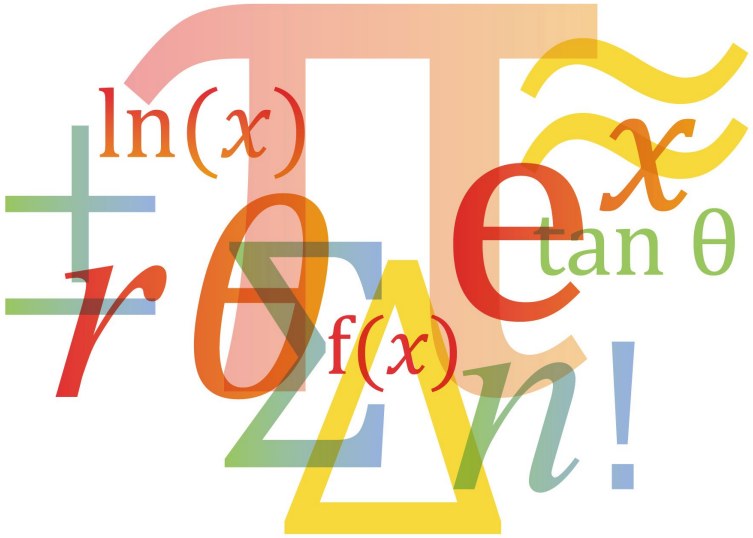
Cambridge Assessment International Education

# Teacher Pack

## Exponential and Logarithmic Functions

**Cambridge International AS & A Level**

Mathematics 9709

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Cambridge Pathway

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# Contents

**Icons used in this pack:**

 **Teacher preparation**

 **Lesson plan**

 **Lesson resource**

 **Lesson reflection**

# Introduction

This pack will help you to develop your learners' skills in mathematical thinking and mathematical communication, which are essential for success at AS & A Level and in further education.

Mathematical thinking and communication will be developed by focussing on:

1. Conceptual understanding – the 'why' behind the 'what'
2. Strategic competence – forming and solving problems
3. Adaptive reasoning – explanations, justifications and deductive reasoning

Throughout all activities, the learners will also develop:

- Procedural fluency – know when, how and which rules to use
- Positive disposition – believe maths can be learned, applied and is useful
- Their skills in writing mathematically – writing working & proofs

These link to the course Assessment Objectives (AOs) which you can find in detail in the syllabus:

**A01 Knowledge and understanding**

**A02 Application and communication**

Each *Teacher Pack* contains one or more lesson plans and associated resources, complete with a section of preparation and reflection.

**Each lesson is designed to be an hour long but you should adjust the timings to suit the lesson length available to you and the needs of your learners.**

**Important note**
Our *Teacher Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other topics.

***This content is designed to give you and your learners the chance to explore a more active way of engaging with mathematics that encourages independent thinking and a deeper conceptual understanding. It is not intended as specific practice for the examination papers.***

The *Teacher Packs* are designed to provide you with some example lessons of how you might deliver content. You should adapt them as appropriate for your learners and your centre. A single pack will only contain at most four lessons, it will **not** cover a whole topic. You should use the lesson plans and advice provided in this pack to help you plan the remaining lessons of the topic yourself.

# Lesson preparation

This *Teacher Pack* will cover the following syllabus content.

| Candidate should be able to: | Notes and examples |
|---|---|
| • understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base) | |
| • understand the definition and properties of $e^x$ and $\ln x$, including their relationship as inverse functions and their graphs | Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of $k$. |
| • use logarithms to solve equations and inequalities in which the unknown appears in indices | e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$ |
| • use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept. | e.g.<br>$y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$.<br>$y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in $x$ and $\ln y$. |

## Dependencies

For all lesson plans in this *Teacher Pack*, knowledge from the following 9709 syllabus content is useful and/or required.

| Candidate should be able to: | Notes and examples |
|---|---|
| • understand the terms function, domain, range, one-one function, inverse function and composition of functions | |
| • understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations. | e.g. to determine the set of values of *k* for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve. |

## Prior knowledge and skills

For all lessons, it is assumed that learners have already completed Cambridge IGCSE™ Mathematics 0580, or a course at an equivalent level. See the syllabus for more details of the expected prior knowledge for taking Cambridge International AS & A Level Mathematics 9709.

When planning any lesson, make a habit of always asking yourself the following questions about your learners' prior knowledge and skills:

- Do I need to re-teach this or do learners just need some practice?
- Is there an interesting activity that will efficiently achieve this?

## Key learning objectives

The following list represents the main underlying concepts that you should make sure your learners have understood by the end of this topic.

- understand the relationship between logarithms and indices
- interconvert the exponential form of equation to logarithmic form
- use the laws of logarithms
- combine different logarithms in same base into single logarithm and separate logarithm
- solve logarithmic equation
- Sketch the graph of exponential and logarithmic functions
- Use logarithms to solve equations and inequalities
- Determine when to reverse the sign when taking the logarithm on both sides of inequality
- use logarithms to transform a given relationship to linear form
- determine unknown constants by considering the gradient and/or intercept

## Why this topic matters

Exponential and logarithmic functions are **fundamental** in **AS and A-Level Mathematics** (and beyond) because they model **real-world phenomena**, enable **problem-solving in advanced topics**, and provide **critical mathematical tools** for analysis.

Exponential functions describe processes where change is proportional to current value, such as:

- Population growth (biology)
- Radioactive decay (physics)
- Compound interest (economics)

Exponential equations appear in:

- Newton's Law of Cooling (temperature change).
- RC circuits (charge decay in capacitors).

## Key terminology and notation

Your learners will need to be confident with the following terminology and notation.

**Exponential Function**: a function of the form of $y = a^x$ (where $a > 0, a \neq 1$)
**Logarithmic Function**: a function which is the inverse function of exponential function, of the form of $y = \log_a x$ (where $a > 0, a \neq 1$)
**Natural Logarithm ($\ln x$)**: logarithm with base of $e \approx 2.718$, Euler's number.

## Lesson progression

Lesson 1 covers the first two bullet points. Lesson 2 focusses on the law of logarithm which is the basic skill learners need for the further learning. Lesson 3 emphasises the exponential and logarithmic function with an explanation of how to obtain logarithmic function from inverse functions of exponential function. Lesson 4 aims to investigate the linearisation of exponential and power model using the skills learned in lesson 2 and 3.

## Going forward

This topic links to integration, differentiation and differential equations.

# Lesson 1: Logarithm

| | |
|---|---|
| **Preparation** | |
| **Resources** | • White Board<br>• Lesson 1 slides<br>• Worksheet 1a, 1b, and 1c |
| **Learning objectives** | By the end of the lesson:<br>• all learners should be able to understand the relationship between logarithms and indices and interconvert the exponential form of equation to logarithmic form |

## Dependencies

Learners need to know the power and indices law from IGCSE mathematics.

## Common misconceptions

| Misconception | Problems this can cause | An example way to resolve the misconception |
|---|---|---|
| Learners often regard the $\log_a b$ as log(a) times b or log(a^b). It is very important to emphasize on this point. | They would not understand the law of logarithm and do not understand what logarithm means. | Emphasise that logarithm is made of 2 numbers: one is base and the other one is argument.<br>Tell them base can be omitted only if the base is 10 or e. |

| Timings | Activity |
|---|---|
| 5 min | **Starter/Introduction**<br>Teach this lesson use the Lesson 1 slides.<br>As a starter, ask the learner this question, if a type of algae needs 30 days to fully cover the pond, how many days it need to cover half of the pond? |
| 45 min | **Main lesson**<br>Part I:<br>Ask the learner to recall the exponent law. Give them some examples and ask them how to simplify the exponent. If nobody mentioned about the negative or fractional exponents, point out and ask them. Show them the **slide 2** and ask them to note down if needed. **Worksheet 1a** can be used for a diagnostic test for prior knowledge.<br>Part II:<br>**Slide 3-7** covers the definition of the logarithm and exponential and logarithmic forms of equations. It is not required for students to know the argument and base but knowing the terminology will help teacher to communicate. |

| Timings | Activity |
|---|---|
| | Converting the logarithmic form and exponential form of equations is the key skill of this lesson. Allocate enough time to show them how to convert and if one example on **slide 6** is not enough, add more examples if needed.<br><br>The special base of 10 and e can be link to their calculator and ask them to familiarise themselves with the $e$, which is a irrational number like $\pi$.<br><br>After slide 7, **worksheet 1b** can be distributed to the students – this could be a think-pair-share activity.<br><br>Part III:<br><br>Show them **slide 8** and solicit 3 students to answer the question on converting the exponential form to the logarithmic form. The first 2 are easy but the 3rd one is a bit challenging. After the students share their answer, the **slide 9** which contain the basic properties of logarithm could be shown.<br><br>Give them 1 example on **slide 10** and explain to learners that the importance of convert the argument into exponent first. Distribute them with **worksheet 1c**.<br><br>**Challenges**: There are several tricky questions in the **worksheet 1c** and ask them not to use calculator. For example, $\log_5 125$ should be 3 instead of 5. Possible learner responses in discussions should be highlighted to help break up blocks of text. |
| 5 min | **Plenary**<br>Wrap up: **Slide 12**<br><br>Ask them to think the range for the Argument.<br><br>Ask them to think why the base and argument should be positive? What happen if there is negative base? |

| **Reflection** | Reflect on your lesson, use the **Lesson reflection** notes to help you. |
|---|---|

# Lesson plan 2: Law of Logarithm

| | |
|---|---|
| **Preparation** | |
| **Resources** | • White Board<br>• Lesson 2 slides<br>• Worksheet 2a, 2b |
| **Learning objectives** | By the end of the lesson:<br>• ***all*** learners should use the laws of logarithms and combine different logarithms in same base into single logarithm<br>• most learners should use the laws of logarithms and combine different logarithms in same base into single logarithm and separate logarithm<br>• some learners should use the laws of logarithms, combine different logarithms in same base into single logarithm and separate logarithm, and solve logarithmic equation |

## Dependencies

Learners need to know the power and indices law from IGCSE mathematics. Learners are required to understand the material in lesson 1.

| Timings | Activity |
|---|---|
| 5 min | **Starter/Introduction**<br>Teach this lesson use the Lesson 2 slides.<br>Solicit some students to recall the definition of logarithm and logarithm properties |
| 45 min | **Main lesson**<br>Part I: Law of logarithms<br>This part could be used to prepare the learners for the command word: show (**slide 3-6**). There are different approaches to show the law of logarithm and this is an approach which could be understood by the students.<br>**Challenge**: ask the learners to show the law of logarithm with the hint given in the slides. Or using leading questions to guide the learners to think about the next step.<br>**Support**: for less capable learners, this show is too challenging for them. Ignore this show part and only use the leading questions to guide the learners.<br>Explain the example 1 and example 2 (**slide 7-9**) in details – make sure all learners know how to combine into single logarithm. This skill needs to be used in the next part of this lesson – solve the logarithmic equation.<br>Give learners **worksheet 2a**, suggested time is 10~15 minutes.<br>Part II: Logarithmic equations |

| Timings | Activity |
|---|---|
| | Solving the logarithmic equations is a very common type of A level math question. Detailed procedures are provided in **slide 10** and teachers could modify these procedures to cater to their own learners' needs.<br><br>Example should be explained in detail to align with the 4 steps. Make sure to ask your learners to note down the procedures as check the solution to satisfy argument>0 is often ignored by learners.<br><br>Give learners **worksheet 2b**, and ask them to show full procedures. |
| 5 min | **Plenary**<br>**Slide 12**<br>Address any common type of mistakes in **worksheet 2b**. |

| **Reflection** | Reflect on your lesson, use the **Lesson reflection** notes to help you. |
|---|---|

# Lesson plan 3: Graph of Exponential and Logarithmic Functions

| | |
|---|---|
| **Preparation** | |
| **Resources** | • White Board<br>• Lesson 3 slides<br>• Worksheet 3a, 3b |
| **Learning objectives** | By the end of the lesson:<br>• ***all*** learners should sketch the graph of exponential and logarithmic functions, use logarithm to solve basic exponential equations<br>• most learners should sketch the graph of exponential and logarithmic functions, use logarithm to solve exponential equations and basic inequalities<br>• some learners should sketch the graph of exponential and logarithmic functions, use logarithm to solve basic exponential equations and inequalities, and determine when to reverse the sign when taking the logarithm on both sides of inequality. |

## Dependencies

Learners need to know the power and indices law from IGCSE mathematics. Learners are required to understand the material in lesson 1 and 2.

| Timings | Activity |
|---|---|
| 5 min | **Starter/Introduction**<br><br>Teach this lesson use the Lesson 3 slides.<br><br>Solicit some students to recall the definition of function and properties of inverse functions |
| 45 min | **Main lesson**<br><br>Part I: Graph of Exponential and Logarithmic Functions<br><br>Present the students with the **slide 2** with the range of values of $a$ and ask them why the other values of $a$ is not meaningful?<br><br>Inquiry-based learning activity:<br><br>● Ask students to draw $f(x) = 2^x$ and $g(x) = 3^x$ with Desmos (**slide 3**)<br>● Ask them what did they see in common<br>● Then ask them to sketch the $y = e^x$ on the same graph of $f(x) = 2^x$ and $g(x) = 3^x$.<br>● Explain the transformation of $y = \left(\frac{1}{2}\right)^x$ from $y = 2^x$ and then ask them to sketch $y = \left(\frac{1}{2}\right)^x$ (**slide 4 and 5**)<br>● Ask them to summarise what they see for the exponential function. Display **slide 6** when most students finished.<br>● Display **slide 7** and ask the students to find the inverse function of $y = e^x$ and then display **slide 8** and ask them to sketch $y = e^x$ |

| Timings | Activity |
|---|---|
| | **Challenge:** Given the property of inverse function, based on the slide 6, deduce the property of exponential function.<br>**Support:** Directly display **slide 9** if this step is not applicable.<br><br>Part II: Logarithmic equations and inequalities<br><br>Display **slide 12**. Go over this example with details with students and guide them step by step.<br><br>Give them the **worksheet 3a** and ask them to finish question 1.<br><br>Display **slide 13.** Go over the example on slide 13 with detail and guide them step by step.<br><br>Then, display **slide 14**, ask them what the difference between the slides. Ask them why this happens? Answer is when $a < 1$, the function $\log_a x$ is a decreasing function. Therefore, the direction of inequality should change.<br><br>Give them the **worksheet 3b** and ask them to finish question 1.<br><br>Support: If no time left, the **worksheet 3b** can be postponed to the next lesson. If the learner is very struggled with the inequality, the worksheet 3b can be used as extra examples and design your own worksheet 3b. |
| 5 min | **Plenary**<br>Address any question arise during worksheet 3a and 3b.<br><br>Leave the uncompleted worksheet as homework<br><br>Use exit card to check whether the learner has grasped most of the concept in this lesson. |

| **Reflection** | Reflect on your lesson, use the **Lesson reflection** notes to help you. |
|---|---|
| | **About inquiry based learning activity, what went well and even better if?**<br>How the learners react with the difficulty of question in worksheet 3b. |

# Lesson plan 4: Linearisation Using Logarithms

| | |
|---|---|
| **Preparation** | |
| **Resources** | • White Board<br>• Lesson 4 slides<br>• Worksheet 4 |
| **Learning objectives** | By the end of the lesson:<br>• ***all*** learners should use logarithms to transform a given relationship to linear form<br>• most learners should use logarithms to transform a given relationship to linear form and determine unknown constants by simple method<br>• some learners should use logarithms to transform a given relationship to linear form and determine unknown constants by considering the gradient and/or intercept |

## Dependencies

Learners need to know the power and indices law from IGCSE mathematics. Learners are required to understand the material in lesson 1, 2, and 3.

| Timings | Activity |
|---|---|
| 15 min | **Starter/Introduction**<br>Teach this lesson use the Lesson 4 slides.<br>Review the worksheet 3a and 3b with learners. |
| 35 min | **Main lesson**<br>Part I: Linearisation of Models<br>Present the students with **slide 2-5**.<br>Go over the linearisation of exponential model in detail, including show the students how to draw the line of best fit on the graphing paper, locate points on the graph.<br>Go through each step and review the formula of gradient and expression of lines if necessary.<br>Go through the transformation of logarithmic form and exponential forms. Show students how to simplify the expression. If necessary, review the exponent laws.<br>Present the students with **slide 6**.<br>Challenge: Ask them to perform what they saw on **slide 2** to deduce the linearisation form of power model.<br>Show them Example 2 on **slide 7**. Ask learners to draw the line of best fit and answer the question.<br>Show them **slide 8 and 9** as a hint if it is necessary. |

| Timings | Activity |
|---|---|
| | Part II: **Worksheet 4**<br><br>This worksheet is very challenging. Most students should be able to access the first question.<br><br>Question 2 is adopted from 0606 past paper question. This question might need a detailed explanation. Enough time should be allocated to this question. |
| 5 min | **Plenary**<br>Address any question arises during worksheet 4<br><br>Present **slide 10** and point out that logarithm of other bases also could be used and commonly seen on the exam. (they have already seen it worksheet 4). |

| **Reflection** | Reflect on your lesson, use the **Lesson reflection** notes to help you.<br>How the learners react with the difficulty of question in worksheet 4?<br>How the learners react with the power model? |
|---|---|

# Planning your own lessons

You now need to plan lessons to cover the following bullet points:

| Candidate should be able to: | Notes and examples |
|---|---|
| • use algebraic methods to solve problems involving lines and circles | Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry.<br>Implicit differentiation is not included. |
| • understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations. | e.g. to determine the set of values of $k$ for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve. |

Follow the structure of the *Skills Pack*, and use techniques from the 'How to' guides, to create your own engaging lessons to cover these bullet points. Consider what preparation you need for each lesson: what prior knowledge is needed, what are the key objectives, what are the dependencies, what common misconceptions are there, and so on.

Below, we have provided an outline of some activities and approaches you might like to try.

Lesson *x*: Solving problems with lines and circles
Common misconceptions:
Starter: You could try …
Main: You could use resource xxxxx and have a class discussion about *y* ….
Plenary: You could try …

Lesson *y*: The relationship between a graph and its algebraic equation
Common misconceptions:
Starter: You could try …
Main: You could use resource xxxxx and have a class discussion about *y* ….
Plenary: You could try …

You will find some other activity suggestions in the Scheme of Work.

# Lesson reflection

As soon as possible after the lesson you need to think about how well it went.

One of the key questions you should always ask yourself is:

*Did all learners get to the point where they can access the next lesson? If not, what will I do?*

Reflection is important so that you can plan your next lesson appropriately. If any misconceptions arose or any underlying concepts were missed, you might want to use this information to inform any adjustments you should make to the next lesson.

It is also helpful to reflect on your lesson for the next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time. There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.

**To help you reflect on your lesson, answer the most relevant questions below.**

*Were the lesson objectives realistic?*
*What did the learners learn today? Or did they learn what was intended? Why not?*
*What proportion of the time did we spend on the most important topics?*
*Were there any common misconceptions?*
*What do I need to address next lesson?*
*What was the learning atmosphere like?*
*Did my planned differentiation work well?*
*How could I have helped the lowest achieving learners to do more?*
*How could I have stretched the highest achieving learners even more?*
*Did I stick to timings?*
*What changes did I make from my plan and why?*

## Summary evaluation

What two things went really well? (Consider both teaching and learning.)

What two things would have improved the lesson? (Consider both teaching and learning.)

What have I learned from this lesson about the class or individuals that will inform my next lesson?

# Worksheets and answers

| ‘ | Worksheet | Answers |
|---|---|---|
| **For use with *Lesson 1*** | | |
| **A:** Evaluate Exponents | x | x |
| **B:** Exponent and Logarithmic Form | x | x |
| **C:** Basic Property of Logarithm | x | x |
| **For use with *Lesson 2*:** | | |
| **D:** Law of Logarithm | x | x |
| **E:** Logarithmic Equation | x | x |
| **For use with *Lesson 3*:** | x | x |
| **F:** Exponential Equation | x | x |
| **G:** Logarithmic Inequalities | x | x |
| **For use with Lesson plan 4:** | | |
| **H:** Linearisation of Models | x | x |

# Worksheet A: Evaluate Exponents

Evaluate with procedures:

a. $5^{-2}$

b. $\left(\frac{2}{3}\right)^{-3}$

c. $8^{\frac{2}{3}}$

d. $\left(\frac{27}{64}\right)^{-\frac{1}{3}}$

e. $\left(\frac{1}{9}\right)^{-\frac{3}{2}}$

# Worksheet B: Exponent and Logarithmic Form

1. Convert $2^3 = 8$ to logarithmic form.

2. Convert $e^x = 7$ to logarithmic form.

3. Convert $\left(\frac{1}{3}\right)^{-2} = 9$ to logarithmic form.

4. Convert $log\ A = 3$ to exponential form. (Hint, the base of $log\ A$ is 10)

5. Convert $\log_2 c = 5$ to exponential form.

6. Convert $\log_x 3 = q$ to exponential form.

# Worksheet C: Basic Property of Logarithm

Evaluate the following:

1. $\log_2 32$

2. $\log_5 125$

3. $\log_6 \sqrt[3]{6}$

4. $\log\left(\frac{1}{10}\right)$

5. $\log_2 \frac{1}{8}$

# Worksheet D: Law of Logarithm

1. Combine the following into a single logarithm:

$$2\log_3 x + \log_3 y - \frac{1}{3}\log_3 z$$

2. Combine the following into single logarithm:

$$3\ln a - 2\ln b + 4\ln c$$

3. Given that $\log_2 x = a$, $\log_2 y = b$, and $\log_2 z = c$;
   Write $\log_2\left(\frac{\sqrt[3]{y}}{x^2 z}\right)$ in terms of $a$, $b$, and $c$.

4. (challenging question, for high achiever)
   If $\log 2 = r$ and $\log 3 = s$ , express in terms of $r$ and $s$
   (a) $\log 16$

   (b) $\log 18$

   (c) $\log 13.5$

   (d) $\log 2.7$

   (e) $\log 5$

(Hint: this is a logarithm in the base of 10)

# Worksheet E: Logarithmic Equation

Show the full procedures:

1. Solve for $x$ in $\log_5(2x-1)=2$

2. Solve for $x$ in $\log_3(x+1)+\log_3(x-1)=2$

3. Solve for $x$ in $\log_2(x)+\log_2(x+6)=4$

# Worksheet F: Exponential Equation

Question 1:

Solve $15^{2a-1} = 27^{3a+2}$ for $a$. Express your answer in terms of $\ln 3$ and $\ln 5$.

Question 2:

Solve $3 \times 6^{x-1} = 2 \times 3^{x+2}$, giving your answer in the form $x = \frac{\ln a}{\ln b}$, where $a,\ b \in \mathbb{Z}$.

# Worksheet G: Logarithmic Inequalities

Question 1:

Solve $\left(\frac{1}{3}\right)^{2x+1} \leq 27$ for $x$.

Question 2:

Solve $3 \times 5^{3x+1} > 21$, for range of possible values of $x$.

# Worksheet H: Linearisation of Models

1. Variables $x$ and $y$ are such that, when $\log y$ is plotted against $x^3$, a straight line graph passing through the points (6, 7) and (10, 9) is obtained. Find $y$ as a function of $x$.

2. The table shows values of the variables $x$ and $y$, which are related by an equation of the form $y = Ab^{x^2}$, where $A$ and $b$ are constants.

| $x$ | 1 | 1.5 | 2 | 2.5 |
|---|---|---|---|---|
| $y$ | 2.0 | 11.3 | 128 | 2896 |

(a) Use this data to obtain table of $\ln y$ and $x^2$

(b) Draw a straight line graph of $\ln y$ and $x^2$

(c) Use your graph to estimate the value of $A$ and $b$, correct to 1 significant figure.

(adopt from 0606 March 23 Paper 12 Q5)

# Worksheet A: Answers

Evaluate with procedures:

a. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b. $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3} = \frac{1}{\frac{8}{27}} = \frac{27}{8}$

c. $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

d. $\left(\frac{27}{64}\right)^{-\frac{1}{3}} = \left(\sqrt[3]{\frac{27}{64}}\right)^{-1} = \left(\frac{3}{4}\right)^{-1} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

e. $\left(\frac{1}{9}\right)^{-\frac{3}{2}} = 9^{\frac{3}{2}} = \left(\sqrt{9}\right)^3 = 3^3 = 27$

# Worksheet B: Answers

1. Convert $2^3 = 8$ to logarithmic form.

$$2^3 = 8 \Leftrightarrow \log_2 8 = 3$$

2. Convert $e^x = 7$ to logarithmic form.

$$e^x = 7 \Leftrightarrow \log_e 7 = x \ (x = \ln 7)$$

3. Convert $\left(\frac{1}{3}\right)^{-2} = 9$ to logarithmic form.

$$\left(\frac{1}{3}\right)^{-2} = 9 \Leftrightarrow \log_{\frac{1}{3}} 9 = -2$$

(Note that: the logarithm can produce a negative number)

4. Convert $log\ A = 3$ to exponential form. (Hint, the base of $log\ A$ is 10)

$$log\ A = 3 \Leftrightarrow \log_{10} A = 3 \Leftrightarrow A = 10^3 = 1000$$

5. Convert $\log_2 c = 5$ to exponential form.

$$\log_2 c = 5 \Leftrightarrow c = 2^5 = 32$$

6. Convert $\log_x 3 = q$ to exponential form.

$$\log_x 3 = q \Leftrightarrow x^q = 3$$

# Worksheet C: Answers

Evaluate the following:

$$\log_2 32 = \log_2 2^5 = 5$$

$$\log_5 125 = \log_5 5^3 = 3$$

$$\log_6 \sqrt[3]{6} = \log_6 6^{\frac{1}{3}} = \frac{1}{3}$$

$$\log\left(\frac{1}{10}\right) = \log_{10} 10^{-1} = -1$$

$$\log_2 \frac{1}{8} = \log_2 8^{-1} = \log_2 (2^3)^{-1} = \log_2 2^{-3} = -3$$

# Worksheet D: Answers

1. Combine the following into a single logarithm:

$$2\log_3 x + \log_3 y - \frac{1}{3}\log_3 z$$

$$= \log_3 x^2 + \log_3 y - \log_3 z^{\frac{1}{3}}$$

$$= \log_3 x^2 y - \log_3 \sqrt[3]{z}$$

$$= \log_3 \frac{x^2 y}{\sqrt[3]{z}}$$

2. Combine the following into single logarithm:

$$3\ln a - 2\ln b + 4\ln c$$

$$= \ln a^3 - \ln b^2 + \ln c^4$$

$$= \ln \frac{a^3}{b^2} + \ln c^4$$

$$= \ln \frac{a^3 c^4}{b^2}$$

3. Given that $\log_2 x = a$, $\log_2 y = b$, and $\log_2 z = c$;
   Write $\log_2 \left(\frac{\sqrt[3]{y}}{x^2 z}\right)$ in terms of $a$, $b$, and $c$.

$$\log_2 \left(\frac{\sqrt[3]{y}}{x^2 z}\right) = \log_2 \sqrt[3]{y} - \log_2 (x^2 z)$$

$$= \log_2 y^{\frac{1}{3}} - (\log_2 x^2 + \log_2 z)$$

$$= \frac{1}{3}\log_2 y - 2\log_2 x - \log_2 z$$

$$= \frac{1}{3}b - 2a - c$$

4. (challenging question, for high achiever)
   If $\log 2 = r$ and $\log 3 = s$ , express in terms of $r$ and $s$
   (a) $\log 16$, (b) $\log 18$, (c) $\log 13.5$ (d) $\log 2.7$ (e) $\log 5$

$$\log 16 = \log 2^4 = 4\log 2 = 4r$$

$$\log 18 = \log(2 \times 9) = \log 2 + \log 9 = r + \log 3^2 = r + 2\log 3 = r + 2s$$

$$\log 13.5 = \log \frac{27}{2} = \log 27 - \log 2 = \log 3^3 - r = 3s - r$$

$$\log 2.7 = \log \frac{27}{10} = \log 3^3 - \log 10 = 3s - \log_{10} 10 = 3s - 1$$

$$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - r$$

   (Hint: this is a logarithm in the base of 10)

# Worksheet E: Answers

1. Solve for $x$ in $\log_5(2x-1)=2$

$$\log_5(2x-1)=2 \Leftrightarrow 2x-1=2^5$$

$$2x-1=32$$

$$2x=33$$

$$x=\frac{33}{2}$$

2. Solve for $x$ in $\log_3(x+1)+\log_3(x-1)=2$

$$\log_3(x+1)(x-1)=2$$

$$\log_3(x^2-1)=2$$

$$x^2-1=3^2$$

$$x^2=10$$

$$x=\pm\sqrt{10}$$

When $x=-\sqrt{10}$, the $\log_3(x+1)+\log_3(x-1)=\log_3\left(-\sqrt{10}+1\right)+\log_3\left(-\sqrt{10}-1\right)$

$$-\sqrt{10}+1<0$$

$$-\sqrt{10}-1<0$$

Argument should be positive.

$x=-\sqrt{10}$ is ignored.

$$x=\sqrt{10}$$

3. Solve for $x$ in $\log_2(x)+\log_2(x+6)=4$

$$\log_2(x)(x+6)=4$$

$$\log_2(x^2+6x)=4$$

$$x^2+6x=2^4$$

$$x^2+6x-16=0$$

$$(x+8)(x-2)=0$$

$$x_1=-8 \text{ or } x_2=2$$

Argument should be positive: $x_1=-8$ is ignored.

$$x=2$$

# Worksheet F: Answers

Question 1:

Solve $15^{2a-1} = 27^{3a+2}$ for $a$. Express your answer in terms of $\ln 3$ and $\ln 5$.

$$15^{2a-1} = 27^{3a+2}$$

$$\ln(15^{2a-1}) = \ln(27^{3a+2})$$

$$(2a-1)\ln 15 = (3a+2)\ln 27$$

$$(2a-1)\ln(3\times 5) = (3a+2)\ln 3^3$$

$$(2a-1)(\ln 3 + \ln 5) = 3(3a+2)\ln 3$$

$$(2a-1)\ln 3 + (2a-1)\ln 5 = (9a+6)\ln 3$$

$$2a\ln 3 - \ln 3 + 2a\ln 5 - \ln 5 = 9a\ln 3 + 6\ln 3$$

$$2a\ln 3 + 2a\ln 5 - 9a\ln 3 = 6\ln 3 + \ln 5 + \ln 3$$

$$2a\ln 5 - 7a\ln 3 = 7\ln 3 + \ln 5$$

$$a(2\ln 5 - 7\ln 3) = 7\ln 3 + \ln 5$$

$$a = \frac{7\ln 3 + \ln 5}{2\ln 5 - 7\ln 3}$$

Question 2:

Solve $3\times 6^{x-1} = 2\times 3^{x+2}$, giving your answer in the form $x = \frac{\ln a}{\ln b}$, where $a,\ b \in \mathbb{Z}$.

$$3\times 6^{x-1} = 2\times 3^{x+2}$$

$$\ln(3\times 6^{x-1}) = \ln(2\times 3^{x+2})$$

$$\ln 3 + \ln 6^{x-1} = \ln 2 + \ln 3^{x+2}$$

$$\ln 3 + (x-1)\ln 6 = \ln 2 + (x+2)\ln 3$$

$$\ln 3 + x\ln 6 - \ln 6 = \ln 2 + x\ln 3 + 2\ln 3$$

$$x\ln 6 - x\ln 3 = \ln 2 + \ln 6 + 2\ln 3 - \ln 3$$

$$x(\ln 6 - \ln 3) = \ln 2 + \ln 6 + \ln 3$$

$$x\ln 2 = \ln(2\times 6\times 3)$$

$$x = \frac{\ln 36}{\ln 2}$$

# Worksheet G: Answers

Question 1:

Solve $\left(\frac{1}{3}\right)^{2x+1} \leq 27$ for $x$.

$$\left(\frac{1}{3}\right)^{2x+1} \leq 27$$

$$2x + 1 \geq \log_{\frac{1}{3}}(27)$$

$$2x + 1 \geq \log_{\frac{1}{3}}(3^3)$$

$$2x + 1 \geq \log_{\frac{1}{3}}\left(\left(\frac{1}{3}\right)^{-3}\right)$$

$$2x + 1 \geq -3$$

$$2x \geq -4$$

$$x \geq -2$$

Alternative method:

$$\left(\frac{1}{3}\right)^{2x+1} \leq 27$$

$$(3^{-1})^{2x+1} \leq 27$$

$$3^{-2x-1} \leq 27$$

$$-2x - 1 \leq \log_3 27$$

$$-2x - 1 \leq 3$$

$$-2x \leq 4$$

$$x \geq -2$$

Question 2:

Solve $3 \times 5^{3x+1} > 21$, for range of possible values of $x$.

$$3 \times 5^{3x+1} > 21$$

$$5^{3x+1} > 7$$

$$3x + 1 > \log_5 7$$

$$3x > \log_5 7 - 1$$

$$x > \frac{1}{3}(\log_5 7 - 1)$$

# Worksheet H: Answers

1. Variables $x$ and $y$ are such that, when $\log y$ is plotted against $x^3$, a straight line graph passing through the points (6, 7) and (10, 9) is obtained. Find $y$ as a function of $x$.

$$m = \frac{9-7}{10-6} = \frac{2}{4} = \frac{1}{2}$$

$$\log y - 7 = \frac{1}{2}(x^3 - 6)$$

$$\log y = \frac{1}{2}x^3 + 4$$

$$y = 10^{\frac{1}{2}x^3+4} = 10^{\frac{1}{2}x^3} \times 10^4 = 10000 \times 10^{\frac{1}{2}x^3}$$

2. The table shows values of the variables $x$ and $y$, which are related by an equation of the form $y = Ab^{x^2}$, where $A$ and $b$ are constants.

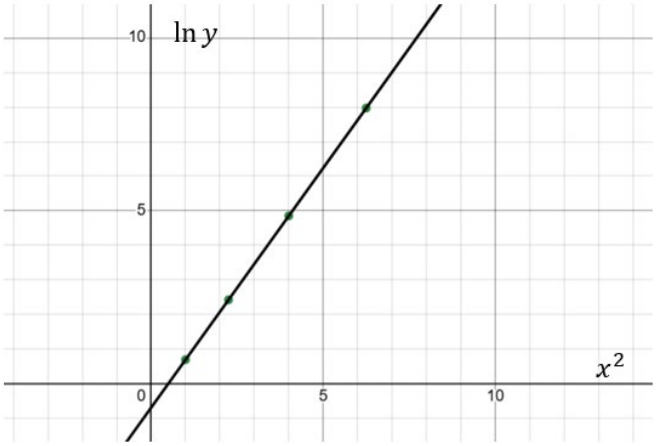
| $x$ | 1 | 1.5 | 2 | 2.5 |
|---|---|---|---|---|
| $y$ | 2.0 | 11.3 | 128 | 2896 |

(a) Use this data to obtain table of $\ln y$ and $x^2$
(b) Draw a straight line graph of $\ln y$ and $x^2$
(c) Use your graph to estimate the value of $A$ and $b$, correct to 1 significant figure.
(adopt from 0606 March 23 Paper 12 Q5)

(a)

| $x^2$ | 1 | 2.25 | 4 | 6.25 |
|---|---|---|---|---|
| $\ln y$ | 0.69315 | 2.425 | 4.852 | 7.9711 |

(b)



(c)

From the graph, $m = 1.386$ (1.2-1.5), y-intercept is -0.694

$$\ln y = 1.386x^2 - 0.694$$

$$y = e^{1.386x^2 - 0.694}$$

$$y = e^{1.386x^2} \times e^{-0.694}$$

$$y = 0.500e^{1.39x^2} \approx 0.5e^{x^2} \ (1\ sig.fig.)$$

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