Cambridge Assessment International Education

Teaching Pack

Venn diagrams

Cambridge IGCSE™

Mathematics 0580

This *Teaching Pack* can also be used with the following syllabuses:
- Cambridge IGCSE™ (9–1) Mathematics **0980**
- Cambridge IGCSE™ International Mathematics **0607**
- Cambridge O Level Mathematics **4024**

Version 2.0

Cambridge Pathway

Contents

Icons used in this pack:

Lesson

Video

Assessment opportunity

Introduction

This pack will help you to develop your learners' mathematical skills as defined by assessment objective 1 (AO1 Knowledge and understanding of mathematical techniques) in the course syllabus.

Important note
Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other skills.

This content is designed to give you and your learners the chance to explore mathematical skills. It is not intended as specific practice for exam papers.

This is one of a range of *Teaching Packs*. Each pack is based on one mathematical topic with a focus on specific mathematical techniques. The packs can be used in any order to suit your teaching sequence.

In this pack you will find the lesson plans and worksheets for learners you will need to successfully complete the teaching of this mathematical skill.

Skill: Venn diagrams

This *Teaching Pack* links to the following syllabus content (see syllabus for detail):

- C1.2 Understand notation of Venn diagrams
- E1.2 Use language, notation and Venn diagrams to describe sets and represent relationships between sets

For assessments from 2025	
• C1.2	Understand and use set language, notation and Venn diagrams to describe sets.
• E1.2	Understand and use set language, notation and Venn diagrams to describe sets and represent relationships between sets.

The pack covers the following mathematical skills, adapted from **AO2: Reason, interpret and communicate mathematically when solving problems** (see syllabus for assessment objectives):

- making logical deductions, making inferences and drawing conclusions from given mathematical data
- interpreting and communicating information accurately and changing from one form of presentation to another
- assessing the validity of an argument and critically evaluating a given way of presenting information

For assessments from 2025 **AO1: Knowledge and understanding of mathematical techniques**
• understand and use mathematical notation and terminology
AO2: Analyse, interpret and communicate mathematically
• make logical inferences and draw conclusions from mathematical data or results • communicate methods and results in a clear and logical form • interpret information in different forms and change from one form or representation to another

Prior knowledge

Knowledge from the following syllabus topics is useful for development of skills in this topic.

- C1.1 / E1.1 Identify and use natural numbers, integers (positive, negative and zero), prime numbers, square and cube numbers, common factors and common multiples, rational and irrational numbers (e.g. π, $\sqrt{2}$), real numbers, reciprocals.
- C2.1 Use letters to express generalised numbers and express basic arithmetic processes algebraically. Substitute numbers for words and letters in formulae. Rearrange simple formulae. Construct simple expressions and set up simple equations.
- C2.5 Derive and solve simple linear equations in one unknown. Derive and solve simultaneous linear equations in two unknowns.

- C8.3 / E8.3 Understand that the probability of an event occurring = 1 – the probability of the event not occurring.

For assessments from 2025	
• C/E1.1	Identify and use: natural numbers, integers (positive, zero and negative) prime numbers, square numbers, cube numbers, common factors, common multiples, rational and irrational numbers and reciprocals.
• C/E2.1	Know that letters can be used to represent generalised numbers. Substitute numbers into expressions and formulas.
• C/E2.5	Construct expressions, equations and formulas. Solve linear equations in one unknown. Solve simultaneous linear equations in two unknowns.
• C/E8.1	Understand that the probability of an event not occurring = 1 – the probability of the event occurring.

Going forward

The knowledge and skills gained from this *Teaching Pack* can be used for when you teach learners about probability.

- C8.1 / E8.1 Calculate the probability of a single event as either a fraction, decimal or percentage.

For assessments from 2025	
• C/E8.1	Calculate the probability of a single event.

Before you begin

This *Teaching Pack* includes a **Teacher Introduction** video to which you should refer before using the resources in this pack. The video is available to watch in Resource Plus within the topic section relevant to this **Teaching Pack**.

The video introduces the resources available for teaching this topic and explains how they can be used to successfully deliver the topic to your learners. In particular, the video highlights typical learner misconceptions and common errors this *Teaching Pack* will help you to overcome.

Common misconceptions: Venn diagrams

Learners often count the wrong sections as they make assumptions about the exclusivity of some information. Learners really struggle with OR, thinking that if a learner studies Geography or History they cannot study both.

Lesson 1: Using Venn diagrams

Resources	• Whiteboard • Lesson 1 presentation • Worksheets 1a, 1b, 1c.
Learning objectives	By the end of the lesson: • ***all*** learners should be able understand how to use Venn diagrams to classify objects and interpret Venn diagrams

Timings	Activity
5 min	**Starter / Introduction** Teach this lesson using Lesson 1 presentation. Using slide 3, which is designed to promote a whole class discussion on how Venn diagrams work, ask learners where (and why) they would put each of the shapes a, b and c. The slide animation provides the answer.
40 min	**Main lesson** Now present learners with slide 4 which uses a Venn diagram to categorise a group of people into those with black hair and those wearing a tie. Ask the following questions: • Where do we place each person? • Where do we place people with neither black hair or a tie? Use the following slides 5 and 6 to teach the notation for the number in set A, denoted n(A) and the universal set, $\mathscr{E}$ Next, look at the iPhone question, slide 7, and ask learners to answer each of the questions on the slide. Pupils often struggle with the ruthless logic of interpreting set notation and Venn diagrams by assuming information, especially around the use of the inclusive/exclusive OR. In particular they are likely to miscount by not including all the sections. E.g. part c) How many people did not own an iPhone. Give each learner Worksheet 1a Interpreting Venn diagrams to test their understanding of how to read Venn diagrams. Finish this section of the lesson with an activity using Worksheet 1b Venn diagram puzzles (Easier) and/or Worksheet 1c Venn diagram puzzles (Harder). Learners use the descriptions to enter values into the Venn diagrams. **Differentiation:** Worksheet 1c is harder, using more complex notation and can be used with more capable learners, or those that wish to practice further.

Plenary

Using slides 8-10 with learners:

Slide 8 is a plenary question to provide an assessment opportunity for the lesson.

This is followed by a set of true and false questions (slide 9 and 10) that could be used as a whole class activity using hands or whiteboards. These are designed to assess pupil's understanding of the language of logic.

Lesson 2: Formal notation used with Venn diagrams (core)

Resources	• Whiteboard • Lesson 2 presentation • Worksheets 2a, 2b, 2c.
Learning objectives	By the end of the lesson: • ***all*** learners should be able to understand the formal notation used with Venn diagrams.

Timings	Activity
5 min	**Starter/Introduction** Teach this lesson using Lesson 2 presentation. Introduce learners to slide 3 and ask them to suggest which animals from around the World would appear where the question mark appears in the Venn diagrams. This activity is designed to be engaging and to get pupils thinking. Ask learners to suggest animals that might appear in the other sections of the Venn diagrams.
40 min	**Main lesson** Begin by working through slide 4 with learners. This should progress without pausing into the main teaching points about notation (slide 5) as it provides the perfect opportunity to introduce the notation for intersection and union. Learners are generally happy with intersection ∩ as it works in a similar way to the word AND in English. They are less happy about U because it is an inclusive OR and in English we tend to use OR in an exclusive way. There is an opportunity with some of the following examples (slides 6-8) to teach more formal notation to describe sets. Follow up by giving learners Worksheet 2a Using Venn diagram notation, which enables learners to practise use of the notation. Next prepare the card sets given on Worksheet 2b Venn diagram match-up. Produce enough sets for groups or individuals to work through this activity. The Venn diagram match-up activity is to look at the link between the worded description, the maths notation and the diagrammatic representation. Pupils often find this more challenging. Finish the main lesson by playing a game with learners using Worksheet 2c Venn diagrams notation game. This game is a competitive two player game to practise using the notation taught in this lesson.

Plenary

Finish with the activity on slide 9: Use mini-whiteboards to check understanding, by doing each of the scaffolded questions one at a time with the whole class.

Lesson 3: Formal notation used with Venn diagrams (extended)

Resources	• Whiteboard • Lesson 3 presentation • Worksheets 3a, 3b.
Learning objectives	By the end of the lesson: • some learners will be able to understand more complex notation associated with Venn diagrams, the empty set and subsets

Timings	Activity
5 min	**Starter/Introduction** Teach this lesson using Lesson 3 presentation. Using slide 3 with learners, pose this as a starter question to ensure the learners are happy with the work from previous lessons.
40 min	**Main lesson** Use the issues that arise from not being able to place numbers in all the sections of the Venn diagram in slide 4 to introduce the concept of the empty set/null set and subsets in the following slides (5-7). Introduce the correct notation and explain what is meant by a proper subset.* Give learners Worksheet 3a Venn diagram notation to complete. This worksheet enables learners to practise use of the notation and terminology. Complete the main lesson by giving each learner Worksheet 3b Venn diagram match-up to complete. The activity encourages pupils to think about how different sets might be drawn to better show their relationships. It does not focus on the notation. **Differentiation:** A good extension is to get pupils to think of other set descriptors that would produce the same diagrams or to create set descriptors that would produce different diagrams to those in the exercise. This can provide practise for writing set descriptors.
15 min	**Plenary** Using mini-whiteboards, work through the remaining problems on slides 8-16 with learners, to check their understanding. Do each of the scaffolded questions one at a time with the whole class.

*Note that proper subsets have been removed from the syllabus in 2025. Simple subsets are considered.

Lesson 4: Constructing Venn diagrams to solve problems

Resources	• Whiteboard • Lesson 4 presentation • Worksheets 4a, 4b.
Learning objectives	By the end of the lesson: • all learners should be able to draw and use Venn diagrams to solve a range of problems.

Timings	Activity
5 min	**Starter/Introduction** Teach this lesson using Lesson 4 presentation. Introduce the problem to learners on slide 3. This links back to the end of lesson 1. Learners often believe that the question is a trick as the numbers do not add up to 30; they are caught in the logic of assuming that a learner doing History cannot do Geography, that they are somehow exclusive to each other. Drawing the Venn diagram without numbers in (the slide animates to show this) is a good way to get pupils started. When this initial misconception has been overcome many learners can answer the question using trial and error. You should demonstrate that trial and error is not necessary, by calculating the intersection as the excess. This can be formalised using algebra with the right group. The important teaching point here is that learners need to avoid making assumptions about exclusivity and start with the overlap, working outwards. The next example reinforces this idea further.
40 min	**Main lesson** Another example is given on slide 4, this time with three sets, but stepped through to emphasise the points described above. Challenge learners to provide the answer. Follow up with an activity using Worksheet 4a Problem solving with Venn diagrams. On this worksheet, scaffolded questions get steadily more difficult, and require learners to complete and then draw their own Venn diagrams. **Differentiation:** There are some more interesting problems later on connected to alternative question types involving algebra. **Differentiation:** Complete the main lesson with an activity (could be used as an extension activity) using Worksheet 4b Problem solving puzzle.
15 min	**Plenary** Present and work through slide 5 with learners, which is a demonstration of a related problem that uses algebra skills. This relates to the second half of the worksheet used previously in this lesson.

Lesson 5: Calculating simple probabilities using Venn diagrams

Resources	• Whiteboard • Lesson 5 presentation • Worksheet 5a.
Learning objectives	By the end of the lesson: • all learners should be able to calculate simple probabilities using a Venn diagram.

Timings	Activity
5 min	**Starter/Introduction** Teach this lesson using Lesson 5 presentation. Introduce learners to the problem displayed on slide 3 and work through it with them in class. Learners should be able to use their understanding of Venn diagrams from the previous lessons to answer this first question.
40 min	**Main lesson** Introduce slide 4 to learners. There is a second question which provides an assessment opportunity and to ensure that learners are happy with probability before attempting the worksheet that follows. Give learners Worksheet 5a Probability with Venn diagrams. This contains scaffolded questions that get steadily more difficult and require learners to complete and then draw their own Venn diagrams to solve a range of probability questions.
15 min	**Plenary** Finish the lesson with slide 5, a standard problem to assess pupil progress and to ensure that they start in the middle of the Venn diagram.

Links to websites: Venn diagrams

The following links to websites provide further opportunities to create activities related to this topic are:

Intersection and union of sets:
https://youtu.be/jAfNg3ylZAI

Venn diagrams are introduced by the BBC:
https://www.bbc.com/education/guides/z8nfrdm/revision/1

Sets notation is linked to Venn diagrams by Maths is Fun
https://www.mathsisfun.com/sets/venn-diagrams.html

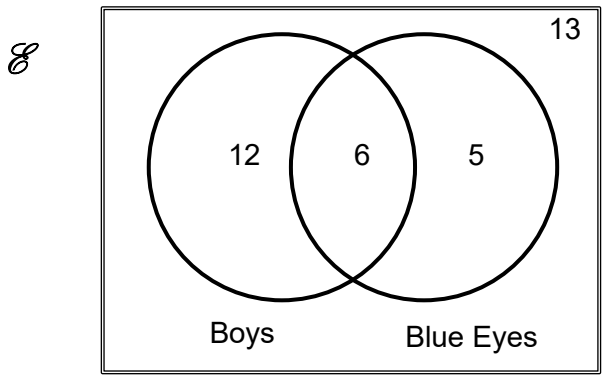
A stretch challenge for Venn diagrams is available via nrich here:
https://nrich.maths.org/7053

Worksheets and answers

	Worksheets	Answers
For use in the *Lesson 1:*		
1a: Interpreting Venn diagrams	**15-17**	**43**
1b: Venn diagram puzzles (Easier)	**18-21**	**44-47**
1c: Venn diagram puzzles (Harder)	**22-24**	**48-50**
For use in *Lesson 2:*		
2a: Using Venn diagram notation	**25-26**	**51**
2b: Venn diagram match-up	**27-28**	**52**
2c: Venn diagrams notation game	**29-30**	
For use in *Lesson 3:*		
3a: Venn diagram notation	**31-33**	**53-54**
3b: Venn diagram match-up	**34-35**	**55-57**
For use in the *Lesson 4:*		
4a: Problem solving with Venn diagrams	**36-37**	**58-59**
4b: Problem solving puzzle	**38**	**60**
For use in the *Lesson 5:*		
5a: Probability with Venn diagrams	**39-42**	**61-63**

Worksheet 1a**:** Interpreting Venn diagrams

1. A group of boys and girls are shown in this Venn diagram



(a) How many boys are there?

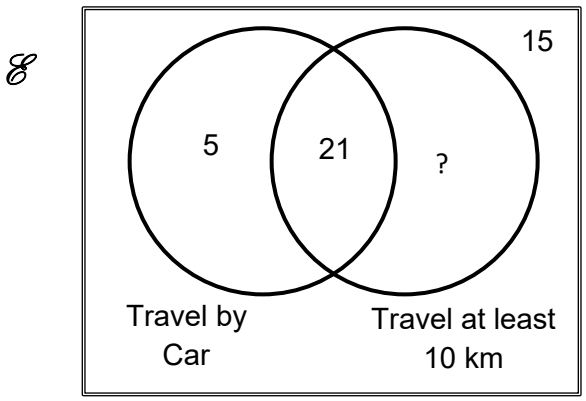
(b) How many boys are there with blue eyes?

(c) How many girls are there?

(d) How many people are in the group?

(e) How many girls have blue eyes?

2. 50 people were asked how far they travelled to work.



(a) How many people travelled by car?

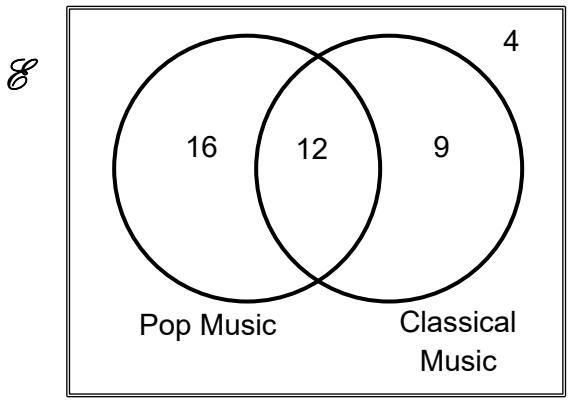
(b) What is the missing number in the Venn diagram?

(c) How many people travelled less than 10 km to work?

(d) How many people travelling less than 10 km travelled by car?

Worksheet 1a**:** Interpreting Venn diagrams *continued*

3. Some people were asked whether they liked pop music or classical music



(a) How many people liked pop music?

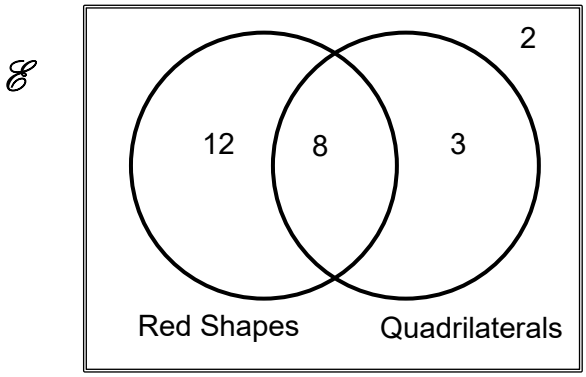
(b) How many people liked both types of music?

(c) How many people liked just one type of music?

(d) How many people liked at least one type of music?

(e) How many people just liked classical music?

4. Some shapes were sorted into two sets.



(a) How many red shapes were there?

(b) How many shapes had 4 sides?

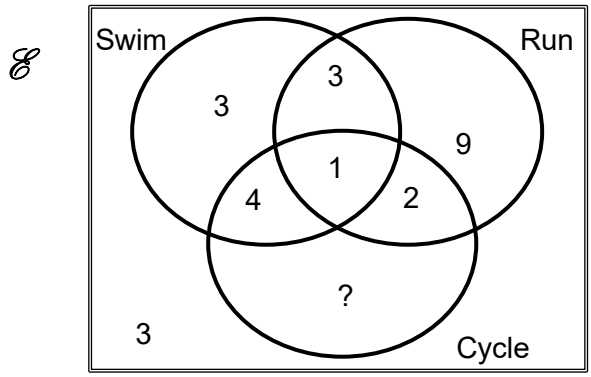
(c) Where would you place a blue triangle in the diagram?

(d) How many shapes were there altogether?

(e) How many red quadrilaterals are there?

Worksheet 1a**:** Interpreting Venn diagrams *continued*

5. 30 people were asked what sports they liked.



(a) Find the missing number.

(b) How many people liked to swim?

(c) How many people liked all three sports?

(d) How many people didn’t like running?

(e) How many people liked exactly two sports?

Worksheet 1b: Venn diagram puzzles (Easier)

For each Venn diagram, think of a number that would fit in each of the labelled regions. If you cannot find a number for a specific region explain why.

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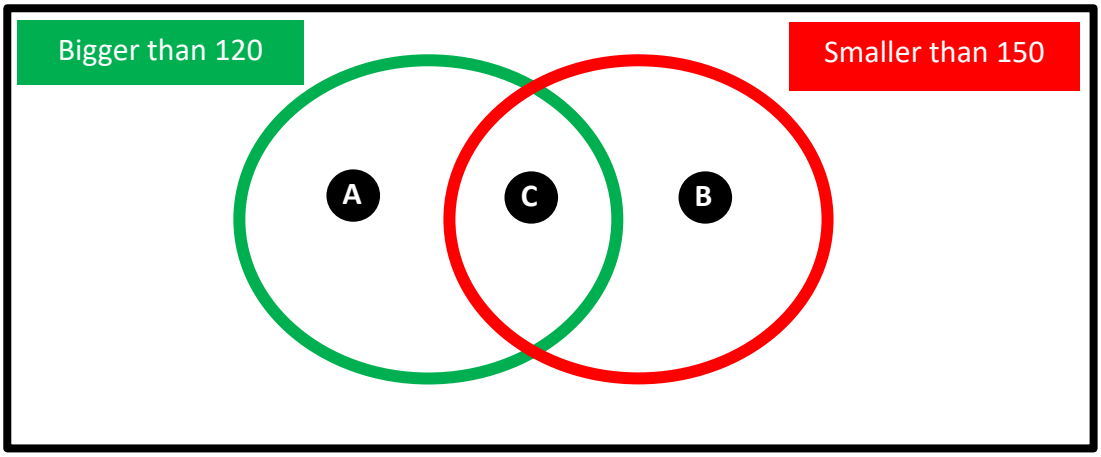

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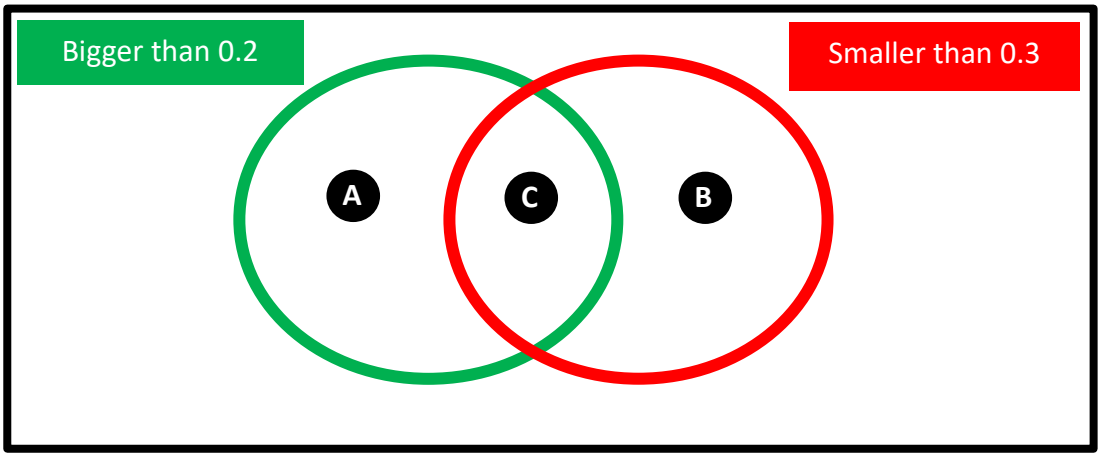

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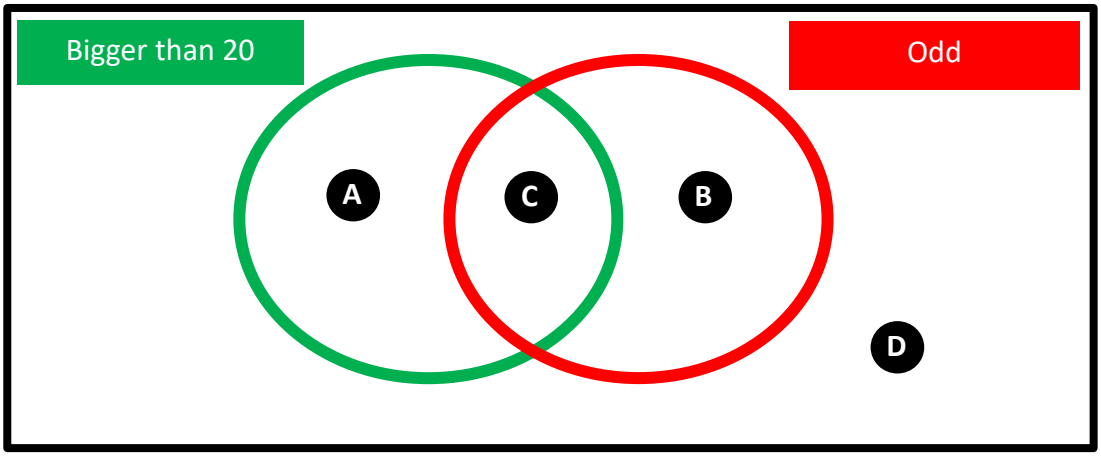

Worksheet 1b: Venn diagram puzzles (Easier) *continued*

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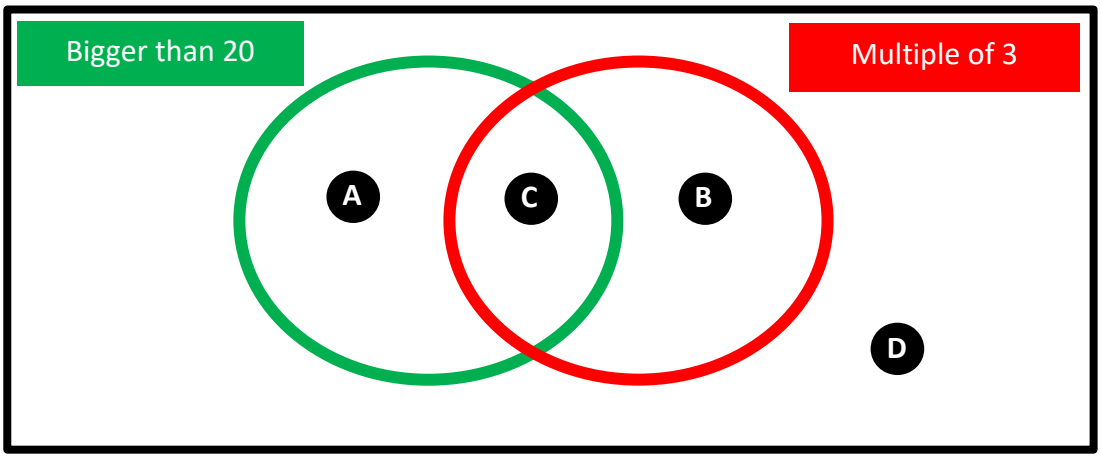

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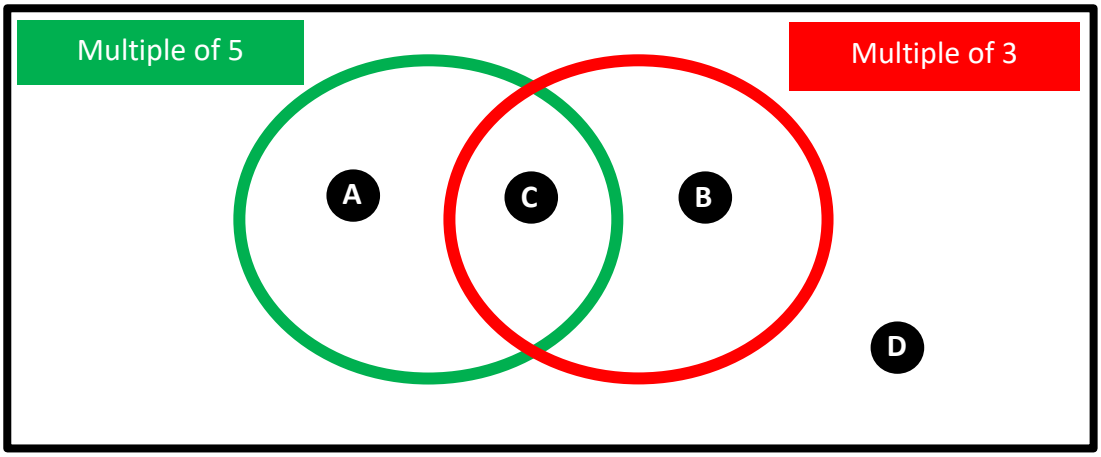

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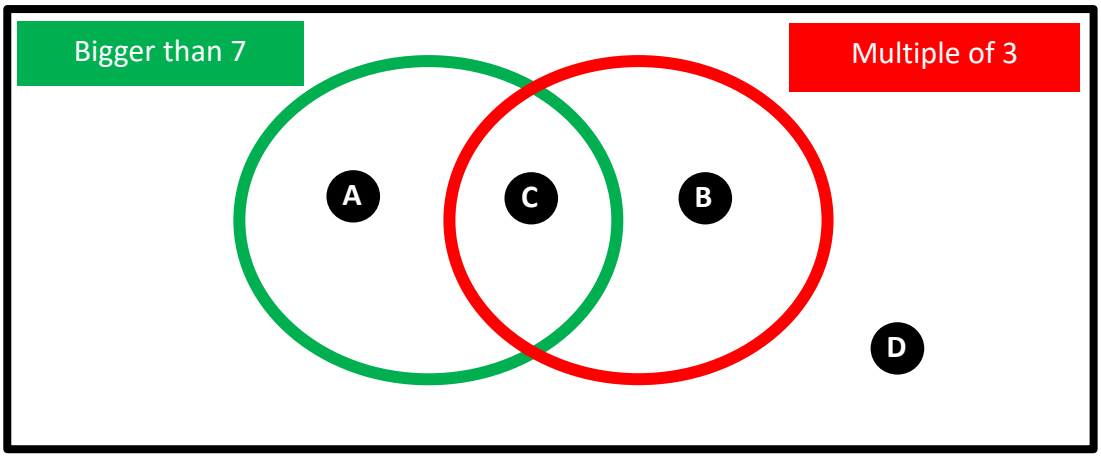

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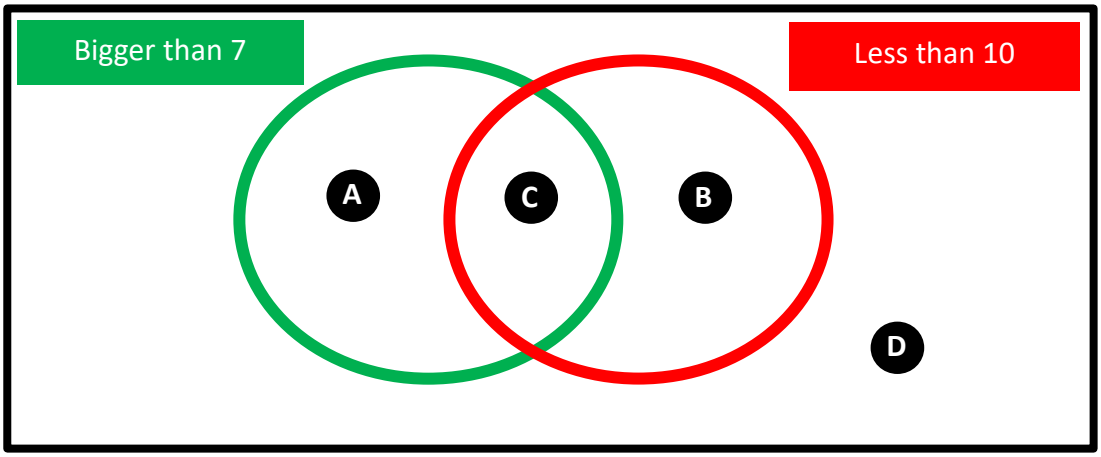

Worksheet 1b: Venn diagram puzzles (Easier) *continued*

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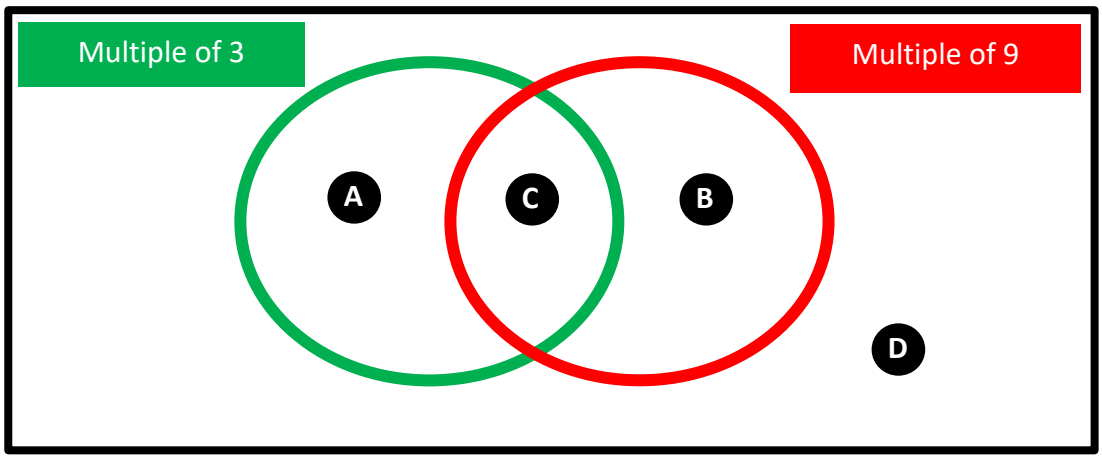

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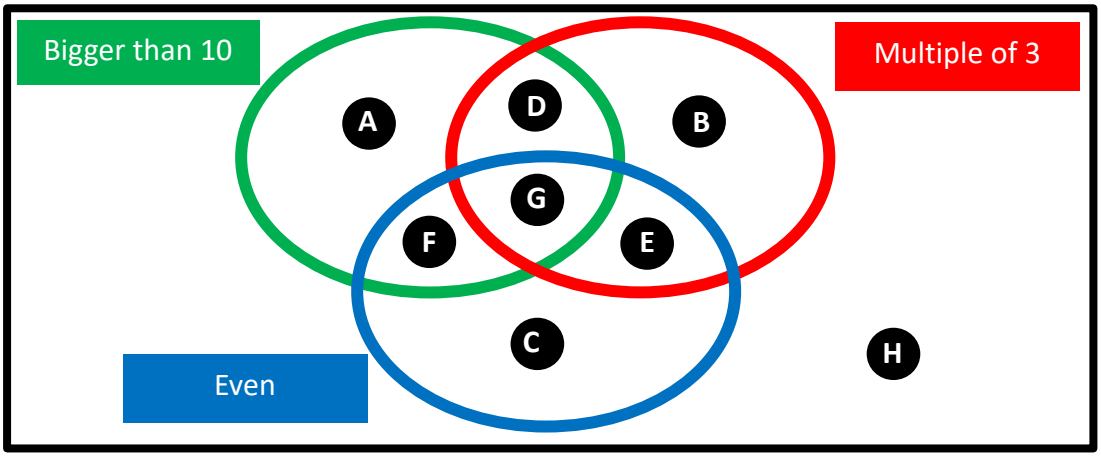

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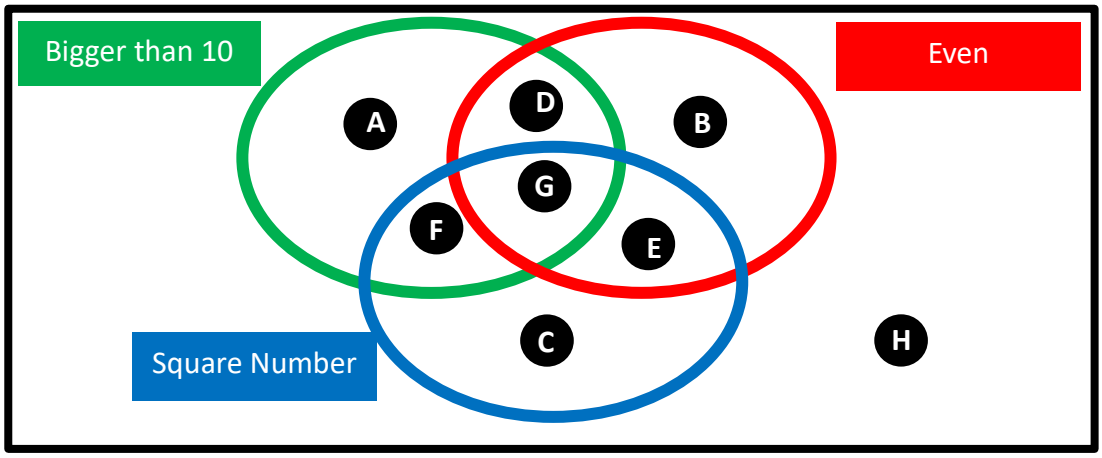

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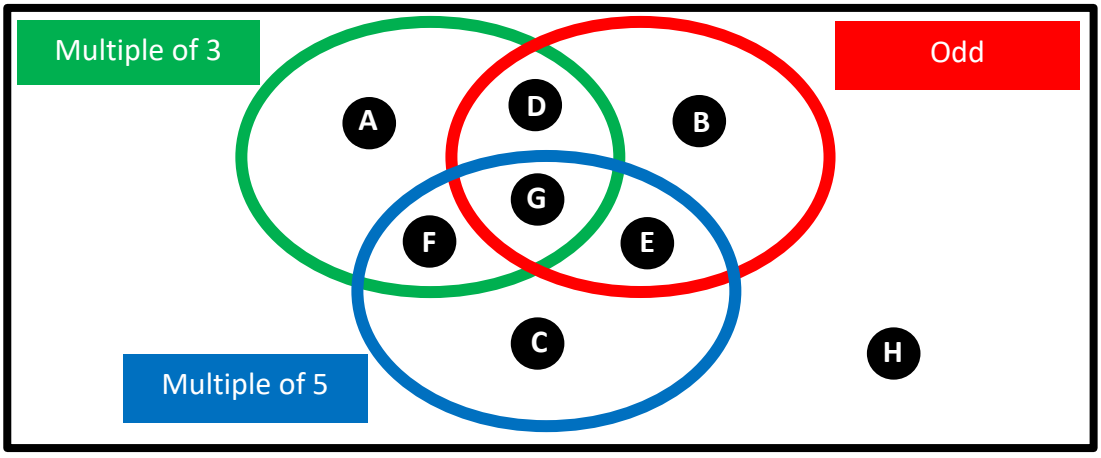

Worksheet 1b: Venn diagram puzzles (Easier) *continued*

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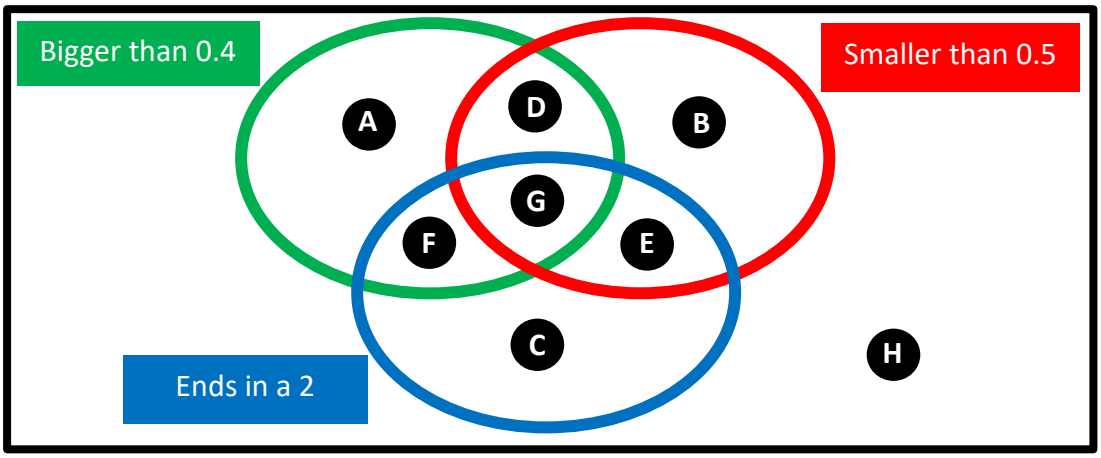

Worksheet 1c: Venn diagrams puzzles (Harder)

For each Venn diagram write an appropriate answer in each section based on the definition for each set. In each case ξ = set of natural numbers.

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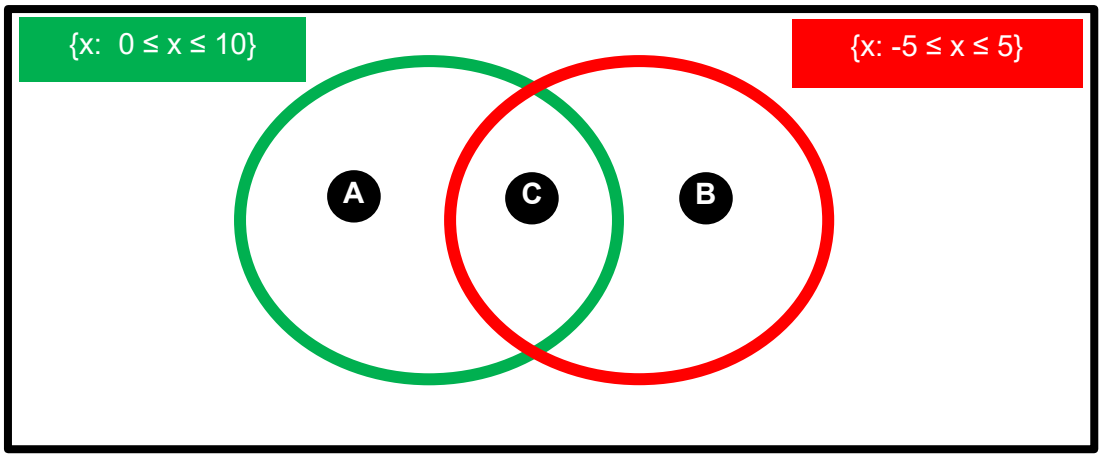

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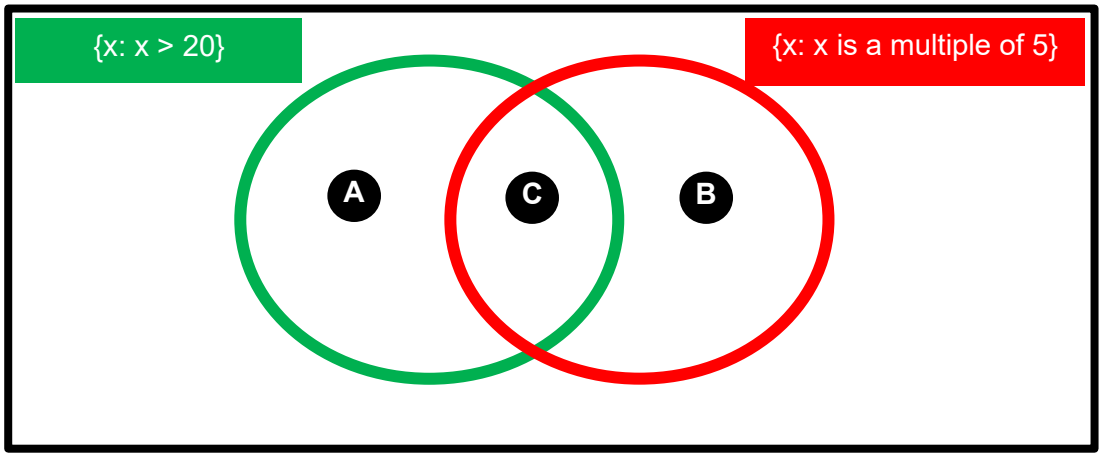

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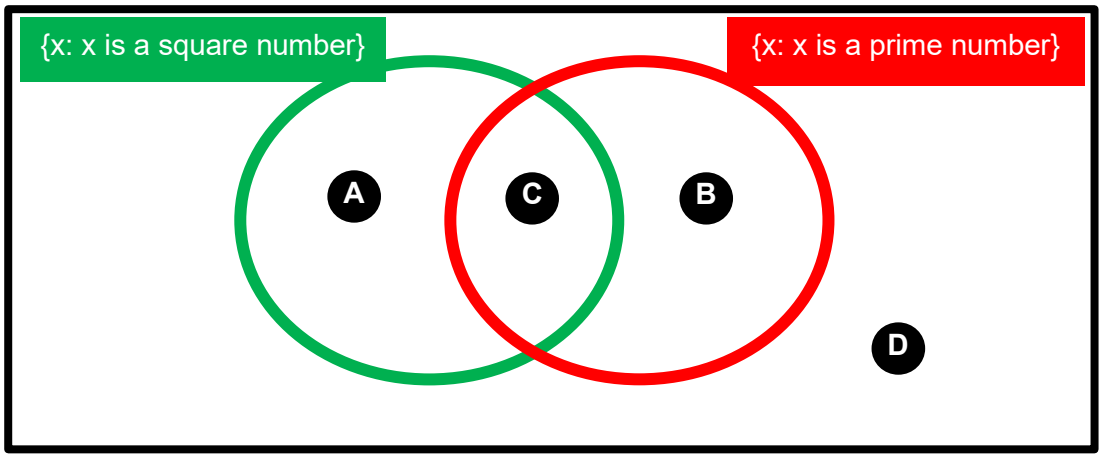

Worksheet 1c: Venn diagrams puzzles (Harder) *continued*

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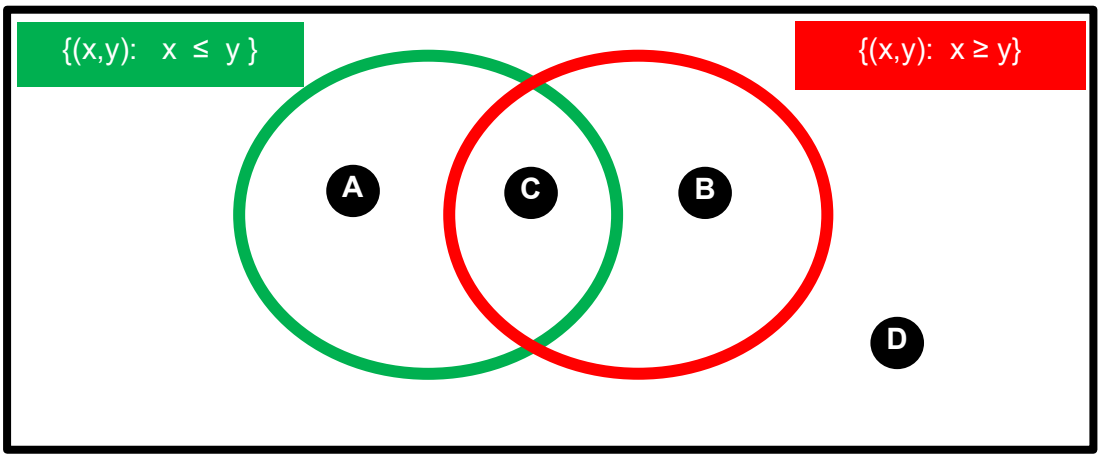

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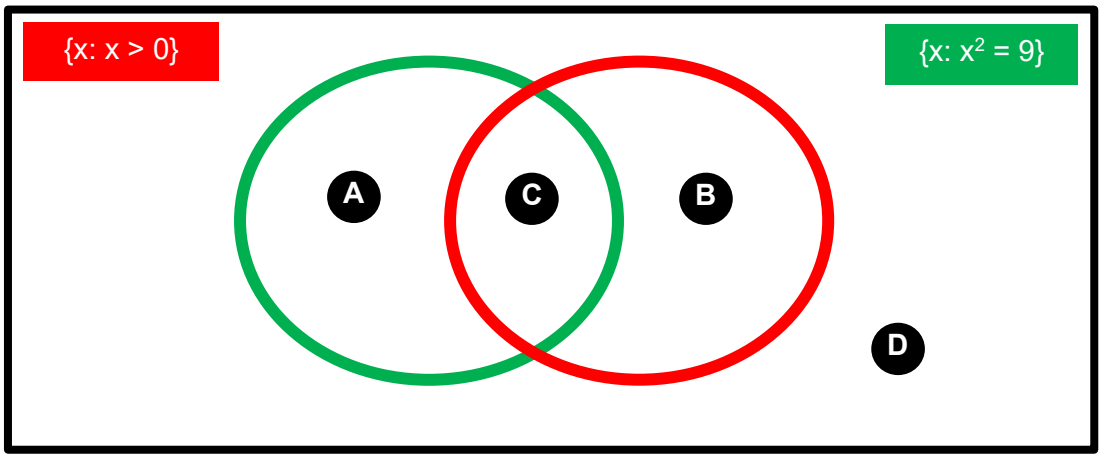

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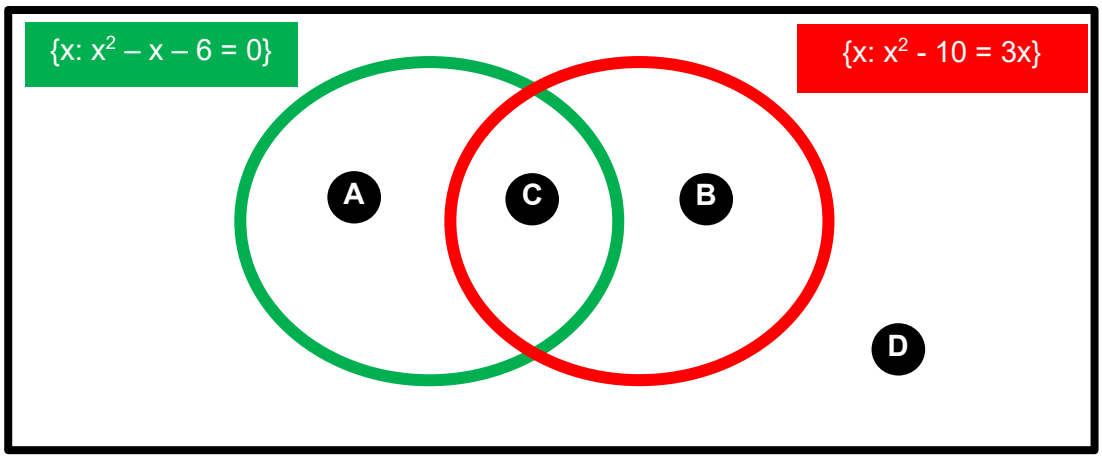

Worksheet 1c: Venn diagrams puzzles (Harder) *continued*

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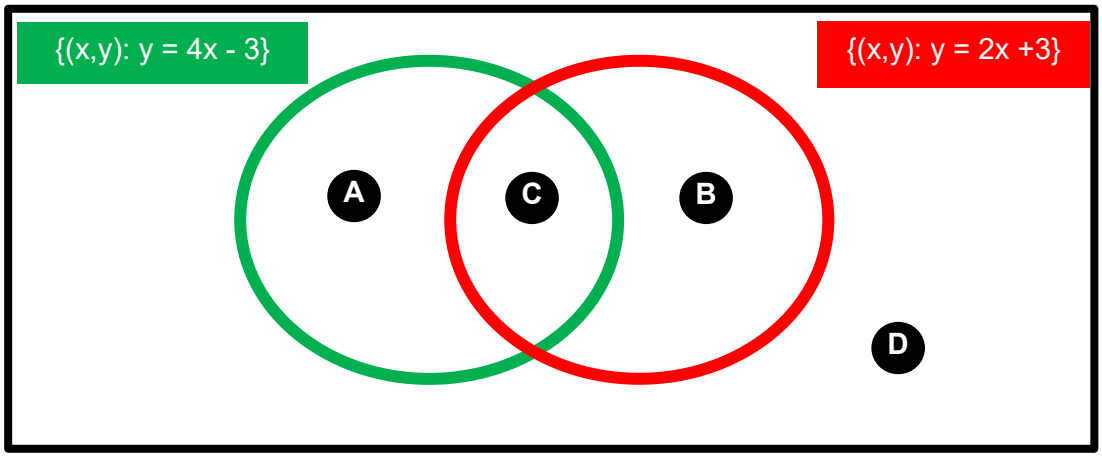

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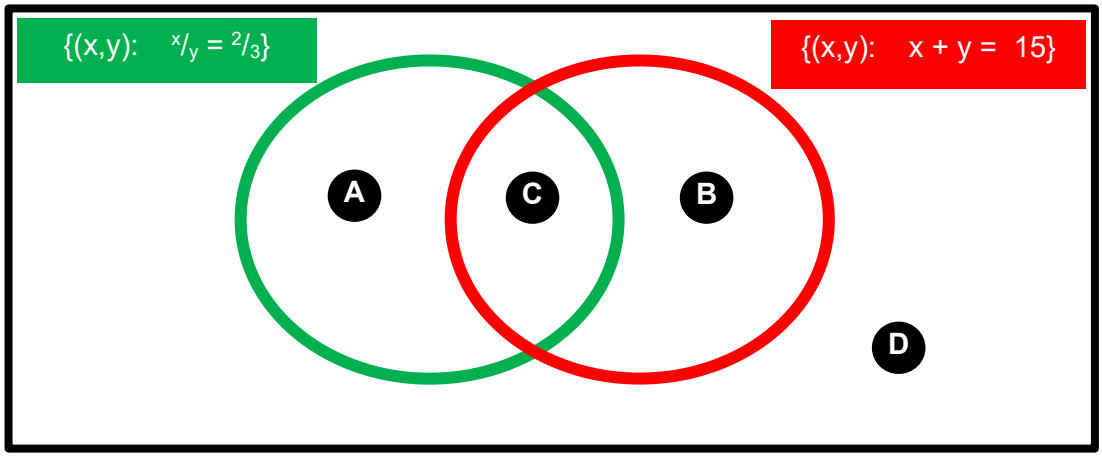


Worksheet 2a: Using Venn diagram notation

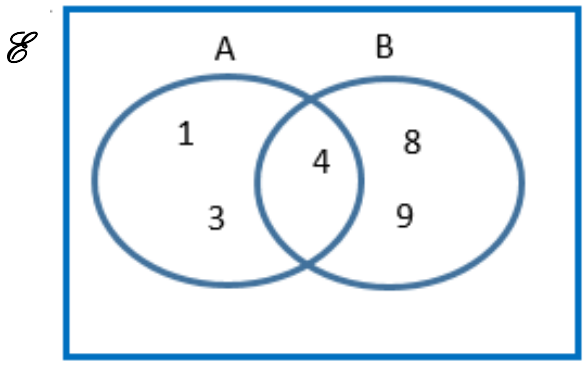
1) Look at the Venn Diagram opposite, find:

a) n(A)

b) n(B′)

c) n(A ∩ B)

d) n(A ∪ B)



2) Look at the Venn Diagram opposite.

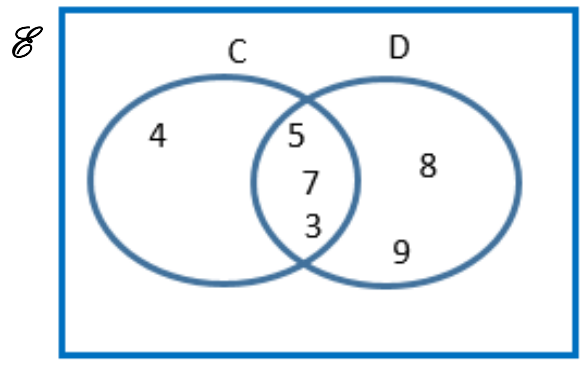
Write down all the elements in:

a) Set C

b) Set D′

c) Set C ∩ D

d) Set C ∪ D



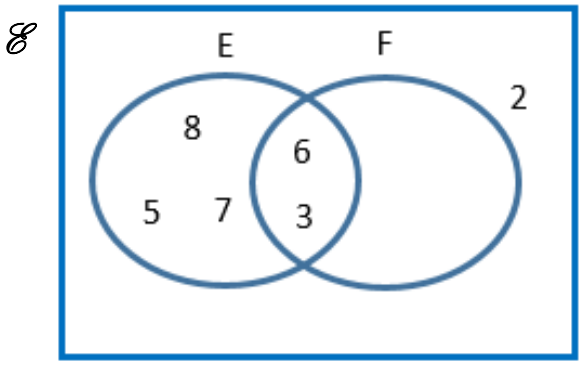
3) Look at the Venn Diagram opposite, find:

a) n(E)

b) n(E ∩ F)

c) n(E ∪ F)

d) n(E’ ∩ F’)



4) Look at the Venn Diagram opposite.
Write down all the elements in:

a) Set G

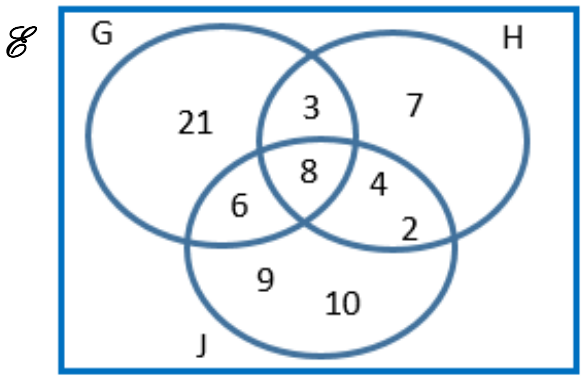
b) Set H ∩ J

c) Set (H ∩ J) ∩ G

d) Set G ∪ H

e) Set (G ∪ H)′ ∩ J

f) Set J′ ∩ H



Worksheet 2a: Using Venn diagram notation *continued*

5) Let $\mathscr{E}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {x: x is a square number}

B = {y: y is an even number}

Find:

a) n(A)

b) n(A ∩ B)

c) n(A ∪ B)

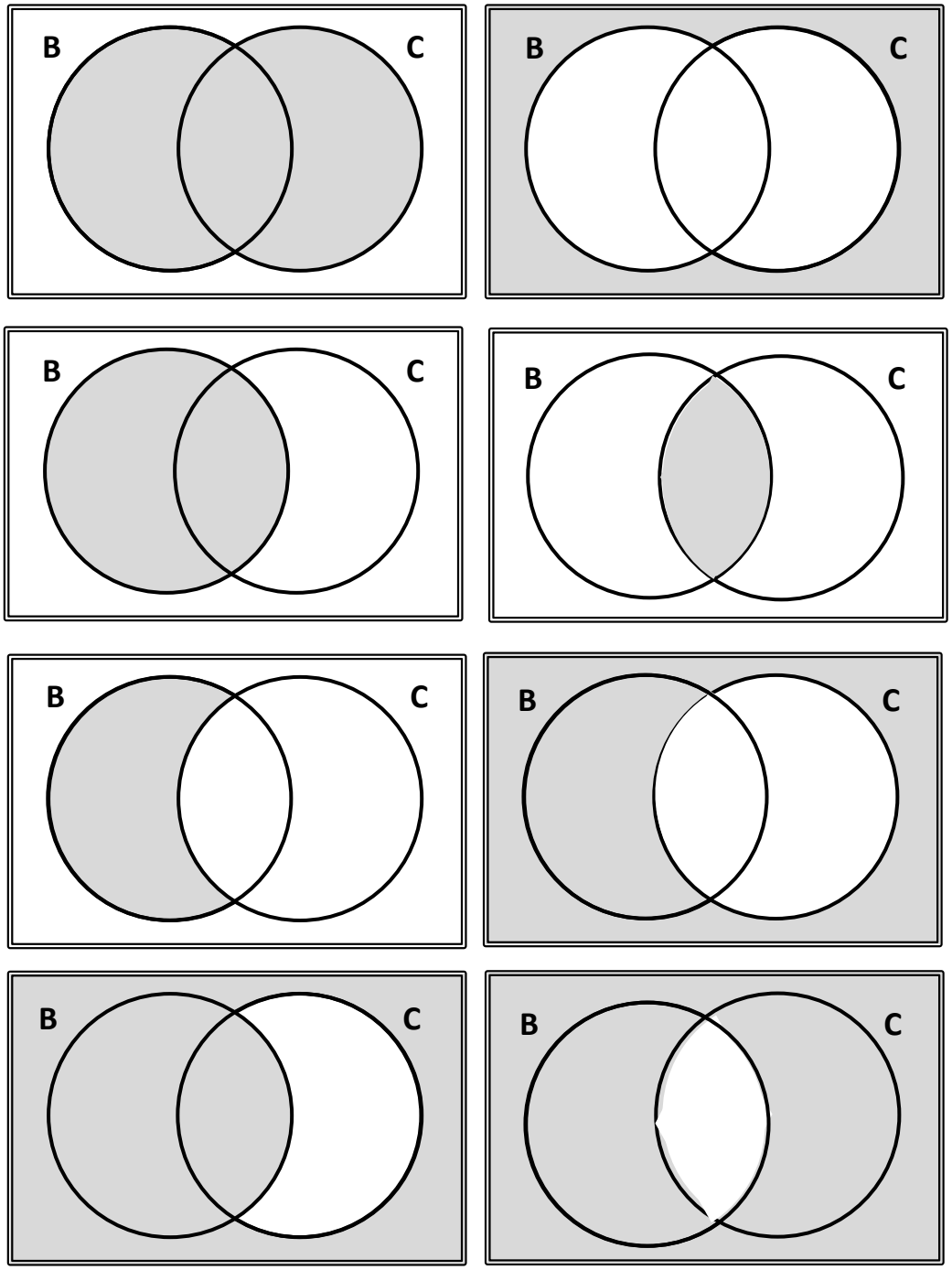
d) Define a set C, so that n(A ∩ C) is 2 and n(B ∩ C) is 0. Describe the set C. Is C unique?

Worksheet 2b: Venn diagram match-up

Cut out the cards and match them so that each set contains a Venn diagram, a worded description and a description using Venn diagram notation:

People that don't own a car	People who own a bicycle	$(B \cup C)'$	B
Everyone apart from those who own just a car	People who own neither a car nor a bicycle	C'	$B \cup C$
People who own just a bicycle	Everyone unless they own both a Bicycle and a Car	$B \cup C'$	$B \cap C'$
People who own both a car and a bicycle	People who own either a car or a bicycle	$(B \cap C)'$	$B \cap C$

Worksheet 2b: Venn diagram match-up *continued*



Worksheet 2c: Venn diagrams notation game

Rules of the Game

Start with the numbers 1 to 20.

Players take it in turns to build a set of numbers using two of the defined sets below and either the Intersection or Union symbol.

If the set that they have created contains no more than three unclaimed numbers, players claim the numbers for themselves. Points are scored according to the value of each number claimed.

Once claimed a number is out of the game and no longer counts towards subsequent sets.

No player can ever claim more than three numbers in a single turn. If a player creates a set that would enable more than three numbers to be claimed then they miss their turn.

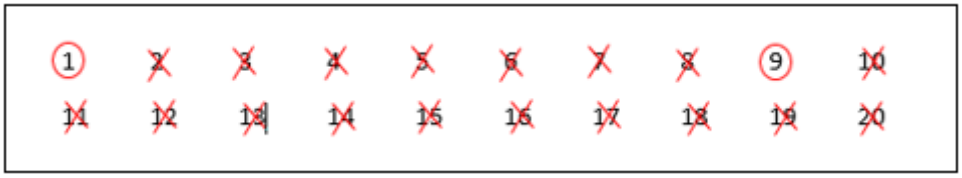
The game ends when no more numbers remain. The winner is the player with the most points.

Sets to use in the game

E = Even Numbers	E′ = Complement of Even Numbers
P = Prime Numbers	P′ = Complement of Prime Numbers
S = Square Numbers	S′ = Complement of Square Numbers
L = Numbers less than 11	L′ = Complement of Numbers Less than 11
F = Factors of 20	F′ = Complement of Factors of 20
M = Multiples of 5	M′ = Complement of Multiples of 5
A = Multiples of 3	A′ = Complement of Multiples of 3

A set is created by choosing two sets above (they may be the same) and either $\cup$ *or* $\cap$.

E.g. E′ ∩ S would give the numbers 1 and 9 scoring 10 points and removing 1 and 9 from the game.



Rule alterations

1) Try experimenting with your own sets

2) Try playing with the numbers from 1 to 30

Worksheet 2c: Venn diagrams notation game *continued*

Player board

	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Worksheet 3a: Venn diagram notation

1) Let $\mathscr{E}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Let A = Prime numbers Let B = Even numbers

Write down all the elements in:

a) $A \cap B$

b) $A \cup B$

c) B'

d) $A \cap B'$

2) Let $\mathscr{E}$ = Natural Numbers

Let W = Factors of 24 Let X = Multiples of 3

Let Y = Multiples of 5 Let Z = Factors of 30

a) Describe the elements in $X \cap Y$

b) Find:

i) n(W)

ii) $n(Z \cap Y)$

iii) n(Z U W)

3) Let $\mathscr{E}$ = {Natural Numbers}

Let P = {1, 2, 3, 4, 5, 6, 7} Let Q = {2, 4, 6, 8}

Let R = {1, 3, 5} Let S = {8, 9, 10}

Draw a Venn diagram to show the sets P, Q, R and S

Worksheet 3a: Venn diagram notation *continued*

4) $\mathscr{E}$ = {Triangles}
R = {Shapes with a right angle} I = {Shapes with two angles the same}
E = {Shapes with three angles the same}

a) Draw a shape in the set R ∩ I

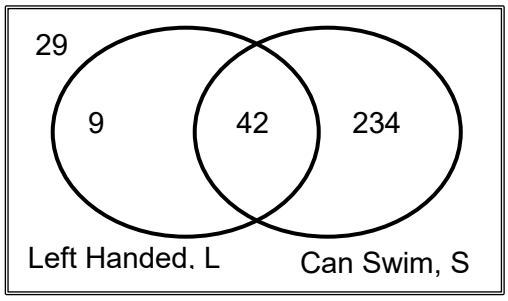
b) Explain why the set R ∩ E is empty

5) $\mathscr{E}$ = {Polygons}
R = {Shapes with all internal angles equal} S = {Shapes with all sides the same length}
F= {Shapes with four sides}

a) Describe the set R ∩ S

b) Describe the set F ∩ (R ∩ S). Is this the same as the set (F ∩ R) ∩ S ?

6) The Venn diagram shows the number of pupils in a primary school who are left handed (L) and whether they can swim (S).



a) How many pupils are in the school?

b) How many pupils are in S′? Describe these pupils.

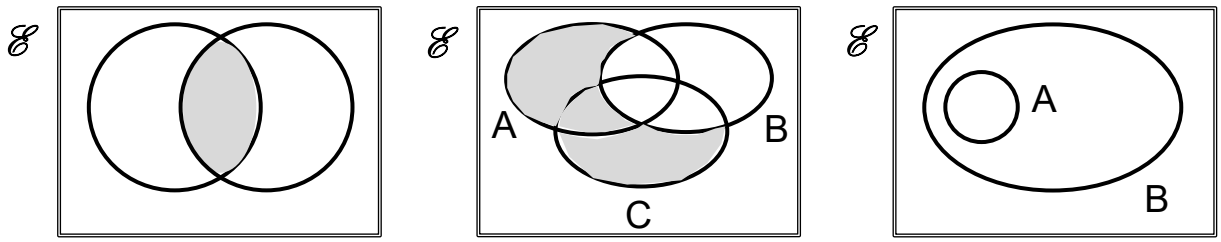
c) How many pupils are in L ∪ S?

d) How many pupils are in L ∩ S? Describe these pupils.

Worksheet 3a: Venn diagram notation *continued*

7) Shade the region represented by

a) $A \cap B$ b) $(A \cup C) \cap B'$ c) $A \cup B'$

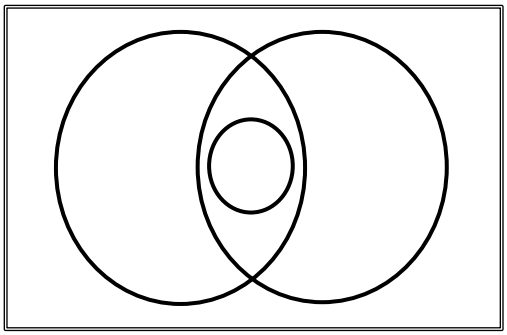


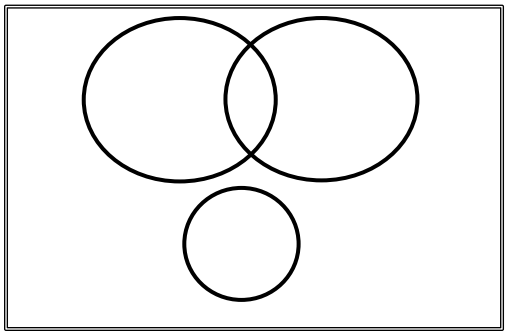
8) A, B and C are sets. Let $A \subset B$, $C \subset B$ and $A \cap C = \emptyset$. Draw a diagram to illustrate these sets.

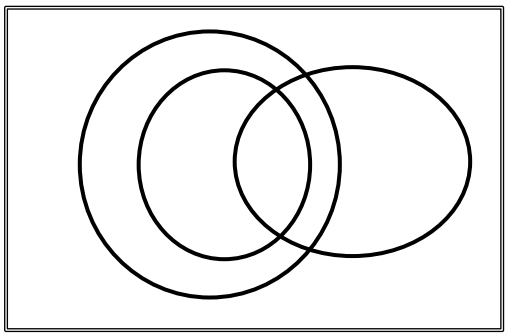
9) How many subsets are there of the set {x: x is a factor of 6}?

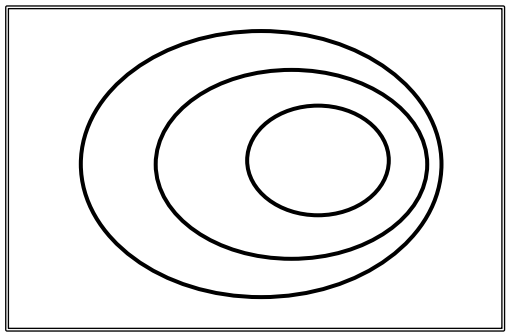
Worksheet 3b: Venn diagram match-up

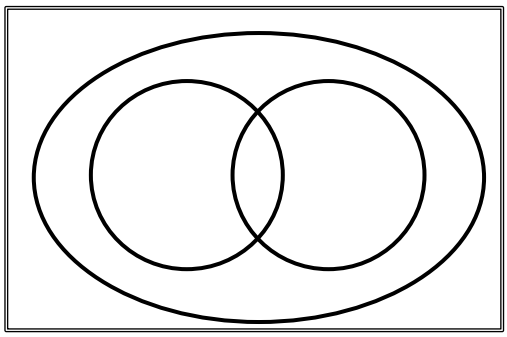
Cut out the cards to make sets for each learner or group. Learners then match the Venn diagram to the relationships.

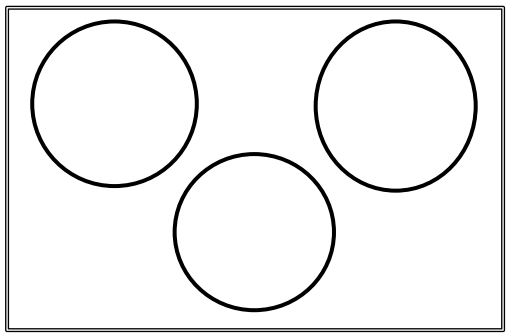


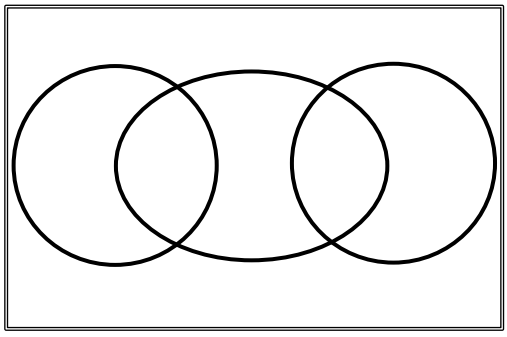


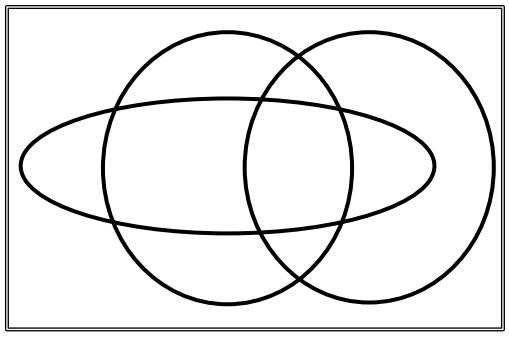


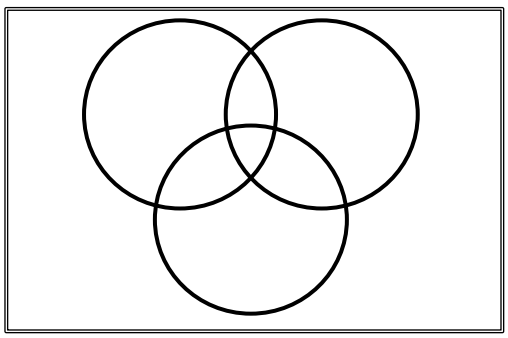


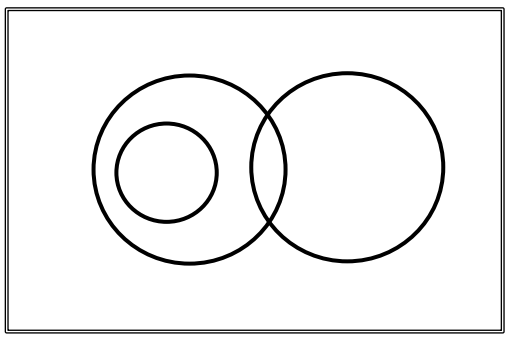












Worksheet 3b: Venn diagram match-up *continued*

A = {x: x is a multiple of 3}

B = {y: y is a multiple of 6}

C = {z: z is a multiple of 5}

A = {x: x is a factor of 7}

B = {y: y is a factor of 5}

C = {z: z is a factor of 70}

A = {x: x < 20}

B = {y: y < 30}

C = {z: z <40}

A = {x: x is a square number}

B = {y: y is a prime number}

C = {z: z is an even number}

A = {x: x is a multiple of 3}

B = {y: y is a multiple of 5}

C = {z: z is a multiple of 7}

A = {x: x is a prime number}

B = {y: y is even}

C = {z: z is a multiple of 4}

A = {x: x is a number that ends in a 2}

B = {y: y is a number that ends in a 3}

C = {z: z is a number that ends in a 5}

A = {x: x is a square number}

B = {y: y is an even number}

C = {z: z is a multiple of 3}

A = {x: x is a factor of 20}

B = {y: y is a factor of 30}

C = {z: z > 40}

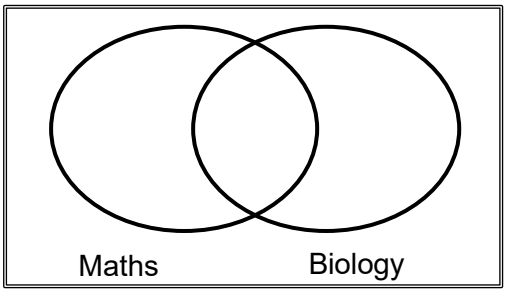
A = {x: x is a factor of 20}

B = {y: y is a factor of 10}

C = {z: z is a factor of 5}

Worksheet 4a: Problem solving with Venn diagrams

1) In a group of 70 learners, 50 study maths, 29 study biology and 13 learners study both maths and biology.

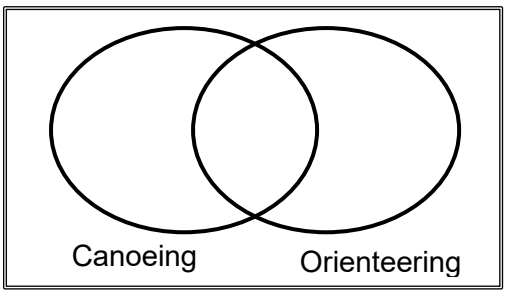


How many learners study:

a) Only maths?

b) Neither maths or biology?

2) Fifty seven people attended at activity centre. On the first day everyone signed up to at least one activity. The first activity was to go orienteering and the second activity was to go canoeing.



If 43 people signed up to orienteering and 37 people signed up to go canoeing, how many people signed up for only one activity?

3) A class of 35 learners were asked to choose an activity. They all chose at least one activity. 24 chose to read and 30 chose to do art. Draw a Venn diagram to show the learner's choices.

a) How many learners chose to do both activities?

b) How many learners chose only to read?

4) At a village fete there are three competitions: The raffle, the best cake competition and the largest pumpkin competition. Six hundred people attended the fete. 170 of the people had not entered any of the competitions. 20 had entered all three competitions, 70 had entered both the raffle and the best cake competition, 40 had entered both the raffle and the largest pumpkin competition. 30 people had entered both the largest pumpkin and the best cake competition. Altogether 50 people had entered the largest pumpkin competition and 100 the best cake competition. How many people entered just the raffle and no other competition?

Worksheet 4a: Problem solving with Venn diagrams *continued*

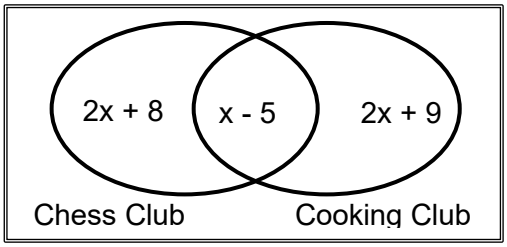
5) In a survey of 200 people, 70 said that they had not used a bus or a train in the past month. 100 said that they had used a bus and 70 said that they had used a train. How many people used both a bus and a train?

6) Fifty people were asked if they had eaten lunch or breakfast on a particular day. Two people ate neither lunch or breakfast and twelve had eaten only lunch. Three quarters of the people who said that they had eaten breakfast had also eaten lunch. How many people ate both lunch and breakfast?

7) A Venn diagram contains two sets. $n(\xi) = 20$, $n(A) = 7$, $n(A \cup B) = 12$, $n(B') = 12$.

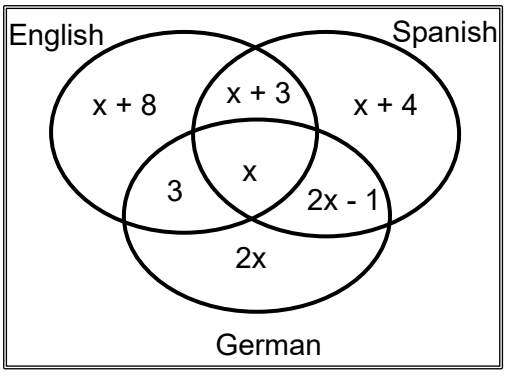
Find $n(A \cap B)$.

8) The Venn diagram shows how many people attended each of two clubs.



If 62 people attended at least one club, what is the value of 'x'?

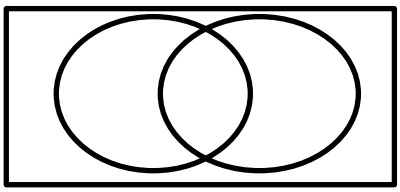
9) A Venn diagram was drawn to show what languages people spoke. The same number of people spoke English as spoke Spanish.



How many people spoke German?

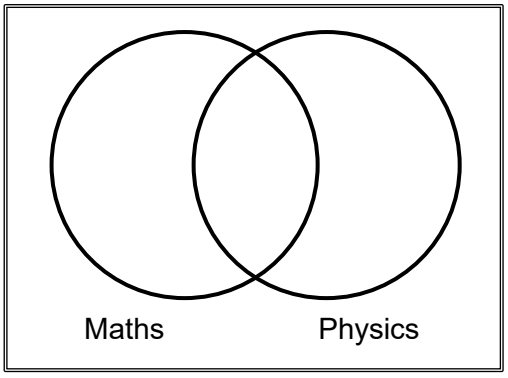
Worksheet 4b: Problem solving puzzle

Seven learners: Asha (f), Carmen (f), Dee (f), Eduardo (m), George (m), Isabelle (f), Juan (m) went to school. During the day some of them received homework. Use the Venn diagram and the clues given to decide who had homework and in which subject.

Juan had less homework than Eduardo	Three learners were set maths homework
Isabelle has no homework	Asha had less homework than Carmen
Only girls received maths homework	Carmen had 2 pieces of homework
Two learners had no homework	Equal numbers of male and female learners were set english homework
Homework was only set in maths and english, and then at most one piece per subject	

Worksheet 5a: Probability with Venn diagrams

1) In a group of 40 learners, 28 study maths, 19 study physics and 12 learners study both physics and maths.

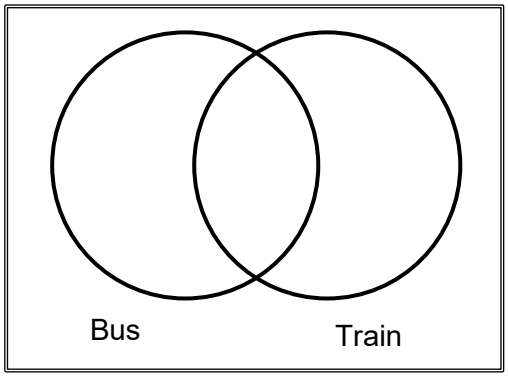


If I select a learner at random, what is the probability that they study:

a) Only maths?

b) Neither maths or physics?

2) 50 people were asked what public transport they had used in the last month. 31 said that they had used a bus and 23 said that they had used a train. 12 had not used public transport at all.



a) Complete the Venn Diagram.

Use the diagram to find the probability that a randomly chosen member of the group:

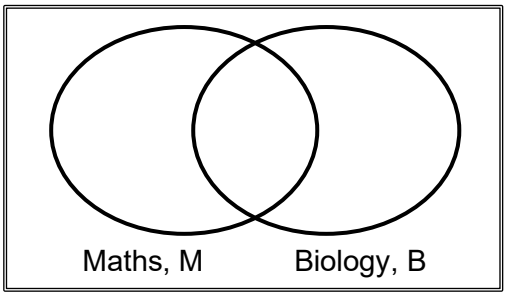
b) Travelled by train in the last month?

c) Travelled by both train and bus in the last month?

Worksheet 5a: Probability with Venn diagrams *continued*

3) In a school sixth form there are 100 pupils. Fifty of these learners study maths (M), 29 study biology (B) and 13 study both maths and biology.

a) Draw a Venn diagram to show this information



If a learner is chosen at random from the sixth form, find:

a) P(M)

b) P(B)

c) P(M′)

d) P(M ∪ B)

e) P(M ∩ B)

f) P(M′ ∩ B)

4) People were asked if they were right or left handed. The group consisted of 50 women and 60 men. 38 of the women were right handed, 13 of the men were left handed. Draw a Venn diagram showing this information. Use the Venn diagram to find the probability that a randomly chosen person from the group was right handed.

5) The probability that an archer hits a target with their first arrow is 0.3. The probability that they hit the target with their second arrow is 0.5. The probability that they hit the target with both of their first two arrows is 0.1. What is the probability that the archer hits the target with:

a) At least one of their first two shots

b) Neither of their first two shots

Worksheet 5a: Probability with Venn diagrams *continued*

6) Passengers with an airline can buy a meal on the flight (M) and buy duty free goods (D).
When a person is selected at random from the passenger list:
$P(M) = 0.5$, $P(M \cap D') = 0.4$ and $P(M' \cap D') = 0.3$
If there are 20 people who buy a meal and duty free goods:

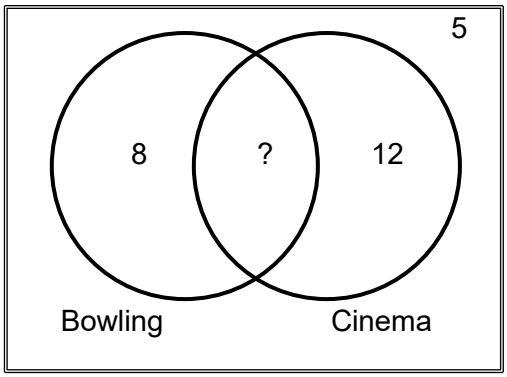
a) Draw a Venn diagram for the passengers on the flight

b) Find $P(M' \cap D)$

c) Find $P(M \cup D)$

7) Jean is organizing two activities as part of a Vacation club. Jean charges \$12 per person to organise the bowling and \$9 per person to organise the cinema trip. Overall she collects \$309.

What is the probability that a randomly selected member of the club has chosen to do both activities?

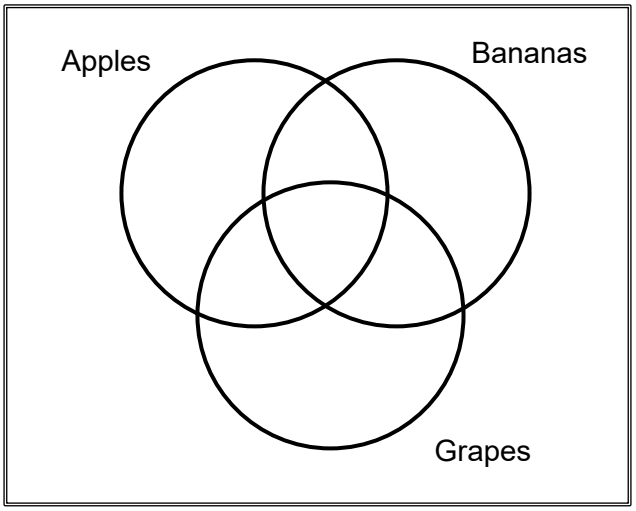


8) In a survey of the UK population 60% of people said that they did at least some Christmas shopping online in 2017. 80% said that they did at least some Christmas shopping on the high street. 10% said that they did not do any Christmas shopping at all. What percentage of the UK population did both online and high street shopping for Christmas?

Worksheet 5a: Probability with Venn diagrams *continued*

9) A group of 100 people were asked if they liked different fruit. 40 said they liked apples, 30 said they liked bananas and 30 said they liked grapes. 5 liked all 3 fruits. 17 liked apples and grapes, 4 liked just apples and 10 liked both grapes and bananas. Everyone liked at least one fruit.

a) Complete the Venn Diagram to show this information



b) If a member of the group is selected at random find the probability that they only like bananas?

10) A group of 60 people were asked what they ate for lunch. $^3/_4$ of the group had a sandwich, $^1/_3$ the group had a packet of crisps. Half of those that had crisps also ate fruit. A third of sandwich eaters also ate crisps. 12 had both sandwiches and fruit and $^1/_{10}$ of the group had all 3 items. Every member of the group ate at least one item.

Find the probability that someone chosen randomly from the group:

a) Ate only one item for lunch?

b) Ate a fruit for lunch?

c) Ate exactly two items for lunch?

Worksheet 1a: Answers

Question 1
a) 18
b) 6
c) 18
d) 36
e) 5

Question 2
a) 26
b) 9
c) 20
d) 5

Question 3
a) 28
b) 12
c) 25
d) 37
e) 9

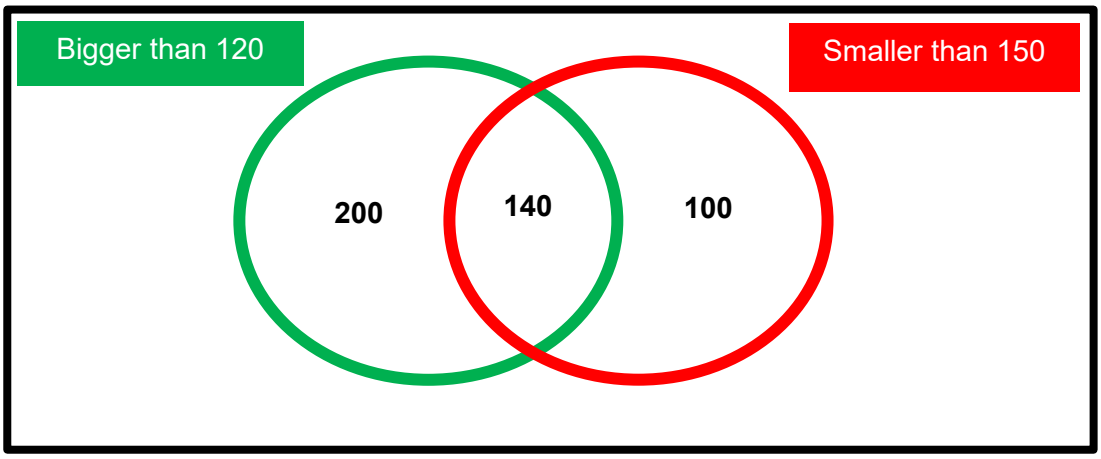
Question 4
a) 20
b) 11
c) outside the rings
d) 25
e) 8

Question 5
a) 5
b) 11
c) 1
d) 15
e) 9

Worksheet 1b: Answers

Note: In most cases there are multiple answers for each section. Much can be learned by getting pupils to share their answers (peer assess) or to look at which sections have an infinite or a finite number of solutions and giving reasons as to why.

1)



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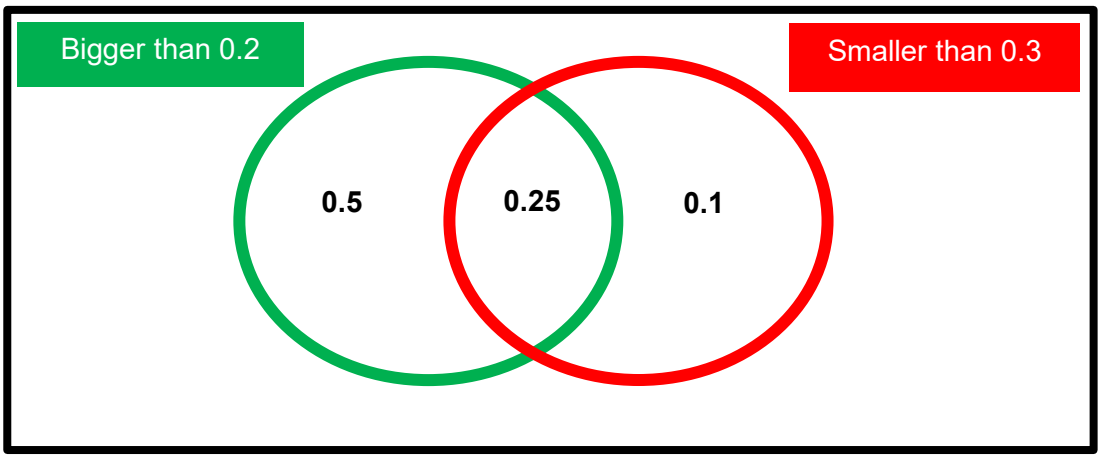
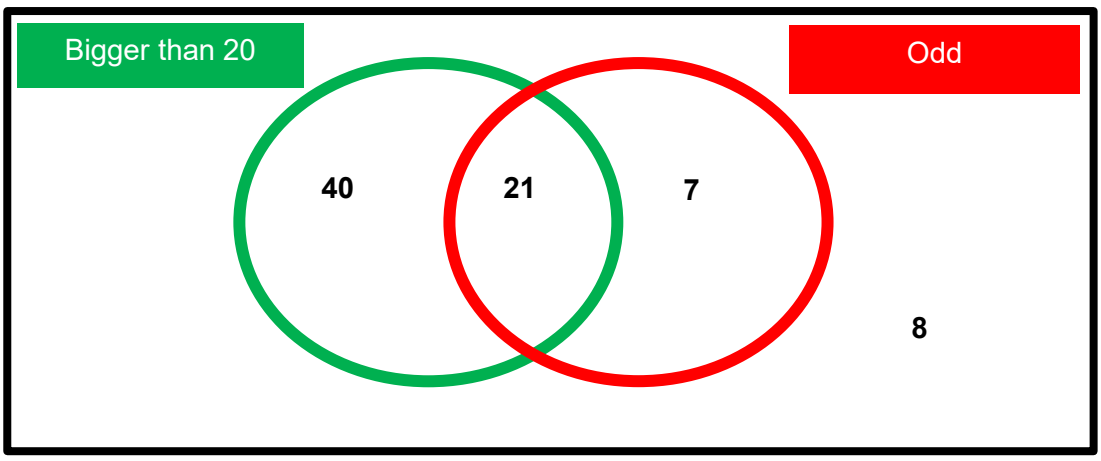
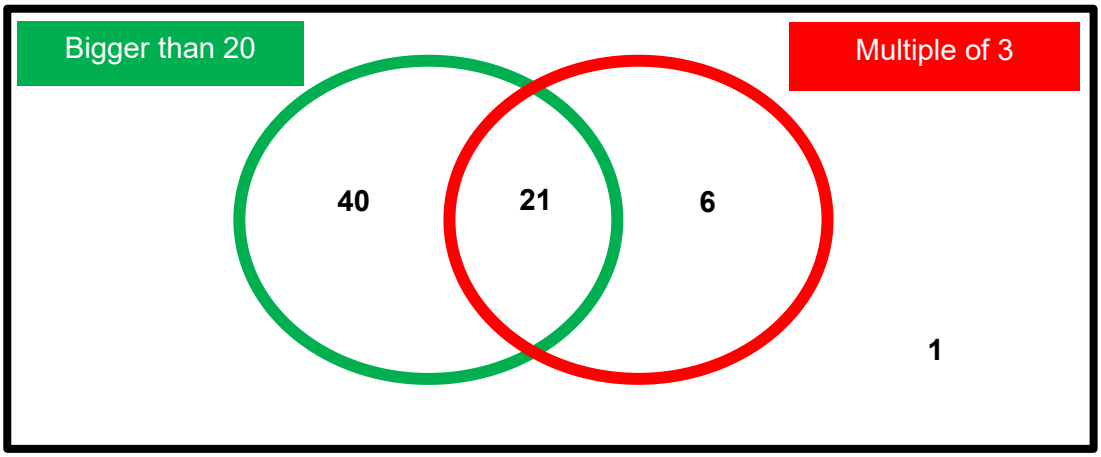


3)



Worksheet 1b: Answers *continued*

4)



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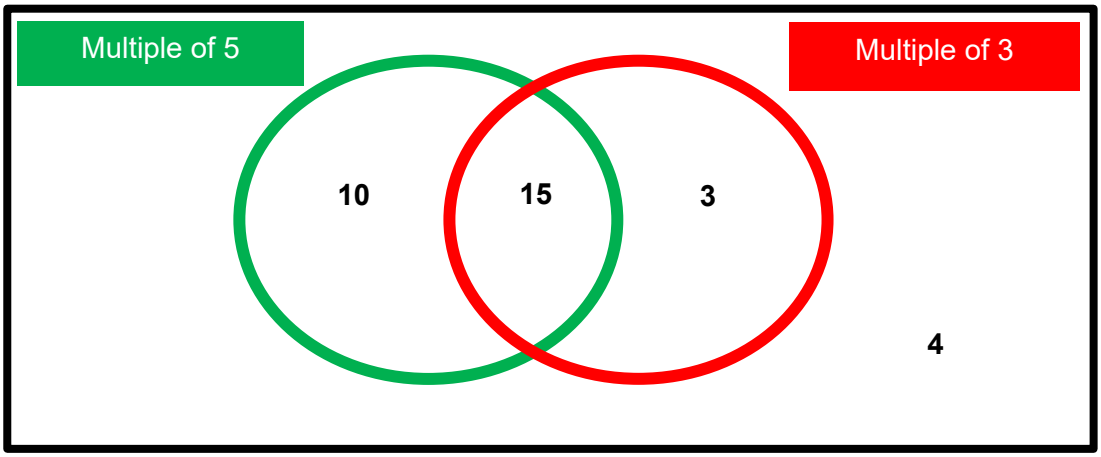
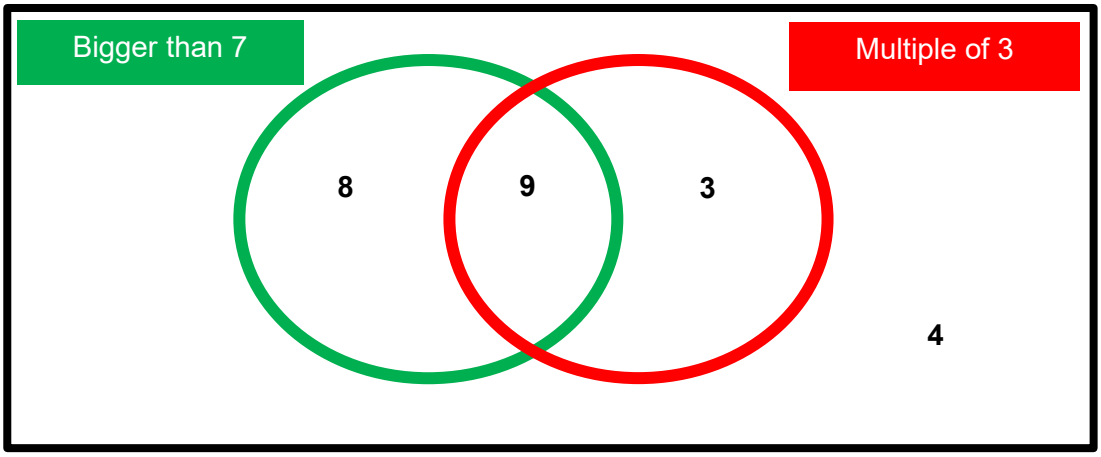


6)



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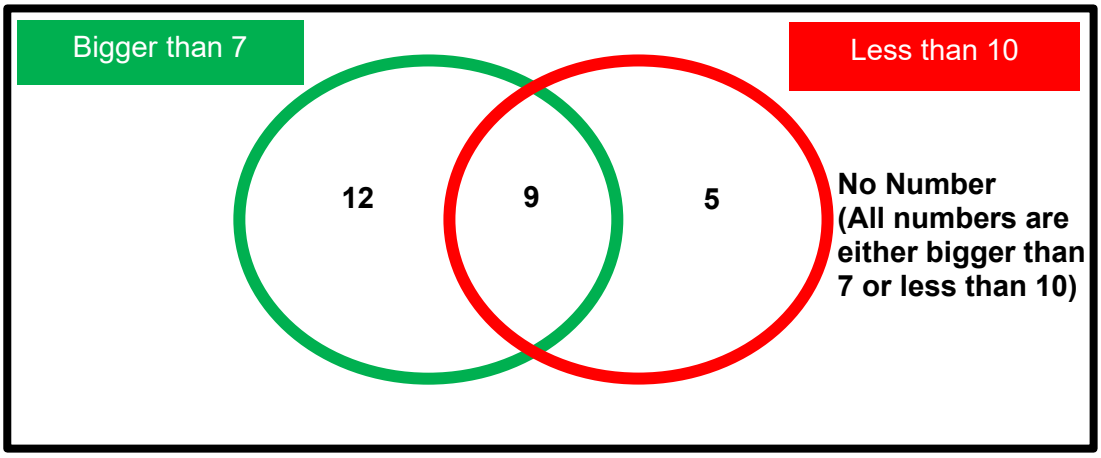
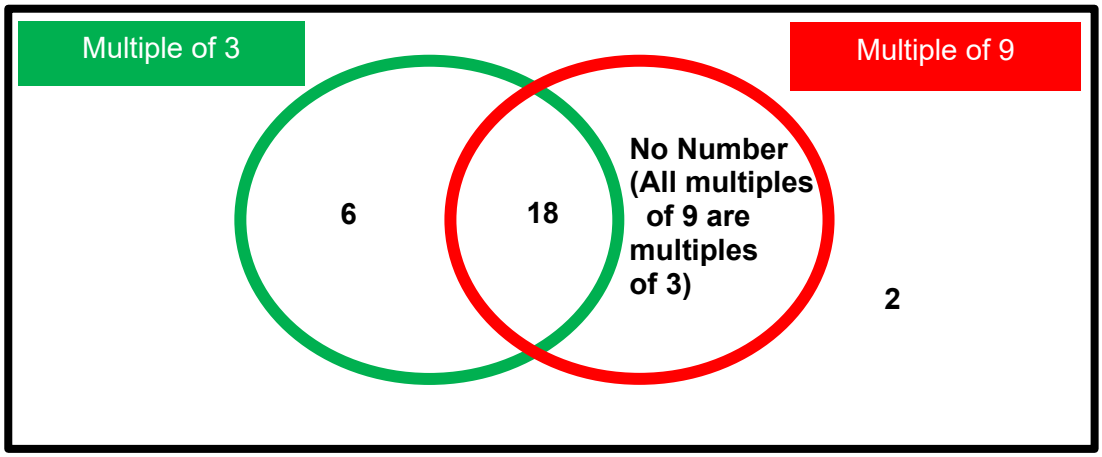


Worksheet 1b: Answers *continued*

8)



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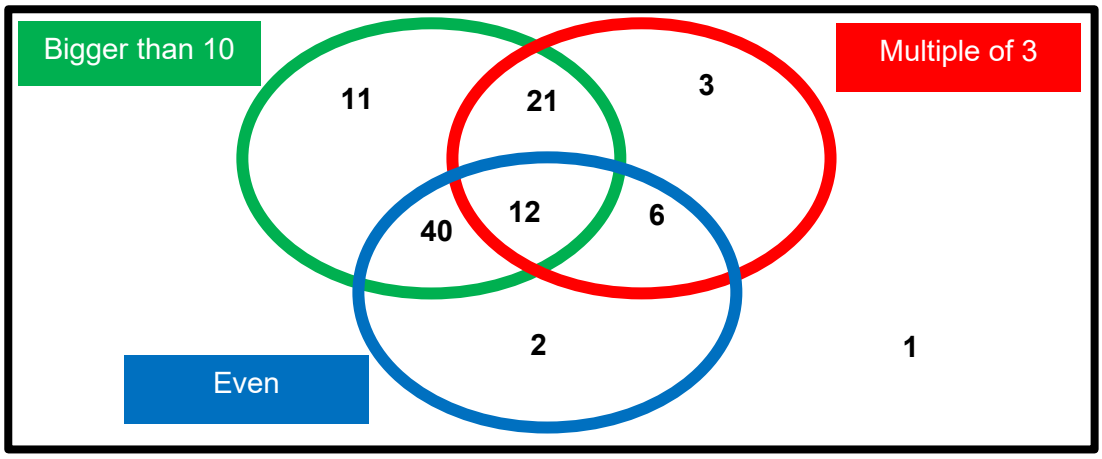

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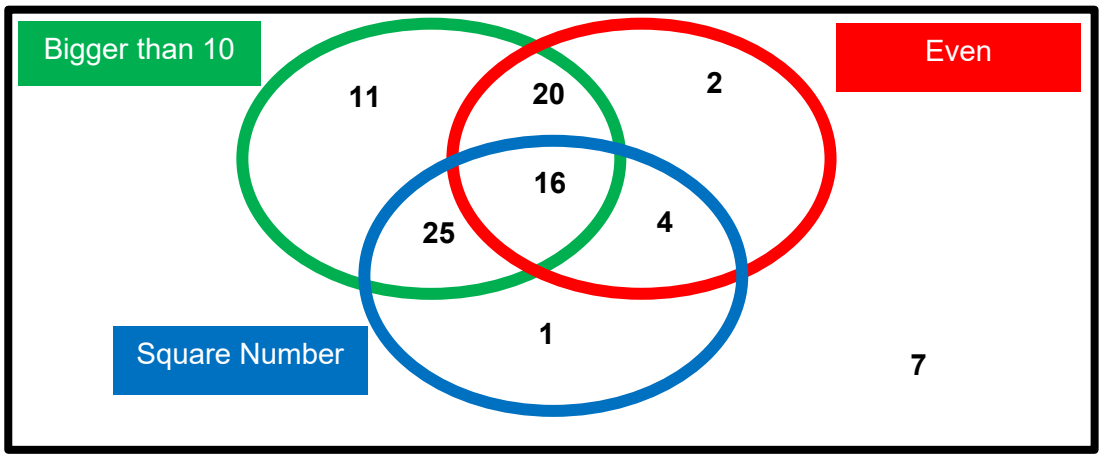

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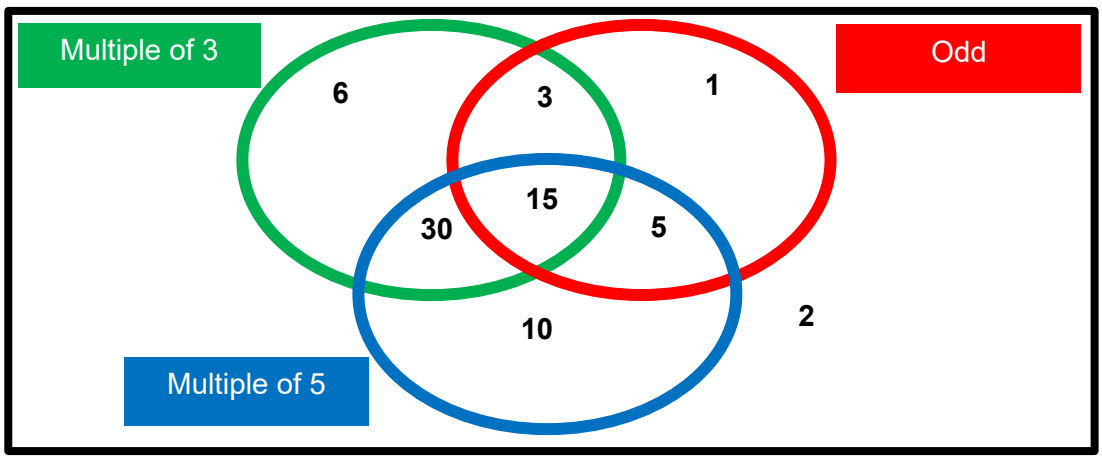
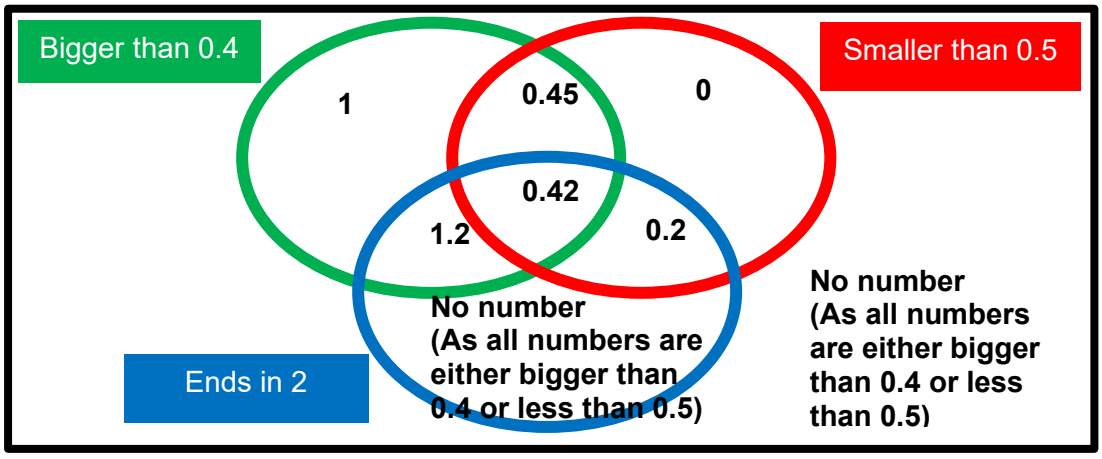


Worksheet 1b: Answers *continued*

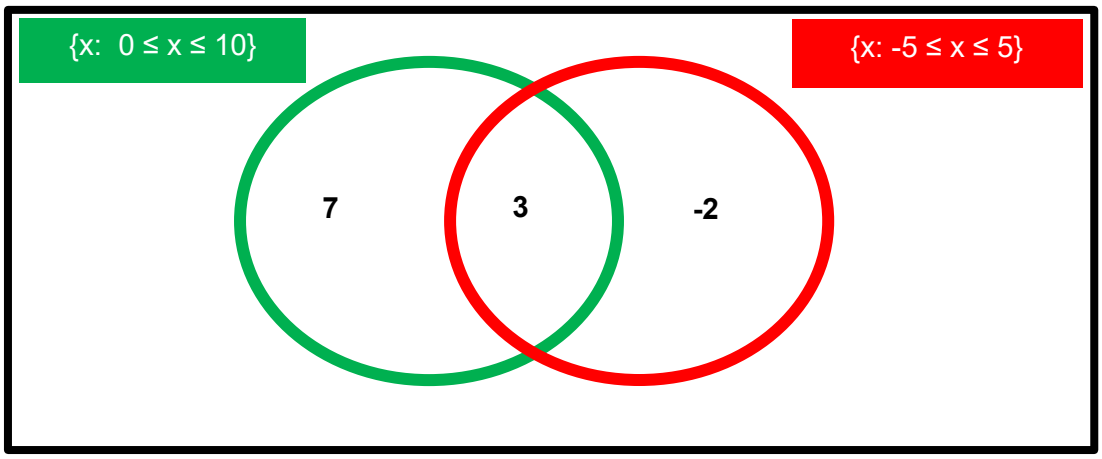
12)



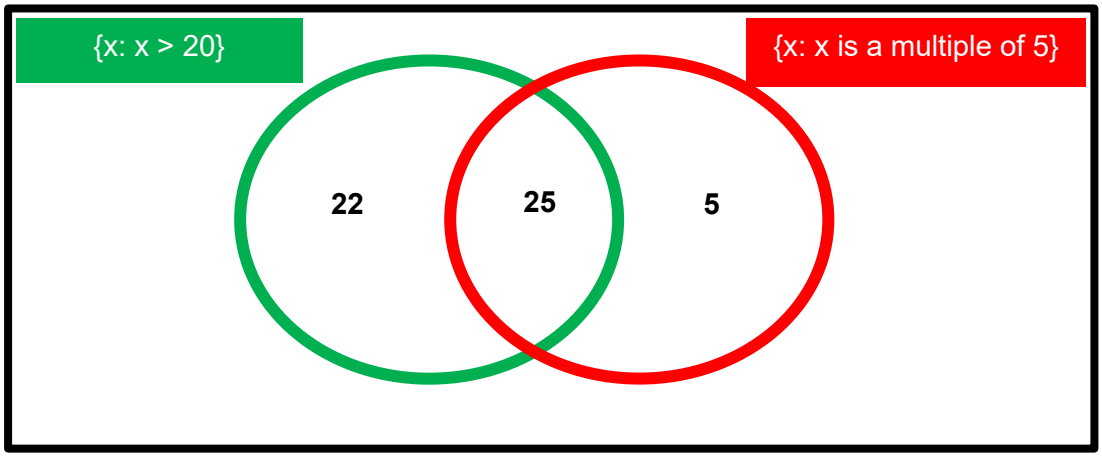
Worksheet 1c: Answers

Note: In most cases there are multiple answers for each section. Much can be learned by getting pupils to share their answers (peer assess) or to look at which sections have an infinite or a finite number of solutions and giving reasons as to why.

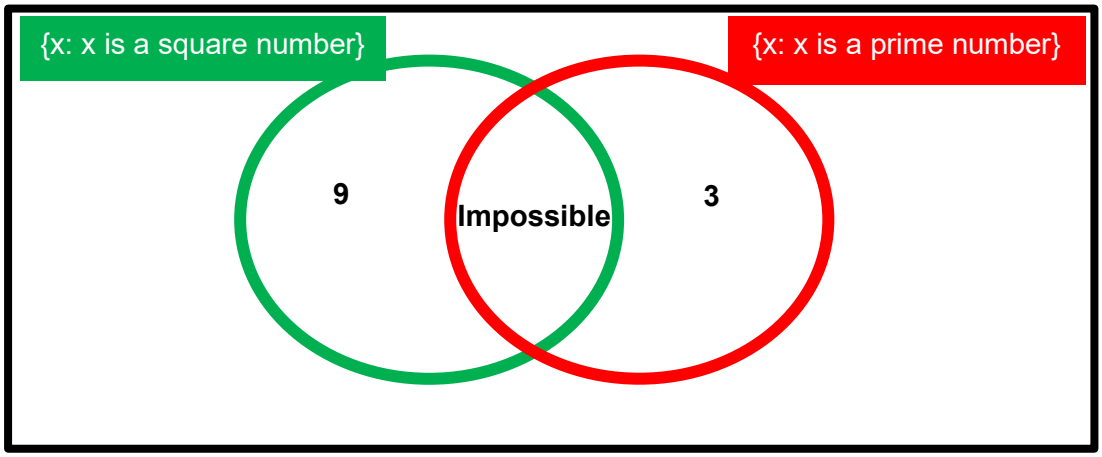
1)



2)

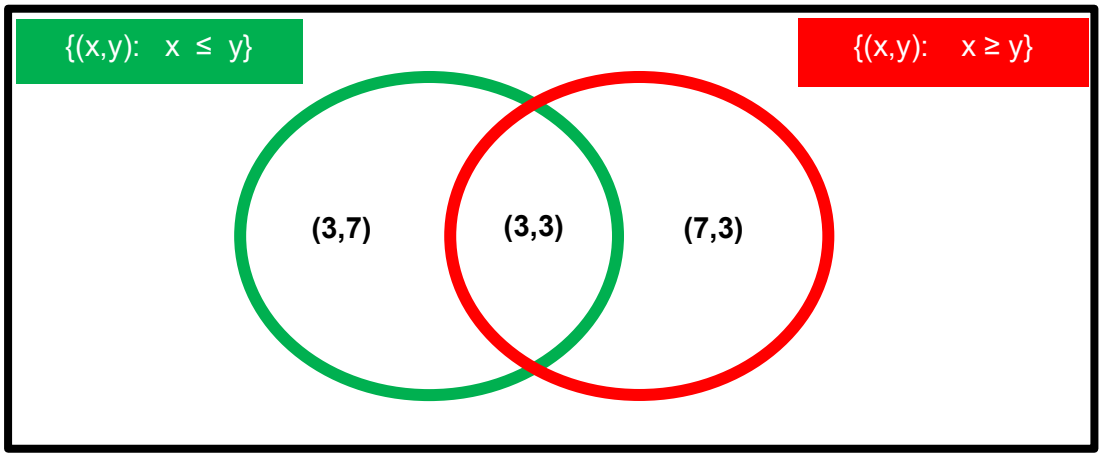


3)

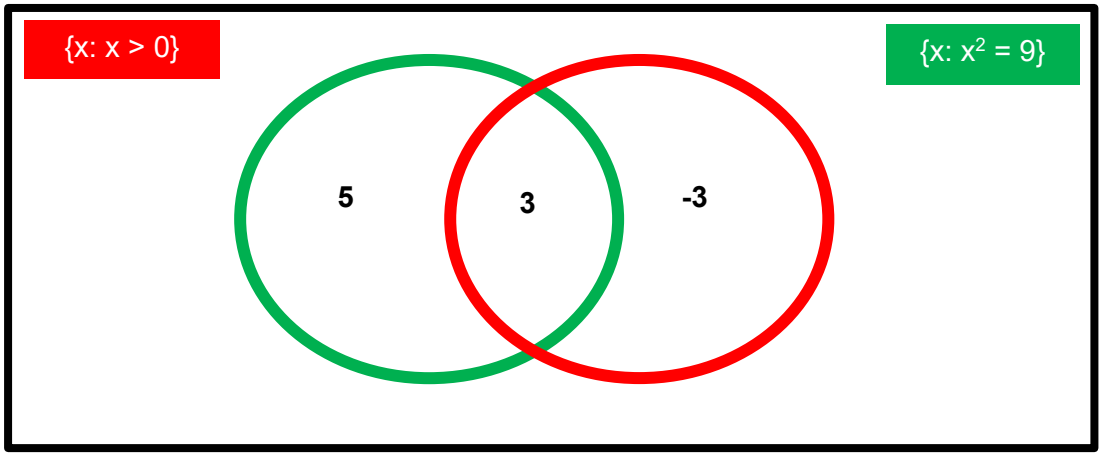


Worksheet 1c: Answers *continued*

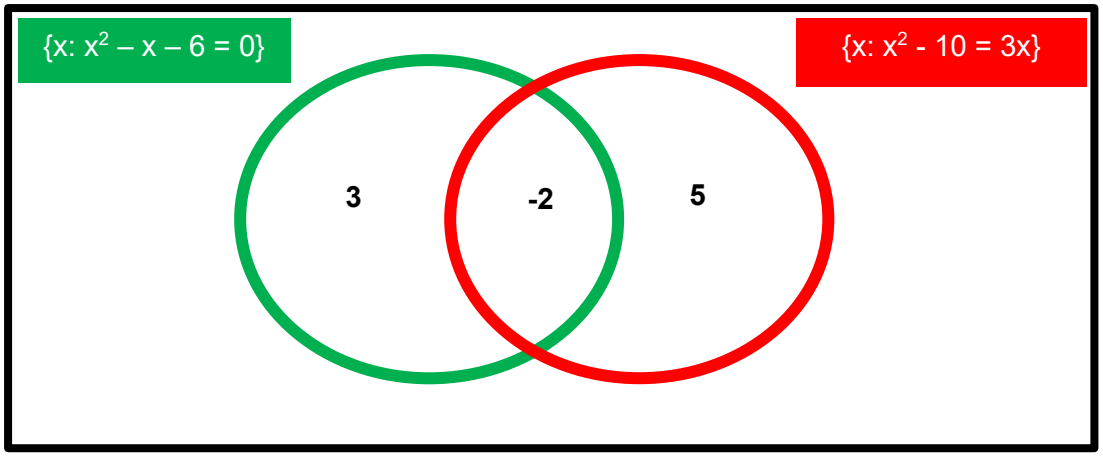
4)



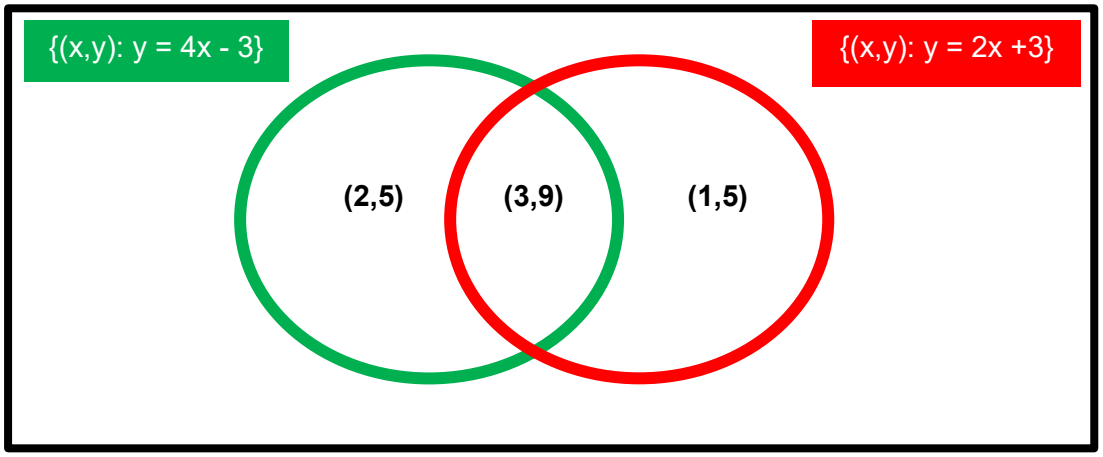
5)



6)

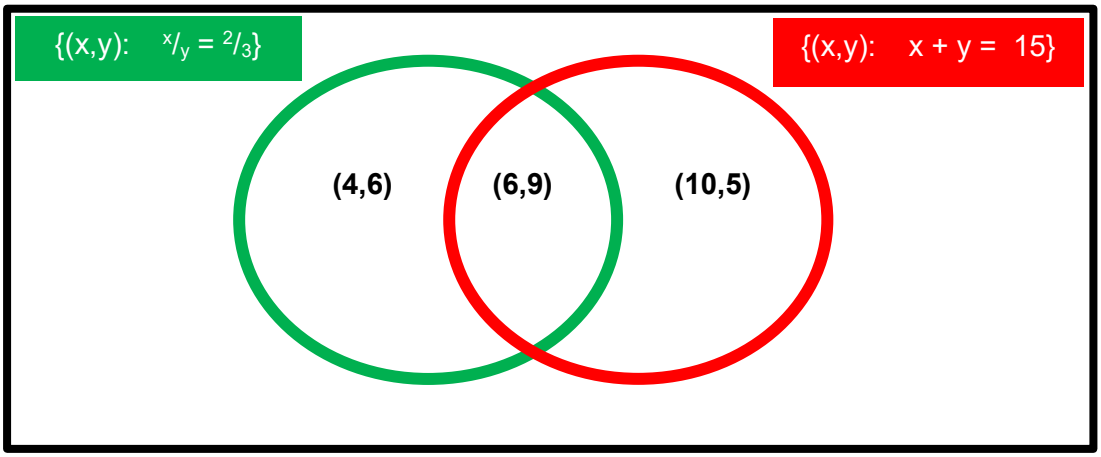


7)



Worksheet 1c: Answers *continued*

8)



Worksheet 2a: Answers

Question 1
a) 3
b) 2
c) 1
d) 5

Question 2
a) 3, 4, 5, 7
b) 4
c) 3, 5, 7
d) 3, 4, 5, 7, 8, 9

Question 3
a) 5
b) 2
c) 5
d) 1

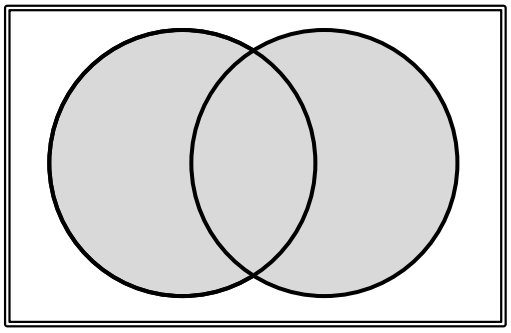
Question 4
a) 3, 6, 8, 21
b) 2, 4, 8
c) 8
d) 2, 3, 4, 6, 7, 8, 21
e) 9, 10
f) 3, 7

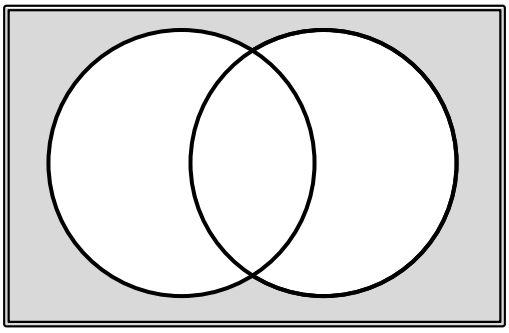
Question 5
a) 3
b) 1
c) 7
d) Odd numbers, {z: z is an odd number}
Not unique as any subset of the odd numbers containing 1 and 9 fits.
E.g. {1,9} {1,3,9} {1,5,9} {1,7,9} {1,3,5,9} {1,3,7,9} {1,5,7,9} {1,3,5,7,9}

Worksheet 2b: Answers

People who own either a car or a bicycle

B ∪ C



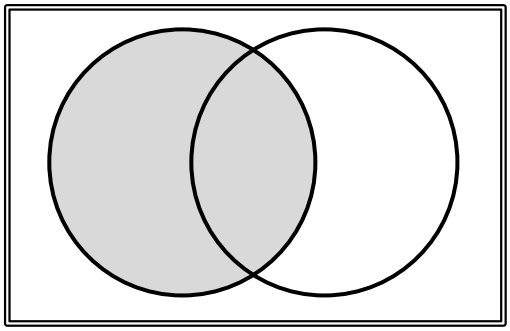


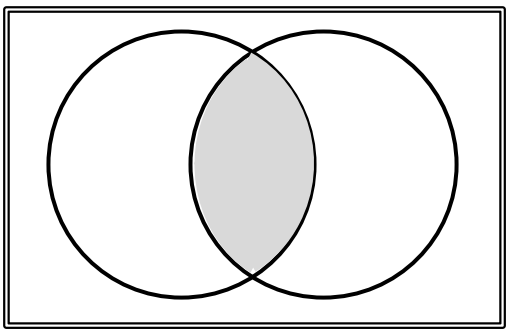
People who own neither a car nor a bicycle

(B ∪ C)’

People who own a bicycle

B



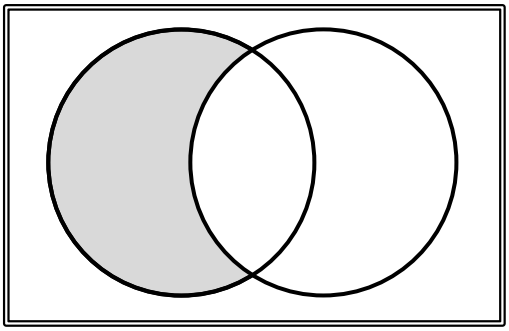


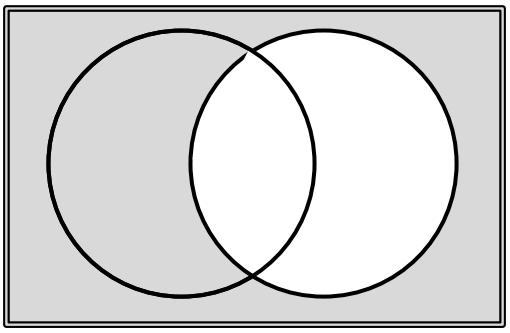
People who own both a car and a bicycle

B ∩ C

People who own just a bicycle

B ∩ C’



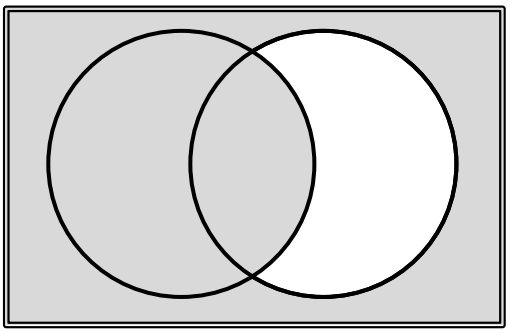


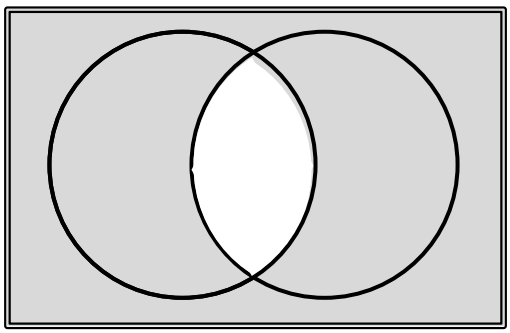
People that don’t own a car

C’

Everyone apart from those who own just a car

B ∪ C’





Everyone unless they own both a Bicycle and a Car

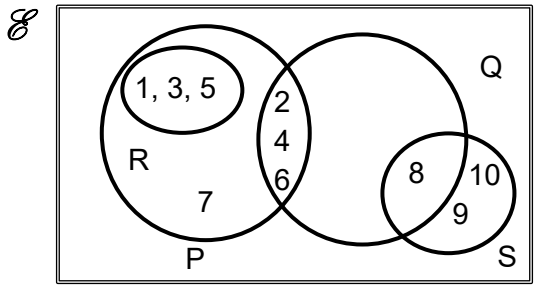
(B ∩ C)’

Worksheet 3a: Answers

Question 1
a) 2
b) 2, 3, 4, 5, 6, 7, 8, 10
c) 1, 3, 5, 7, 9
d) 3, 5, 7

Question 2
a) Common multiples of 3 and 5 or multiples of 15
b) i) 8 ii) 4 iii) 12

Question 3

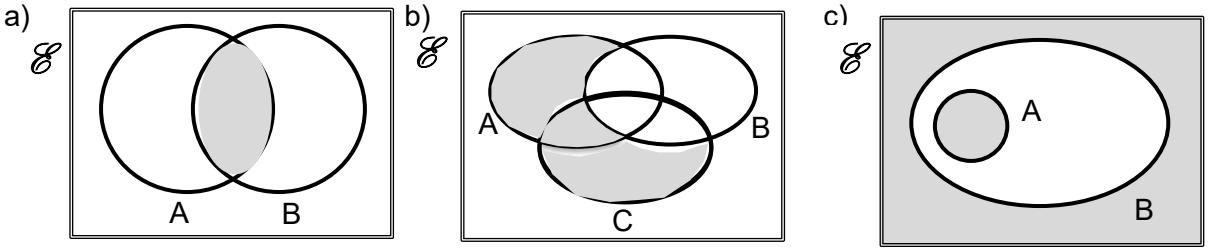


Question 4
a) Any right-angled isosceles triangle
b) An equilateral triangle only has internal angles of 60 degrees

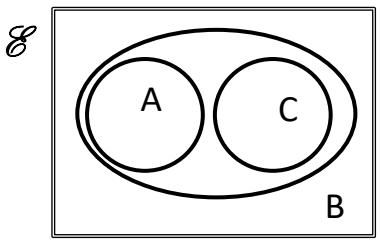
Question 5
a) Regular polygons
b) Square - both are the same

Question 6
a) 314
b) 38 Pupils who cannot swim
c) 285
d) 42 left handed pupils who can swim

Question 7



Question 8



Worksheet 3a: Answers *continued*

Question 9

Factors of 6 are 1, 2, 3 and 6

Proper sub groups: {1} {2} {3} {6}
{1,2} {1,3} {1,6} {2,3} {2,6} {3,6}
{1,2,3} {2,3,6} {1,3,6}

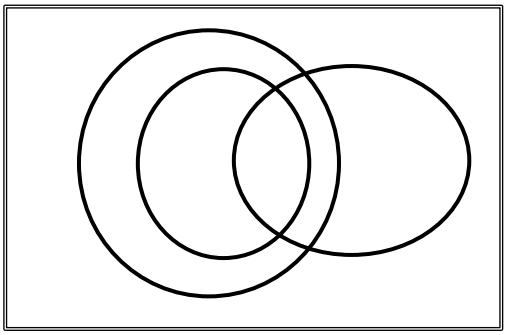
So 13 proper subgroups

Worksheet 3b: Answers

A = {x: x is a multiple of 3}

B = {y: y is a multiple of 6}

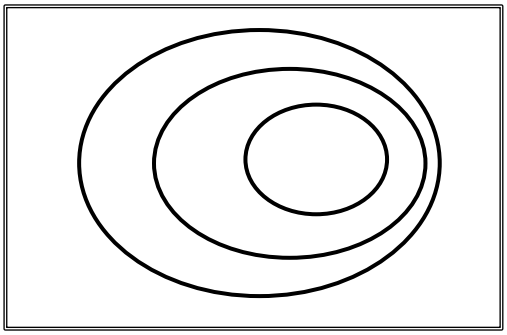
C = {z: z is a multiple of 5}

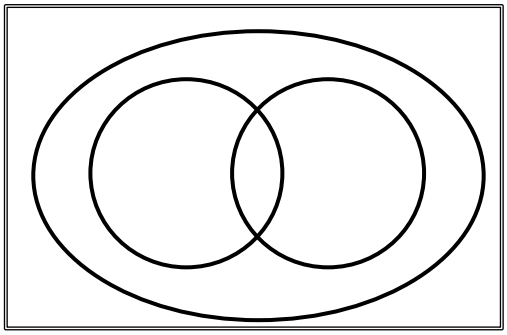


A = {x: x <20}

B = {y: y < 30}

C = {z: z <40}

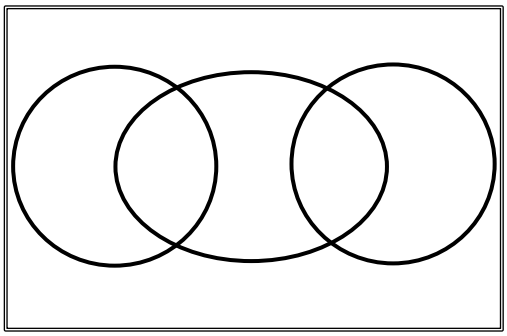




A = {x: x is a factor of 7}

B = {y: y is a factor of 5}

C = {z: z is a factor of 70}

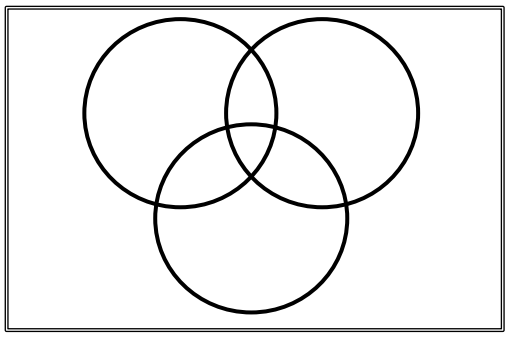


A = {x: x is a square number}

B = {y: y is a prime number}

C = {z: z is an even number}

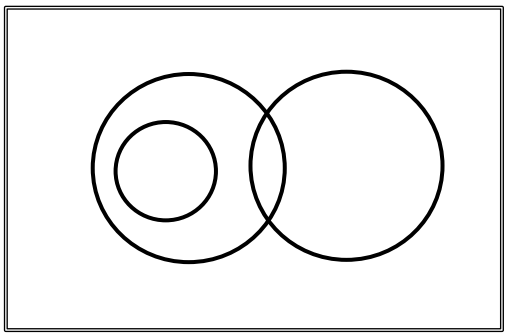
Worksheet 3b: Answers *continued*



A = {x: x is a multiple of 3}

B = {y: y is a multiple of 5}

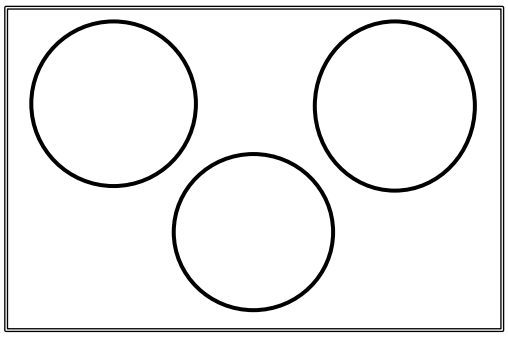
C = {z: z is a multiple of 7}



A = {x: x is a prime number}

B = {y: y is even}

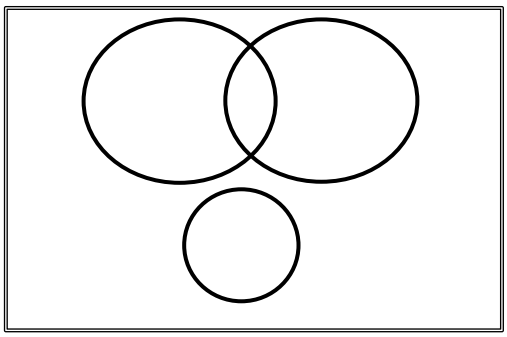
C = {z: z is a multiple of 4}



A = {x: x is a number that ends in a 2}

B = {y: y is a number that ends in a 3}

C = {z: z is a number that ends in a 5}

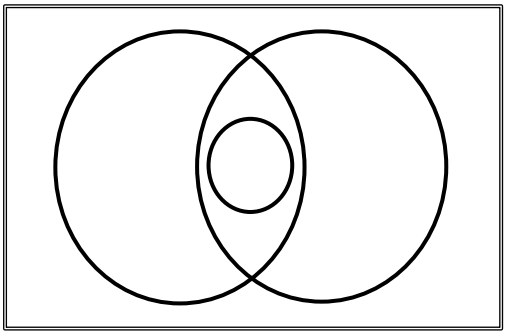


A = {x: x is a factor of 20}

B = {y: y is a factor of 30}

C = {z: z > 40}

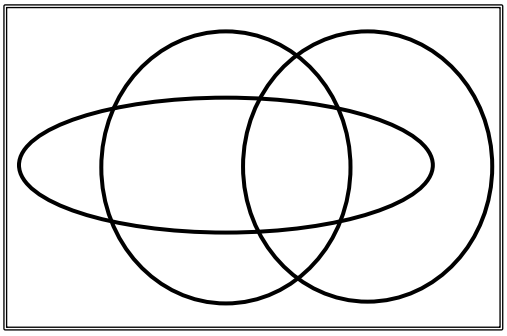
Worksheet 3b: Answers *continued*



A = {x: x is a factor of 20}

B = {y: y is a factor of 10}

C = {z: z is a factor of 5}



A = {x: x is a square number}

B = {y: y is an even number}

C = {z: z is a multiple of 3}

Worksheet 4a: Answers

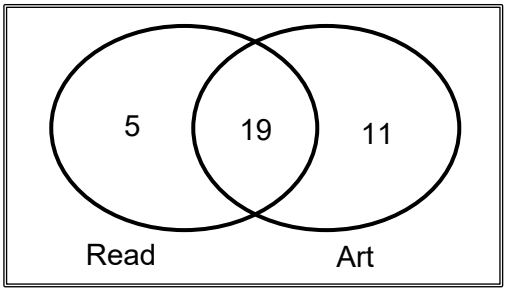
Question 1

a) 37 b) 4

Question 2

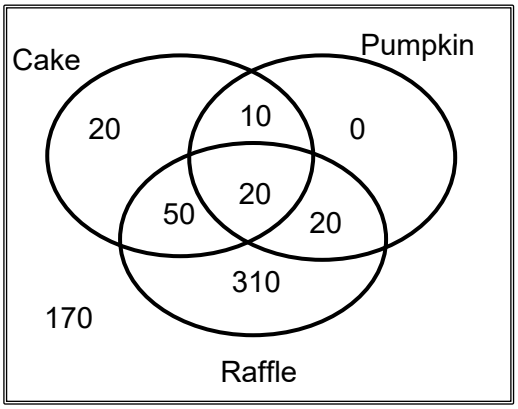
34

Question3



a) 19 b) 5

Question 4



So 310

Question 5

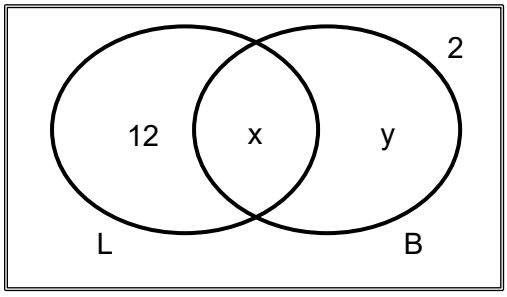
40

Worksheet 4a: Answers *continued*

Question 6

12 + x + y + 2 = 50

x + y = 36



x = ¾ (x + y)

4x = 3x + 3y

x = 3y

Hence 3y + y = 36

Solving gives x = 27, y = 9

27 ate both lunch and breakfast

Question 7

n(A ∩ B) = 3

Question 8

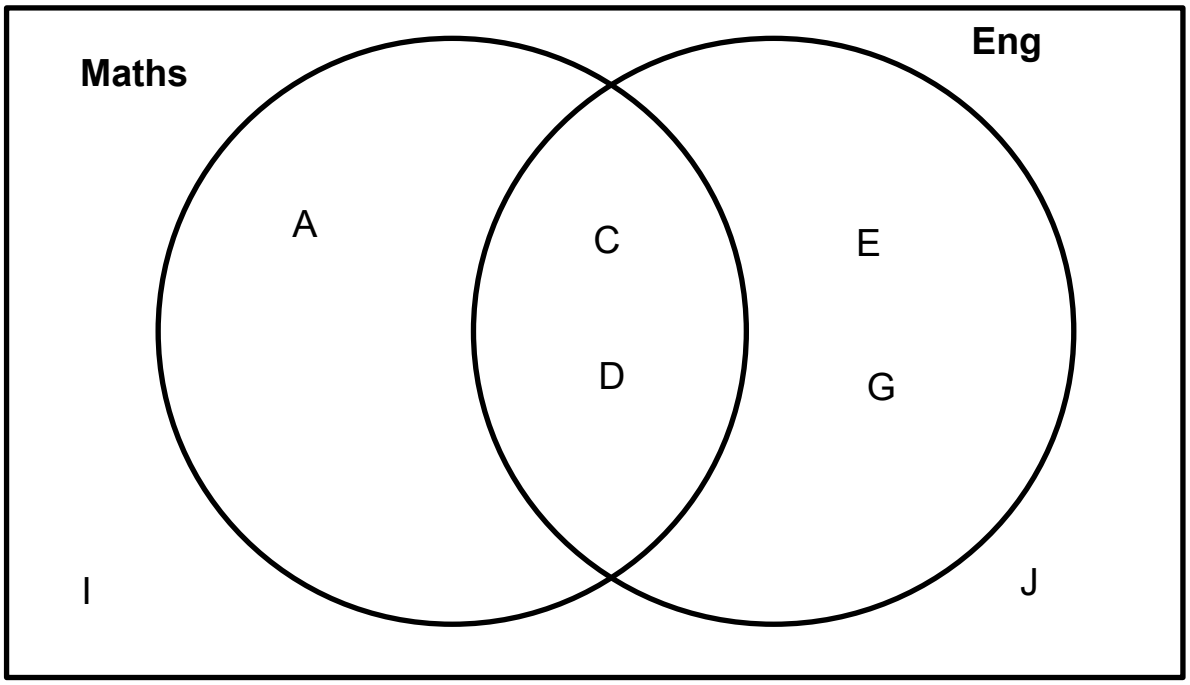
5x + 12 = 62 so x = 10, so 5 attended both clubs.

Question 9

3x + 14 = 5x + 6

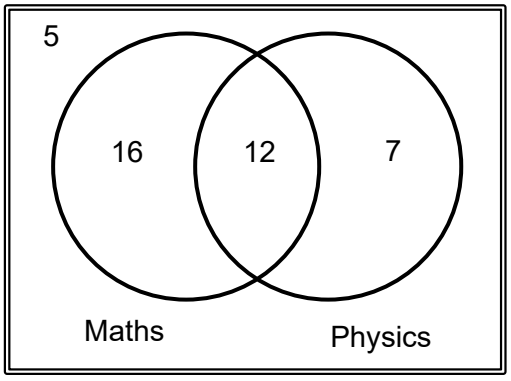
x = 4 so 22 spoke German

Worksheet 4b: Answers



Worksheet 5a: Answers

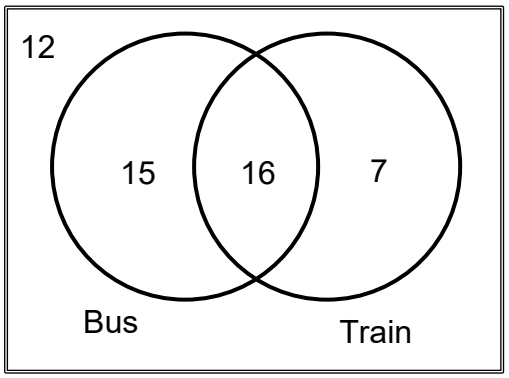
Question 1



a) $^{2}/_{5}$

b) $^{1}/_{8}$

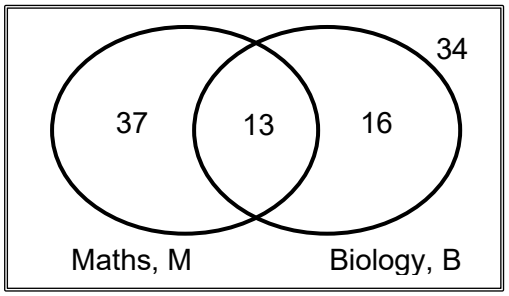
Question 2



b) $^{23}/_{50}$

c) $^{8}/_{25}$

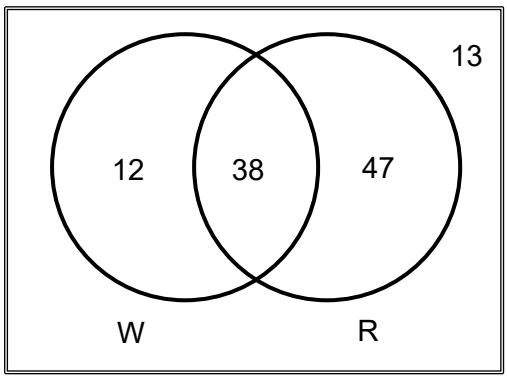
Question 3



a) ½ b) $^{29}/_{100}$ c) ½ d) $^{33}/_{50}$ e) $^{13}/_{100}$ f) $^{4}/_{25}$

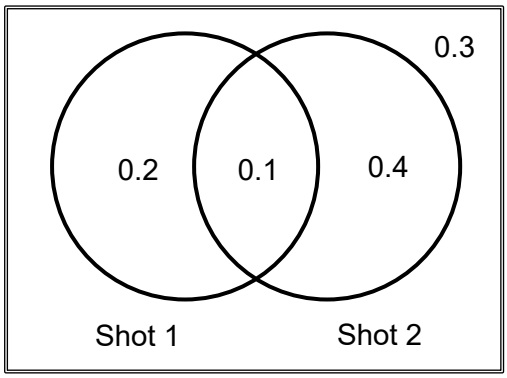
Worksheet 5a: Answers *continued*

Question 4



b) $^{17}/_{22}$

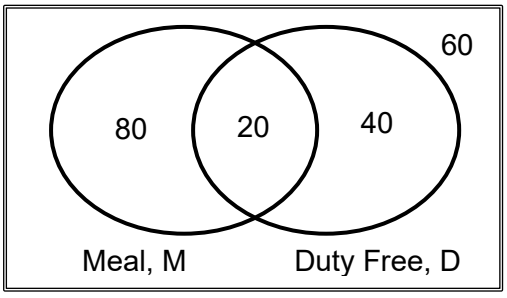
Question 5



a) 0.7

b) 0.3

Question 6



b) 0.2

c) 0.7

Worksheet 5a: Answers *continued*

Question 7

Money from those who booked a single activity = 8 x 12 + 9 x 12 = $204

The learners who booked both activities paid $309 - $204 = $105

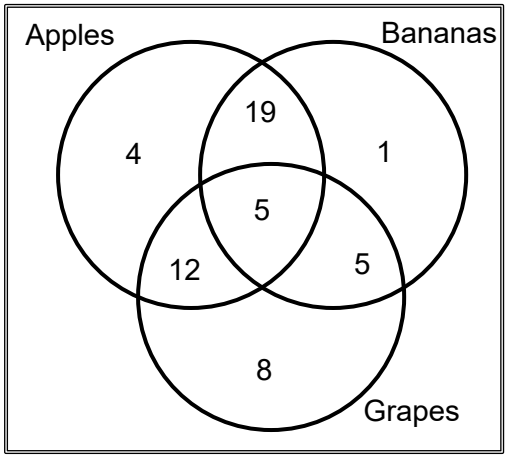
Number of learners who booked both = $105 ÷ 21 = 5

P(Both) = $^1/_6$

Question 8

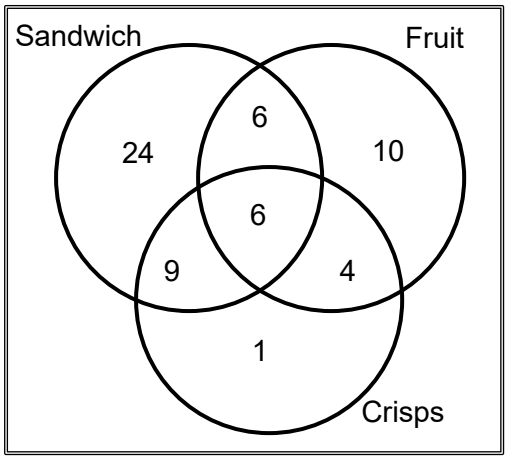
50%

Question 9



b) $^1/_{100}$

Question 10



a) $^7/_{12}$ b) $^{13}/_{30}$ c) $^{19}/_{60}$

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