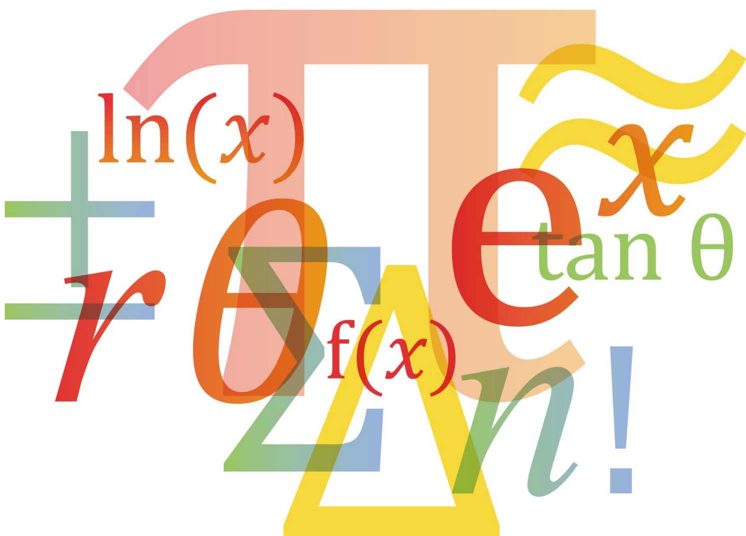
Cambridge Assessment International Education

Teacher Pack

Differentiation

Cambridge International AS & A Level

Mathematics 9709



Version 0.1

Cambridge Pathway

Contents

Icons used in this pack:

Teacher preparation

Lesson plan

Lesson resource

Lesson reflection

Introduction

This pack will help you to develop your learners' skills in mathematical thinking and mathematical communication, which are essential for success at AS & A Level and in further education.

Mathematical thinking and communication will be developed by focussing on:

1. Conceptual understanding – the 'why' behind the 'what'
2. Strategic competence – forming and solving problems
3. Adaptive reasoning – explanations, justifications and deductive reasoning

Throughout all activities, the learners will also develop:

- Procedural fluency – know when, how and which rules to use
- Positive disposition – believe maths can be learned, applied and is useful
- Their skills in writing mathematically – writing working & proofs

These link to the course Assessment Objectives (AOs) which you can find in detail in the syllabus:

A01 Knowledge and understanding

A02 Application and communication

Each *Teacher Pack* contains one or more lesson plans and associated resources, complete with a section of preparation and reflection.

Each lesson is designed to be an hour long but you should adjust the timings to suit the lesson length available to you and the needs of your learners.

Important note

Our *Teacher Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other topics.

This content is designed to give you and your learners the chance to explore a more active way of engaging with mathematics that encourages independent thinking and a deeper conceptual understanding. It is not intended as specific practice for the examination papers.

The *Teacher Packs* are designed to provide you with some example lessons of how you might deliver content. You should adapt them as appropriate for your learners and your centre. A single pack will only contain at most four lessons, it will **not** cover a whole topic. You should use the lesson plans and advice provided in this pack to help you plan the remaining lessons of the topic yourself.

Lesson preparation

This *Teacher Pack* will cover the following syllabus content.

Candidate should be able to:	Notes and examples
• understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords	eg includes consideration of the gradient of the chord joining the points with x coordinates 2 and (2 + h) on the curve $y = x^3$
• Use the chain rule to differentiate composite functions and apply differentiation to gradients, tangents and normals	eg find $\frac{dy}{dx}$ given $y = \sqrt{2x^3 + 5}$
• Apply the chain rule to problems involving connected rates of change	eg given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables

Dependencies

For all lesson plans in this *Teacher Pack*, knowledge from the following 9709 syllabus content is required.

Candidate should be able to:	Notes and examples
• Solve quadratic equations	
• Find the equation of a line from point and gradient	
• use the relationship between gradients of parallel and perpendicular lines	

Prior knowledge and skills

For all lessons, it is assumed that learners have already completed Cambridge IGCSE™ Mathematics 0580, or a course at an equivalent level. See the syllabus for more details of the expected prior knowledge for taking Cambridge International AS & A Level Mathematics 9709.

When planning any lesson, make a habit of always asking yourself the following questions about your learners' prior knowledge and skills:

- Do I need to re-teach this or do learners just need some practice?
- Is there an interesting activity that will efficiently achieve this?

Key learning objectives

The following list represents the main underlying concepts that you should make sure your learners have understood by the end of this topic.

- Gradient of tangent as the limit of gradients of a sequence of appropriate chords
- Apply the chain rule to differentiate composite functions
- Apply differentiation to obtain tangent and normals to curves
- Use the chain for problems involving connected rates of changes

Why this topic matters

This topic introduces and lays the foundations for all further work in calculus. Once the principle of gradient of a curve is understood and rules for differentiating various types of functions have been established, then differentiation can be applied to a wide range of mathematical and real-life contexts.

Key terminology and notation

Your learners will need to be confident with the following terminology and notation.

chord	Line connecting two points on a curve
gradient	How to find the gradient of a straight line
$\frac{dy}{dx}$	For the gradient function

Lesson progression

These 3 lessons are <u>not intended</u> to be delivered one after the other.

<u>Lesson 1</u>. The first lesson should be used as an introduction to differentiation and covers the fundamental concept of the gradient of a curve as the limit of a sequence of chords. It gives learners a dynamic understanding, that will underpin the development of calculus.

Following this first lesson, the rule for differentiating x^n should be introduced and practiced, together with constant multiples, sums and differences of functions.

Learners should then be taught how to apply differentiation to obtain equations of tangents and normals, to locate and identify stationary points, and to determine where functions are increasing and decreasing. These skills are often developed and practised for polynomials, eg to find the equation of the tangent to curve $y = 2x^3 - 4x^2 + 5x - 7$ at the point where $x = 1$.

Practice should be extensive, and learners should be confident with these skills before moving onto lesson 2.

<u>Lesson 2</u>. Introduces the chain rule, a very important result, which gives learners the skills to differentiate a much wider range of functions.

They should then differentiate composites functions and apply as before to tangents, normals, stationary points and increasing/decreasing functions.

<u>Lesson 3</u>. Broadens the use of the chain rule to related rates of change. This lesson includes a visual interpretation of a point moving along a function, along with some real-life examples in simple contexts.

Going forward

Differentiation to obtain gradient function will naturally lead onto integration. Once differentiation and integration processes and applications have been firmly established, learners will be able to develop and apply these skills to a much wider variety of functions as the A level course progresses.

Lesson 1: Gradient of curve from a sequence of chords

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.
Resources	• PowerPoint presentation • Worksheets 1 and 2
Learning objectives	By the end of the lesson: • **all** learners should be able to describe how the gradient of a curve at any point is the limit of the gradient of a sequence of chords, and be able to calculate gradients of chords numerically • **most** learners should be able to find the gradient at a given point on a curve • **some** learners should be able to construct a rigorous argument to find an expression for the gradient of a curve at any point

Dependencies

Learners need to know how to find the gradient of a line segment, either from an informal approach $\frac{rise}{run}$ or using their coordinate geometry knowledge $\frac{y_2-y_1}{x_2-x_1}$. These skills are refreshed during the starter activity.

Timings	Activity
5 mins	**Starter/Introduction** <u>Gradient of a straight line</u> The opening slides show a bicycle on a hill. Ask learners to calculate the gradient of each straight line. This is a good opportunity to use Think Pair Share. Show the answers and discuss methods used. It is important that every learner has a correct process at this stage. The hyperlink "Biking on a Hill" is a graph in DESMOS, which can be used dynamically to reinforce $\frac{rise}{run}$ or to set more examples if necessary.
45 mins	**Main lesson** <u>Gradient of a curve and sequence of chords</u> The next slides explore the gradient of a curve dynamically and can be linked to prior learning from IGCSE 0580 (drawing a tangent by eye). Play/pause video as necessary to establish the dynamic concept that gradient is changing at every point, and gradient of the curve is equal to the gradient of the tangent at that point. Play/pause video inserts to establish the concept that gradient of tangent is the limit of the gradients of a sequence of chords. Can also use the DESMOS graph to reinforce. <u>Gradient of Tangent from a Sequence of Chords</u>

Timings	Activity
	Show the slide and ask students to complete Worksheet1. Think pair share. Students work independently or in small groups to consolidate learning through discussion and activity. Small Changes and approaching a limit Work through the slides to demonstrate how we can take a small change horizontally and then consider what happens when this approaches zero. Set worksheet 2 as Think Pair Share or independent work. Note. Questions increase in difficulty and learners can be directed as appropriate. **Challenge:** The most able learners should be encouraged to explore the more general case eg derive the expression for gradient at any point on a suitable curve, and/or could be asked to explore alternative notations such as $\frac{\delta y}{\delta x}$. Creating lower and upper bounds by considering chords either side of the point is another extension activity. **Support:** Use any of the DESMOS graphs to reinforce/underpin key ideas. Eg Biking on a Hill can be used to set more straight line gradients. Encourage students to do simple sketches at every point if that is helpful.
10 mins	**Plenary** Revisit key issues that have come up through the lesson or Use the True/False slide to explore further issues about gradient

Reflection	Reflect on your lesson, use the **Lesson reflection** notes to help you.

Lesson plan 2: Chain Rule

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.
Resources	• PowerPoint presentation • Worksheets 3 and 4
Learning objectives	By the end of the lesson: • **all** learners should be able to be able to differentiate by applying the chain rule for positive powers of a linear function • **most** learners should be able to apply the chain rule to problems about gradient • **some** learners should be able to apply the chain rule to a wide range of problems involving curves

Dependencies

Learners must have spent a considerable amount of time differentiating powers of x, and applying differentiation to problems involving gradient such as tangents, normals, stationary points, increasing and decreasing functions for polynomials eg cubics, before beginning these lessons on the chain rule.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Not recognising when the function/curve to be differentiated is a composite function	Learners may wrongly assume that when $(2x+3)^5$ is differentiated they only need to multiple by the power and reduce the power by one	Once the starter activity is completed, emphasise that the differentiation rules learned/practised so far are limited to functions of the form x^n. The structure of composite functions can be linked back to specification topic 1.2 Functions, but this important structural understanding is developed through the main lesson resources.

Timings	Activity
5 mins	**Starter/Introduction** Recap differentiating x^n Set Activity 1, which is available in several forms, eg table/list, domino task in Tarsia. Think Pair Share approach is recommended. Revisit the process for differentiating powers of x, by setting Activity 1 in a suitable form. These questions can be adapted/extended as necessary to reinforce the rule: • Multiply by the power • Reduce the power by one

Timings	Activity
45 mins	**Main lesson** <u>Introducing the Chain Rule</u> Work through the slides that introduce the chain rule, and the examples that follow. Be careful to describe the functions appropriately so that learners can understand the descriptions “composite function” and “function of a function”. Discuss the introduction of “u” as a process to breakdown the differentiation into two parts, exploring a wider range of examples if necessary to establish this key idea. Work through each of the 3 examples, modelling the same process each time. Set Worksheet 3 as practice. Think Pair Share or working independently as appropriate. <u>Applications to problems involving gradient</u> Explain that learners will be revisiting applications such as tangents, normals, stationary points, increasing/decreasing functions, from prior learning within the differentiation topic. Ask learners to write a brief method statement in response to one application on the Think Pair Share slide. Discuss/revise methodology as required. Set worksheet 4, where learners apply their skills to problems involving gradients of curves that have to be differentiated using the chain rule. **Challenge:** Some learners can be challenged to complete the process intuitively, without the need to define the 3rd variable “u” explicitly. This is helpful at a later stage when learning to differentiate other functions eg trig, exponentials and natural logs. **Support:** Encourage the use of highlighters or similar to identify the separate functions.
10 mins	**Plenary** Ask learners to discuss / debate each example on the slide. Which examples can be described as function of a function or composite functions, and which cannot? Explore any misconceptions

Reflection	Reflect on your lesson, use the **<u>Lesson reflection</u>** notes to help you.

Lesson plan 3: Connected rates of change

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.

Resources	• PowerPoint presentation • Worksheets 5 and 6

Learning objectives	By the end of the lesson: • **all** learners should be able to solve problems for a point moving along a curve in terms of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ • **most** learners should be able to construct the relevant chain rule for a problem in context • **some** learners should be able to solve problems in a wide range of real-life contexts

Dependencies

Learners need to recall the chain rule from earlier lessons.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Wrong relationship between the 3 variables when setting up the chain rule for connected rates of change	A "wrong" chain rule is likely to lead to an invalid attempt	The True/False slides are designed to reinforce the correct relationship, and further examples can be explored as necessary

Timings	Activity
5 mins	**Starter/Introduction** <u>Rollercoasters</u> Use the opening slides to set a strong visual context for a point moving along a curve. Prompt learners to think about the experience of riding the rollercoaster, to help them describe the rate of change of height at various points along the curve. Reinforce the use of the chain rule from earlier lessons, to connect the 3 variables.
45 mins	**Main lesson** <u>Chain rule for point moving along a curve</u> Work through the example, exploring the use of the chain rule to model connected rates of change. Emphasise alternative approaches and the reciprocal relationships for any two variables. Set worksheet 5 with a Think Pair Share approach. Explore any issues that come up in learners' attempts. <u>Real-life contexts</u> Show the slides with examples of real-world situations for connected rates of change. Emphasise the variables in each case, and how they are related.

Timings	Activity
	Challenge learners to come up with a new situation/context. Explore the gym-ball example by playing the dynamic graph insert (DESMOS can also be used for reinforcement). Emphasise the relationship between the 3 variables: volume, radius, time. Focus on how the rate of change of radius is changing all the time and challenge learners to explain why. Any three variables It is vital at this point to establish a correct structure for the chain rule for connected rates of change. Discuss the example chain rule with "A B C". Set learners the True or False slide. Think Pair Share. Discuss learners' responses to each case, to reinforce the correct structure for a chain rule linking any 3 variables. Show Example 2, being careful to link the situation described to the chain rule. Set Worksheet 6 and support learners to complete at least one of the problems successfully. **Challenge:** Challenge the most able learners to design a new problem in a similar context. **Support:** Help students to identify and define each variable and each rate of change.
10 mins	**Plenary** Revisit key ideas and challenge any misconceptions that have come up or Show the slide and ask learners to develop an argument for any one of the points P, Q, R.

Reflection	Reflect on your lesson, use the **Lesson reflection** notes to help you.

Lesson reflection

As soon as possible after the lesson you need to think about how well it went.

One of the key questions you should always ask yourself is:

Did all learners get to the point where they can access the next lesson? If not, what will I do?

Reflection is important so that you can plan your next lesson appropriately. If any misconceptions arose or any underlying concepts were missed, you might want to use this information to inform any adjustments you should make to the next lesson.

It is also helpful to reflect on your lesson for the next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time. There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.

To help you reflect on your lesson, answer the most relevant questions below.

Were the lesson objectives realistic?
What did the learners learn today? Or did they learn what was intended? Why not?
What proportion of the time did we spend on the most important topics?
Were there any common misconceptions?
What do I need to address next lesson?
What was the learning atmosphere like?
Did my planned differentiation work well?
How could I have helped the lowest achieving learners to do more?
How could I have stretched the highest achieving learners even more?
Did I stick to timings?
What changes did I make from my plan and why?

Summary evaluation

What two things went really well? (Consider both teaching and learning.)

What two things would have improved the lesson? (Consider both teaching and learning.)

What have I learned from this lesson about the class or individuals that will inform my next lesson?

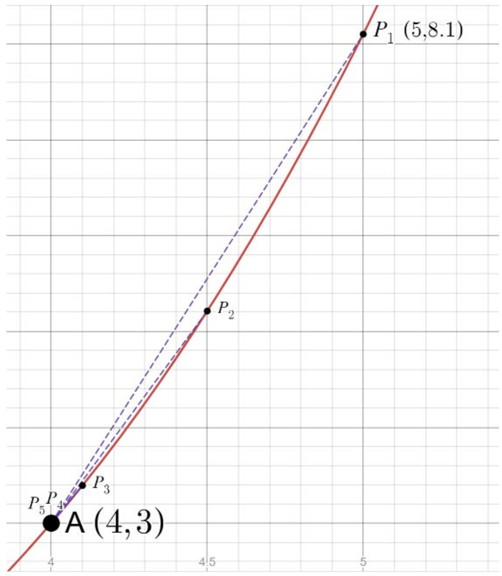
Worksheets and answers

	Worksheet	Answers
For use with *Lesson 1*:		
A: Sequence of chords	**x**	**x**
B: Gradient of Tangent at Point A	**x**	**x**
For use with *Lesson 2*:		
Activity 1: Differentiating Powers of x	**x**	**x**
C: The Chain Rule	**x**	**x**
D: Application to Curves	**x**	**x**
For use with Lesson 3:		
E: Points Moving along Graphs	**x**	**x**
F: Chain Rule Applications	**x**	**x**

Note: Activity 1 can also be done in the form of a card sort or domino chain

Worksheet A: **Sequence of Chords**

$y = \frac{1}{10}x^3 - x + 0.6$



Point	**Coordinates**
A	$(4, 3)$
P_1	$(5, 8.1)$
P_2	$(4.5, \quad)$
P_3	$(4.1, 3.3921)$
P_4	$(4.01, \quad)$
P_5	$(4.001, 3.0038012001)$

Chord	**Gradient**
AP_1	$\frac{8.1 - 3}{1} = 5.1$
AP_2	
AP_3	$\frac{3.3921 - 3}{0.1} = 3.921$
AP_4	
AP_5	$\frac{3.0038012001 - 3}{0.001} = 3.8012001$

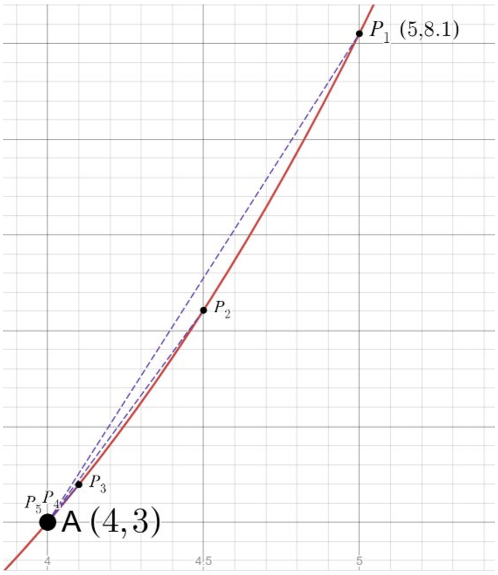
The gradient at point A is: __________

Because: ____

Worksheet A: Answers

$y = \frac{1}{10}x^3 - x + 0.6$

Point	Coordinates
A	$(4, 3)$
P_1	$(5, 8.1)$
P_2	$(4.5, 5.2125)$
P_3	$(4.1, 3.3921)$
P_4	$(4.01, 3.0381201)$
P_5	$(4.001, 3.0038012001)$



Chord	Gradient
AP_1	$\frac{8.1 - 3}{1} = 5.1$
AP_2	$\frac{5.2125 - 3}{0.5} = 4.425$
AP_3	$\frac{3.3921 - 3}{0.1} = 3.921$
AP_4	$\frac{3.0381201 - 3}{0.01} = 3.81201$
AP_5	$\frac{3.0038012001 - 3}{0.001} = 3.8012001$

The gradient at point A is: 3.8

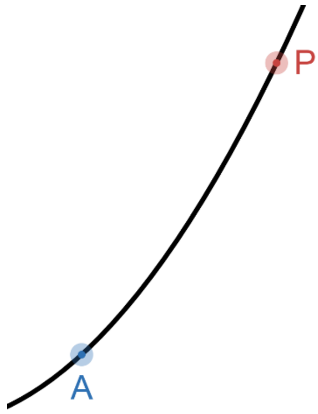
Because: As the chords get closer to point A, the gradients are approaching a limiting value of 3.8

Worksheet B: **Gradient of Tangent at Point A**

Choose one or two of the following problems to work on.

In each question you should obtain an expression for the gradient of the chord.

Then consider the limit as h approaches zero $(h \to 0)$.



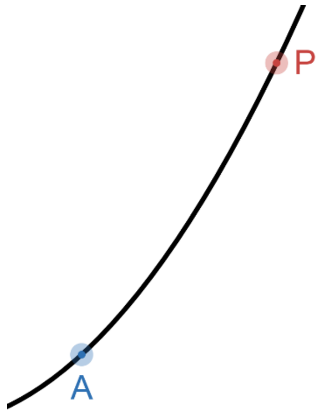
Qn	Curve	Point
1.	$y = x^2$	A: $x = 3$
		P: $x = 3 + h$
2.	$y = 3x^2 + 5$	A: $x = 2$
		P: $x = 2 + h$
3.	$y = x^2 - 4x$	A: $x = 5$
		P: $x = 5 + h$
4.	$y = 2x^2 + 5x$	A: $x = 2$
		P: $x = 2 + h$
5.	$y = x^2 - 3x + 2$	A: $x = 4$
		P: $x = 4 + h$
6.	$y = x^3$	A: $x = 1$
		P: $x = 1 + h$
7.	$y = x^2 + 7x$	A: $x = x$
		P: $x = x + h$

Worksheet B: Answers

Choose one or two of the following problems to work on.

In each question you should obtain an expression for the gradient of the chord.

Then consider the limit as h approaches zero $(h \to 0)$.



Qn	Curve	Point	Gradient of tangent
1.	$y = x^2$	A: $x = 3$ P: $x = 3 + h$	6
2.	$y = 3x^2 + 5$	A: $x = 2$ P: $x = 2 + h$	12
3.	$y = x^2 - 4x$	A: $x = 5$ P: $x = 5 + h$	6
4.	$y = 2x^2 + 5x$	A: $x = 2$ P: $x = 2 + h$	13
5.	$y = x^2 - 3x + 2$	A: $x = 4$ P: $x = 4 + h$	5
6.	$y = x^3$	A: $x = 1$ P: $x = 1 + h$	3
7.	$y = x^2 + 7x$	A: $x = x$ P: $x = x + h$	$2x + 7$

Activity 1: **Differentiating Powers of x**

Match each function with its derivative

function		derivative
$5x^3$		$15x^4$
$3x^5$		$\frac{1}{2\sqrt{x}}$
$\frac{3}{4}x^4$		$-\frac{15}{x^2}$
$\sqrt{x}$		$15x^2$
$\frac{2}{3\sqrt{x}}$		$3x^3$
$\frac{15}{x}$		$-\frac{15}{x^6}$
$\frac{3}{x^5}$		$-\frac{1}{3(\sqrt{x})^3}$

Activity 1: Answers

Match each function with its derivative

function		derivative
$5x^3$		$15x^2$
$3x^5$		$15x^4$
$\frac{3}{4}x^4$		$3x^3$
$\sqrt{x}$		$\frac{1}{2\sqrt{x}}$
$\frac{2}{3\sqrt{x}}$		$-\frac{1}{3\left(\sqrt{x}\right)^3}$
$\frac{15}{x}$		$-\frac{15}{x^2}$
$\frac{3}{x^5}$		$-\frac{15}{x^6}$

Activity 1: **Alternative Version**

This is an alternative form for activity 1. It has been created from a Tarsia generator

$\frac{d}{dx}(5x^3)$	$\frac{1}{2\sqrt{x}}$	$\frac{d}{dx}\left(\frac{15}{x}\right)$	$15x^2$
$-\frac{15}{x^6}$	$\frac{d}{dx}\left(\frac{3}{4}x^4\right)$	$\frac{d}{dx}\left(\frac{2}{3\sqrt{x}}\right)$	$15x^4$
$\frac{d}{dx}(3x^5)$	$-\frac{15}{x^2}$	$\frac{d}{dx}(\sqrt{x})$	*START*
$3x^3$	$-\frac{1}{3(\sqrt{x})^3}$	*FINISH*	$\frac{d}{dx}\left(\frac{3}{x^5}\right)$

					FINISH
					$\frac{d}{dx}\left(\frac{3}{x^5}\right)$
$15x^4$	$\frac{d}{dx}\left(\frac{2}{3\sqrt{x}}\right)$	$-\frac{1}{3(\sqrt{x})^3}$	$3x^3$	$\frac{d}{dx}\left(\frac{3}{4}x^4\right)$	$-\frac{15}{x^6}$
$\frac{d}{dx}(3x^5)$					
$-\frac{15}{x^2}$					
$\frac{d}{dx}\left(\frac{15}{x}\right)$	$15x^2$	$\frac{d}{dx}(5x^3)$	$\frac{1}{2\sqrt{x}}$	$\frac{d}{dx}(\sqrt{x})$	*START*

Worksheet C: The Chain Rule

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Choose some of the following to work on.

In each case find $\frac{dy}{dx}$

<u>Qn</u>

1. $y = (5x + 9)^4$

2. $y = (8 - 2x)^3$

3. $y = (2x^5 + 7)^3$

4. $y = (3x^2 - 5x)^6$

5. $y = \dfrac{2}{5x - 1}$

6. $y = \dfrac{3}{(x^2 + 5x)^2}$

7. $y = \dfrac{1}{\sqrt{x^2 + 3}}$

8. $y = \sqrt{9x^2 - 10x}$

<u>Extension:</u>

For the following curves find the gradient of the tangent at the given point.

9. $y = (4x - 9)^{-2}$

At $x = 3$

10. $y = \dfrac{1}{\sqrt{5 - x^2}}$

At $x = 1$

Worksheet C: **Answers**

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Choose some of the following to work on.

In each case find $\frac{dy}{dx}$

Qn		Ans	Qn		Ans
1.	$y = (5x+9)^4$	$\frac{dy}{dx} = 20(5x+9)^3$	**2.**	$y = (8-2x)^3$	$\frac{dy}{dx} = -6(8-2x)^2$
3.	$y = (2x^5+7)^3$	$\frac{dy}{dx} = 30x(2x^5+7)^2$	**4.**	$y = (3x^2-5x)^6$	$\frac{dy}{dx} = 6(3x^2-5x)(6x-5)$
5.	$y = \frac{2}{5x-1}$	$\frac{dy}{dx} = -10(5x-1)^{-2}$	**6.**	$y = \frac{3}{(x^2+5x)^2}$	$\frac{dy}{dx} = 6(x^2+5x)^{-3}(2x+5)$
7.	$y = \frac{1}{\sqrt{x^2+3}}$	$\frac{dy}{dx} = -x(x^2+3)^{-\frac{1}{2}}$	**8.**	$y = \sqrt{9x^2-10x}$	$\frac{dy}{dx} = (9x^2-10x)^{-\frac{1}{2}}(9x-5)$

Extension:

For the following curves find the gradient of the tangent at the given point.

9.	$y = (4x-9)^{-2}$ At $x = 3$	Gradient $= -\frac{8}{27}$	**10.**	$y = \frac{1}{\sqrt{5-x^2}}$ At $x = 1$	Gradient $= \frac{1}{8}$

Worksheet D: **Application to Curves**

Work on one of the following two questions.

In each case

(a) find the equation of the tangent
(b) find the equation of the normal

at the given point.

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Qn

1. $y = \frac{3}{x^2 - 5}$ at the point $(3, 0.75)$

2. $y = \sqrt{2x - 3}$ at the point $(6, 3)$

Check your answers by drawing the curve, tangent and normal using technology such as a graphing calculator or DESMOS

Extension:

3. Show that the following curve has three stationary points.

$$y = (2x^2 - 5)^4$$

Check your answers by drawing the curve, using technology such as a graphing calculator or DESMOS

Worksheet D: Answers

Work on one of the following two questions.

In each case

(a) find the equation of the tangent
(b) find the equation of the normal

at the given point.

Qn	**Ans**
1. $y = \frac{3}{x^2 - 5}$ at the point $(3, 0.75)$	Tangent $9x + 8y = 33$ oe Normal $8x - 9y = \frac{69}{4}$ oe
2. $y = \sqrt{2x - 3}$ at the point $(6, 3)$	Tangent $3y = x + 3$ oe Normal $3x + y = 21$ oe

Check your answers by drawing the curve, tangent and normal using technology such as a graphing calculator or DESMOS

Extension:

3. Show that the following curve has three stationary points.

$y = (2x^2 - 5)^4$

Stationary points at

$x = 0, \quad x = \pm\frac{\sqrt{10}}{2}$ oe

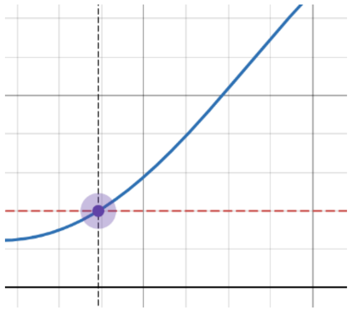
At $\left(-\frac{\sqrt{10}}{2},\ 0\right)$ $(0, 625)$ $\left(\frac{\sqrt{10}}{2},\ 0\right)$

Check your answers by drawing the curve, using technology such as a graphing calculator or DESMOS

Worksheet E: Points Moving along Graphs

Q1

A point P moves along the curve $y = \frac{1}{5}x^2 + \frac{4}{x}$.

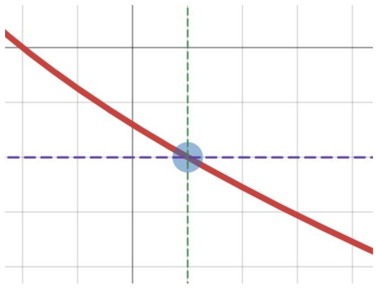


P is moving so that the $\boldsymbol{x}$ **coordinate** is increasing at a rate of 3 units per second.

Find the rate of increase of the y coordinate at the point where $x = 4$.

Q2

A point P moves along the curve $y = 10 - \sqrt{8x + 1}$.

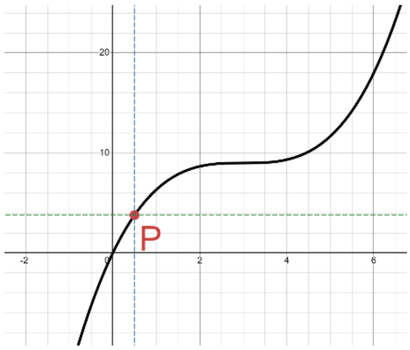


P is moving so that the $\boldsymbol{y}$ **coordinate** is increasing at a rate of 1.5 units per second.

Find the rate of increase of the x coordinate at the point where $y = 3$.

Q3

The curve shown has equation $y = \frac{1}{3}x^3 - 3x^2 + 9x$.



A point P moves along the curve so that the $\boldsymbol{x}$ **coordinate** is increasing at a rate of 2 units per second.

Find the coordinates of the two points on the curve where the $\boldsymbol{y}$ **coordinate** is increasing at a rate of 8 units per second.

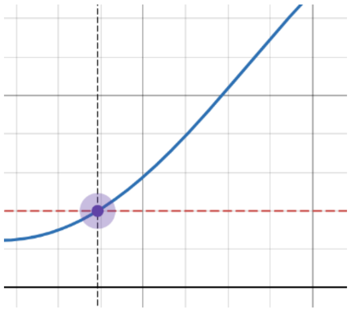
Worksheet E: **Answers**

Q1

A point P moves along the curve $y = \frac{1}{5}x^2 + \frac{4}{x}$.

P is moving so that the $\boldsymbol{x}$ **coordinate** is increasing at a rate of 3 units per second.

Find the rate of increase of the y coordinate at the point where $x = 4$.



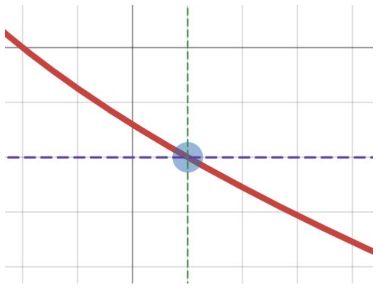
Answer 4.05 units per second

Q2

A point P moves along the curve $y = 10 - \sqrt{8x + 1}$.

P is moving so that the $\boldsymbol{y}$ **coordinate** is increasing at a rate of 1.5 units per second.

Find the rate of increase of the x coordinate at the point where $y = 3$.



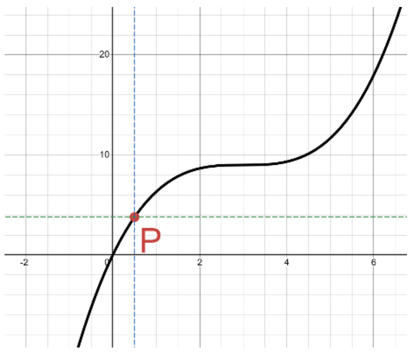
Answer -1.875 units per second (point is moving to the left)

Q3

The curve shown has equation $y = \frac{1}{3}x^3 - 3x^2 + 9x$.

A point P moves along the curve so that the $\boldsymbol{x}$ **coordinate** is increasing at a rate of 2 units per second.

Find the coordinates of the two points on the curve where the $\boldsymbol{y}$ **coordinate** is increasing at a rate of 8 units per second.



Answer: $\left(1, \frac{19}{3}\right)$ and $\left(5, \frac{35}{3}\right)$

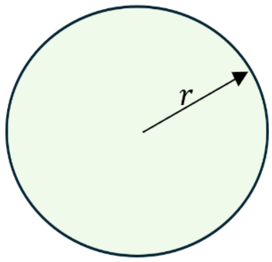
Worksheet F: **Chain Rule Applications**

Work on one or more of the following questions.

Q1

Air is pumped into a gym ball in the shape of a sphere at a constant rate of $1500\ cm^3$ per second.

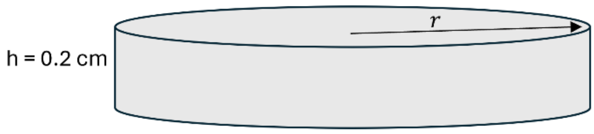
Find the rate at which the radius is increasing when the radius is 25cm.



Q2

Oil drips onto a flat surface at a rate of $12\pi\ cm^3$per second. The oil forms a circular film which can be considered to have a uniform depth of 0.2cm.

Find the rate at which the radius is increasing when the radius is 10cm.



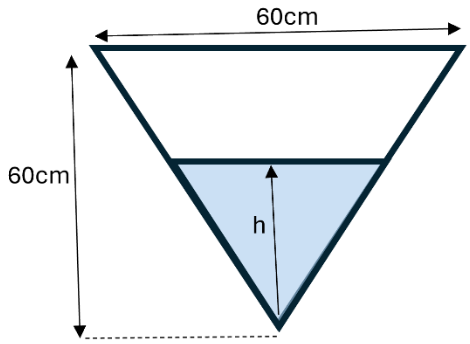
What will be the radius of the oil spill when the rate of increase of radius has slowed to $0.3\ cm$ per second?

Q3

A large water tank 2 metres long, has a triangular cross-section as shown.

Water is pumped into the tank at a rate of $600\ cm^3$ per second.

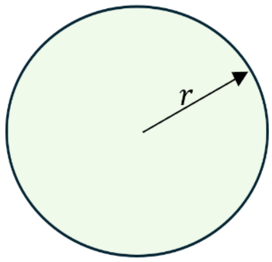
Find the rate at which the height is increasing when the height is $30cm$.



Worksheet F: Answers

Work on one or more of the following questions.

Q1



Air is pumped into a gym ball in the shape of a sphere at a constant rate of $1500\ cm^3$ per second.

Find the rate at which the radius is increasing when the radius is 25cm.

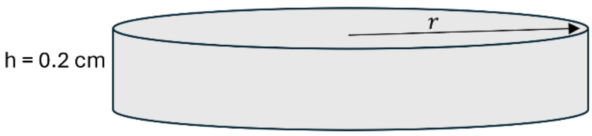
Answer   0.191 cm/s

Q2

Oil drips onto a flat surface at a rate of $12\pi\ cm^3$ per second. The oil forms a circular film which can be considered to have a uniform depth of 0.2cm.

Find the rate at which the radius is increasing when the radius is 10cm.

Answer 3 cm/s



What will be the radius of the oil spill when the rate of increase of radius has slowed to $0.3\ cm$ per second?
Answer 100cm

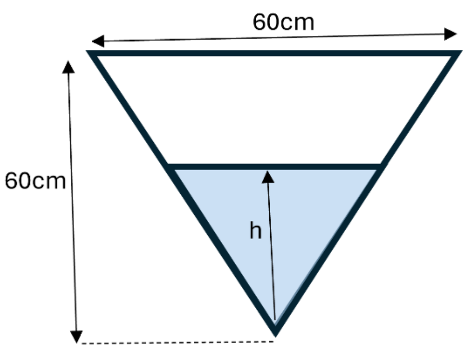
Q3

A large water tank 2 metres long, has a triangular cross-section as shown.

Water is pumped into the tank at a rate of $600\ cm^3$ per second.

Find the rate at which the height is increasing when the height is $30cm$.

Answer : $0.1\ cm$ per second



Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom
t: +44 1223 553554
e: info@cambridgeinternational.org www.cambridgeinternational.org