a) What is the probability that the second tile is smaller than the first tile?
- b) What is the probability that the sum of the two tiles is odd?
- 6) In a bag there are 4 red balls in a bag and some blue balls. I take two balls out of the bag without replacement. The probability that both balls are red is $\frac{2}{7}$. How many blue balls are in the bag?

Worksheet 1b: Answers

- 1) Draw a **sample (possibility) space diagram** to show all the possible outcomes when you spin the spinner shown opposite twice and **multiply the scores together**.

- a) What is the probability of the score being even?

$$\frac{16}{25}$$

- b) What is the probability of the score being below 10?

$$\frac{3}{5}$$

	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

- 2) In a game the player must roll a tetrahedral die (numbered 1, 2, 3, 4) and a normal six sided die (numbered 1, 2, 3, 4, 5, 6).

- a) Draw a **sample (possibility) space diagram** to show the outcomes from rolling the dice.

- b) What is the probability that you get the same score on each die? $\frac{1}{6}$

	1	2	3	4
1	X			
2		X		
3			X	
4				X
5				
6				

- 3) Draw a **sample (possibility) space diagram** to show all the outcomes for a game of rock/scissors/paper.

Does your diagram indicate that the game is fair? *Game is fair, but see lesson*

- 5) A bag contains a red sweet, a blue sweet and a yellow sweet. To win a sweet a player flips a coin, if they get a head then they select a sweet from the bag. Draw a sample (possibility) space to show all the possible outcomes.

What is the probability that the player wins a blue sweet? $\frac{1}{6}$

- 6) How can you draw a **sample (possibility) space diagram** to show the outcomes from tossing a fair coin 3 times? Draw one. Use it to calculate the probability of getting exactly one head.

$$P(1 \text{ head and 2 tails in any order}) = \frac{3}{8}$$

Worksheet 2a: Answers

1) $0.5^6 = 0.015625$

2) $(0.75)^{12} = 0.031676$

3) $1/216 = 0.0046296$

4) $1/169 = 0.005917$

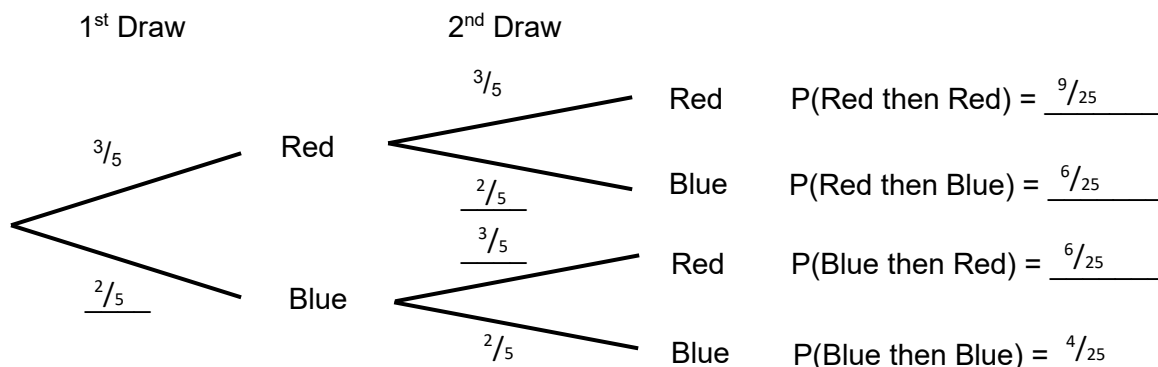
5) $1/120 = 0.0083333$

6) $(2/3)^{10} = 0.0173415$

Worksheet 3a: answers

- 1) A bag contains 5 balls, 3 are red and 2 are blue. An experiment is conducted by taking a ball from the bag at random, noting its colour and then returning it to the bag. A second ball is then drawn randomly from the bag.

Copy and complete the tree diagram below to show the outcomes from the experiment.

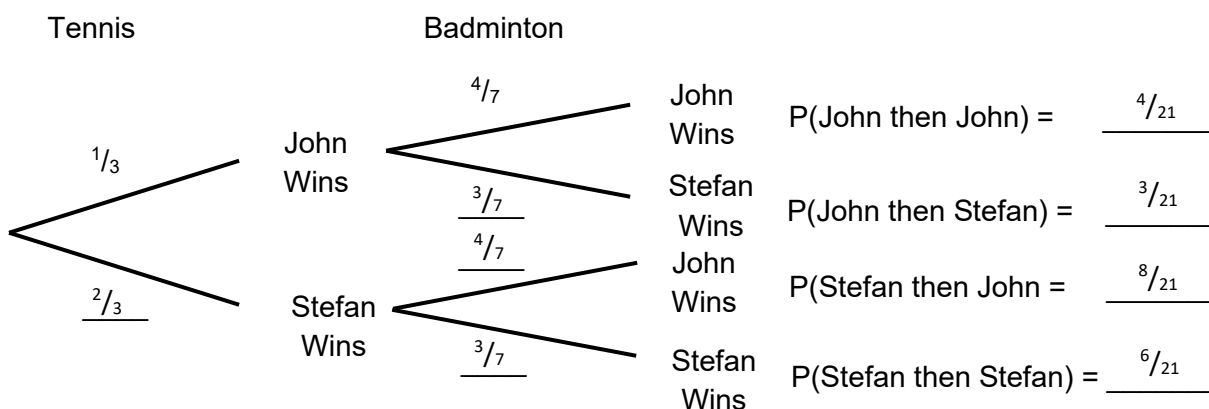


Use your tree diagram to calculate the probability of:

a) Two red balls being drawn from the bag. $\frac{9}{25}$

b) Exactly one blue ball being drawn from the bag. $\frac{12}{25}$

- 2) John and Stefan are playing a tournament in which they will play each other at tennis and then at badminton. They always play to win and no draws are allowed. The probability that John wins the tennis game is $\frac{1}{3}$. The probability that John wins the badminton game is $\frac{4}{7}$. Copy and complete the tree diagram.



Use your tree diagram to calculate the probability that:

b) Stefan wins at both tennis and badminton. $\frac{6}{21}$

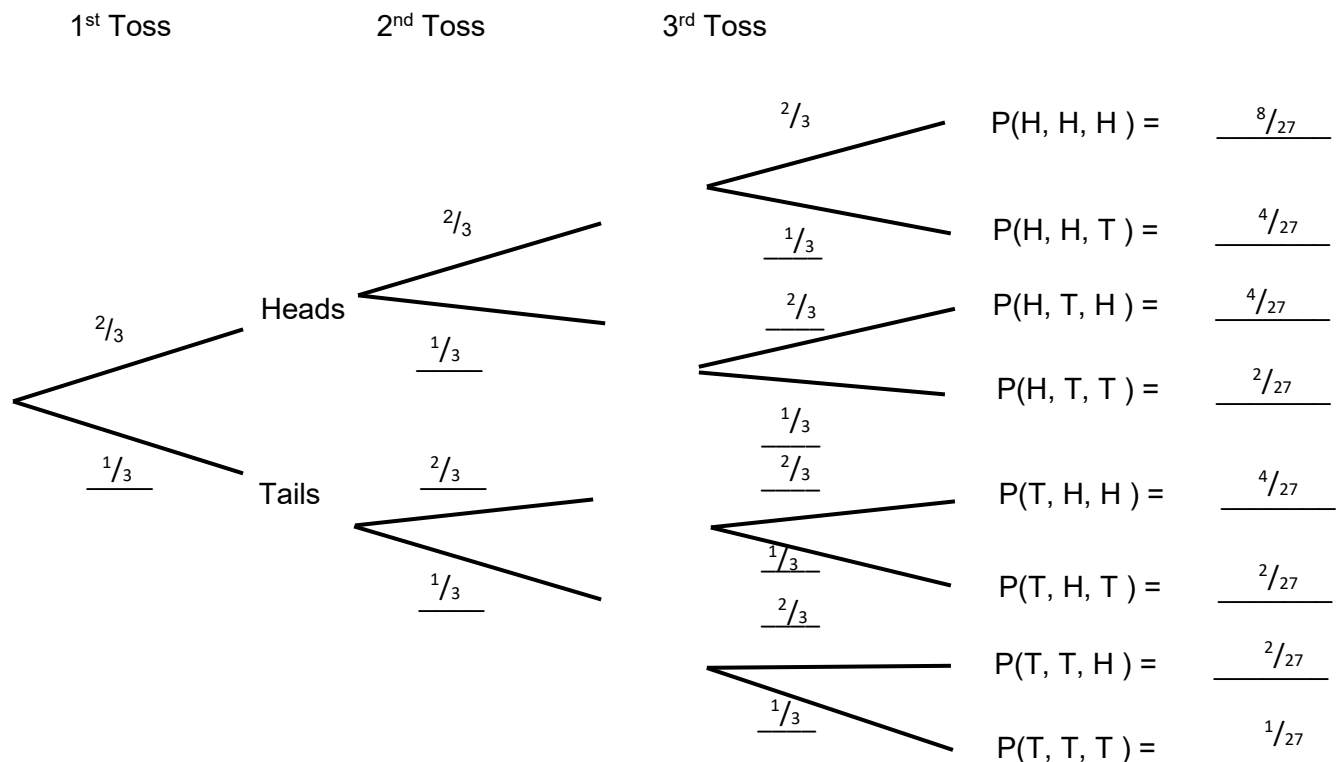
b) John wins at least one of the games played. $\frac{15}{21}$

Worksheet 3a: answers continued

- 3) Experience shows that a bus is late on week day with probability $\frac{1}{5}$. Draw a tree diagram to show the likelihood of the bus being late on two consecutive week days. Use your tree diagram to calculate the probability that the bus is late on exactly one of the two days.

$$\frac{1}{5} \times \frac{4}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{8}{25}$$

- 4) A biased coin has a probability of $\frac{2}{3}$ of landing heads up. An experiment involves tossing the coin three times. Copy and complete the tree diagram below showing the outcomes from the experiment. Use your tree diagram to calculate the probability that from the three tosses exactly 2 heads were obtained. $\frac{12}{27}$



- 5) Asher is going to school tomorrow. If it is raining the probability that she walks to school is 0.3. If it is not raining the probability that she walks to school is 0.7. The probability that it rains is 0.1.

a) Draw a tree diagram to describe this situation.

b) Calculate the probability that Asher walks to school tomorrow.

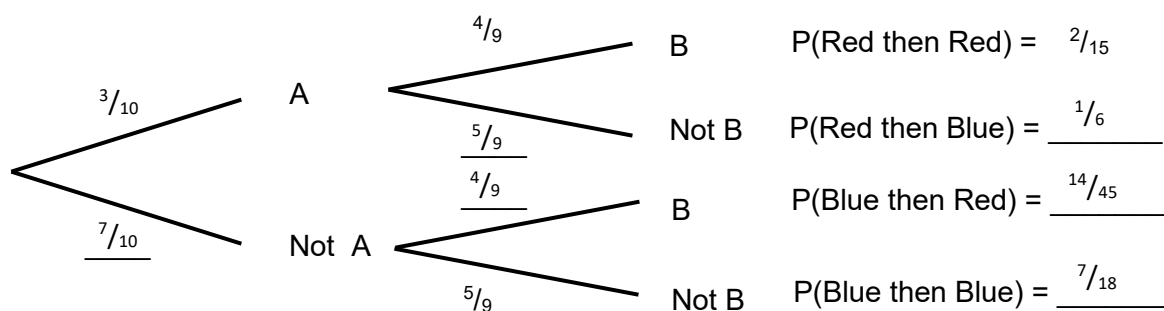
$$0.1 \times 0.3 + 0.9 \times 0.7 = 0.66$$

- 6) The probability that a learner driver passes their driving test is set to be 0.4. If they fail the first test the probability that they pass on the second test is 0.5. If they fail the second test the probability that they pass on the third attempt is 0.3. Draw a tree diagram to show this information. Hence find the probability that the driver passes their test before their fourth attempt.

$$0.4 + 0.6 \times 0.5 + 0.6 \times 0.5 \times 0.3 = 0.79$$

Worksheet 3a: answers *continued*

7) The tree diagram below shows the probability of event B happening after event A.



a) Use the tree diagram to find the probability of A.

$$\frac{2}{15} \div \frac{4}{9} = \frac{3}{10}$$

b) Find all the missing values in the tree diagram.

Worksheet 4a: answers

Question 1

a) $\frac{17}{40}$

b) $\frac{1}{5}$

c) $\frac{8}{17}$

Question 2

a) 15

b) $\frac{5}{7}$

c) $\frac{3}{10}$

d) $\frac{3}{7}$

Question 3

a) $\frac{21}{50}$

b) $\frac{13}{21}$

c) $\frac{7}{18}$

d) $\frac{2}{7}$

Question 4

a) $\frac{2}{5}$

b) $\frac{2}{15}$

c) $\frac{1}{3}$

d) $\frac{2}{3}$

e) $\frac{1}{3}$

Question 5

a) $\frac{3}{4}$

b) $\frac{1}{10}$

Question 6

$\frac{26}{57}$

Worksheet 4b: answers

a) $P(S) = \frac{5}{36}$

b) $P(B) = \frac{1}{2}$

c) $P(R) = \frac{1}{3}$

d) $P(E) = \frac{1}{2}$

e) $P(S \text{ given that } R) = \frac{1}{6}$

f) $P(R \text{ given that } S) = \frac{2}{5}$

g) $P(R \text{ given that } E) = \frac{1}{3}$

h) $P(E \text{ given that } R) = \frac{1}{2}$

i) $P(R \text{ given that } B) = \frac{1}{3}$

j) $P(B \text{ given that } R) = \frac{1}{2}$

k) $P(S \text{ given that } E) = \frac{5}{18}$

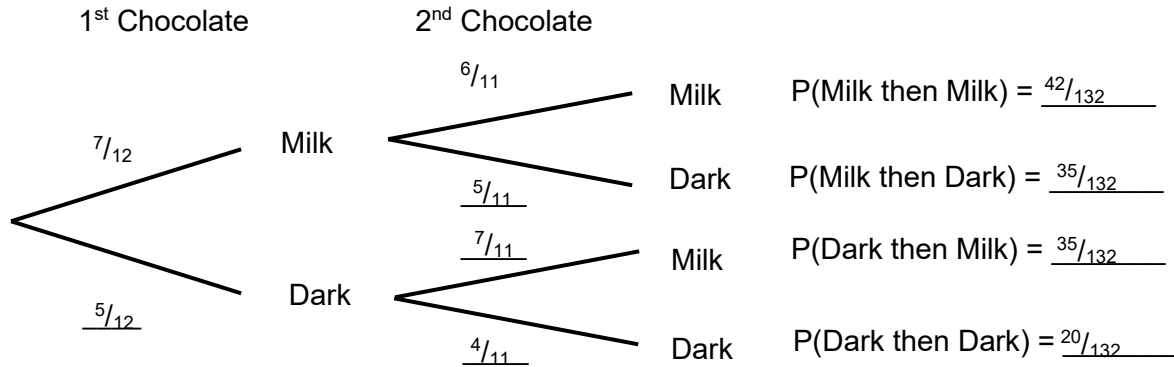
l) $P(E \text{ given that } S) = 1$

m) $P(B \text{ given that } S) = \frac{3}{5}$

n) $P(S \text{ given that } B) = \frac{1}{6}$

Worksheet 5a: answers

- 1) A box contains 12 chocolates, 7 are milk chocolate and 5 are dark chocolate. Two chocolates are taken from the box. Copy and complete the tree diagram below to show the types of chocolates taken from the box.



Use your tree diagram to calculate the probability of getting two different types of chocolate. $\frac{70}{132}$

- 2) A bag contains ten tickets, numbered 1 to 10 inclusively. Two tickets are drawn from the bag at the same time. Draw a tree diagram to show the outcomes from. Find the probability that both tickets are even numbers. $\frac{2}{9}$
- 3) The probability that it rains on a particular day is $\frac{1}{5}$. If it rains the probability that it rains the next day is $\frac{2}{3}$, if it does not rain the probability that it rains remains at $\frac{1}{5}$. Find the probability that it rains exactly once over a three-day period. $\frac{88}{375}$
- 4) Bag A contains 4 red discs and 3 green discs
 Bag B contains 3 red discs and 3 green discs.
 A disc is drawn at random from Bag A and placed in Bag B. A second disc is then drawn from Bag B.
 What is the probability that the disc from Bag B is green? $\frac{24}{49}$
- 5) The following numbered tiles are placed in a bag:



Claudia takes one of the tiles at random. She keeps it and then takes a second tile.

- a) What is the probability that the second tile is smaller than the first tile? $\frac{11}{30}$
- b) What is the probability that the sum of the two tiles is odd? $\frac{3}{5}$
- 7) In a bag there are 4 red balls in a bag and some blue balls. I take two balls out of the bag without replacement. The probability that both balls are red is $\frac{2}{7}$. How many blue balls are in the bag?

3 blue balls

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