

Teaching Pack

Functions

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Mathematics 0580



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	Lesson plan
	Teacher’s notes
	Lesson resources

Introduction: Functions

This *Teaching Materials Pack* focuses on supporting learners to develop a confidence and fluency with the notation and terminology associated with mathematical function notation. Learners will be guided from basics such as substituting into a single function to creating composite functions of two or more functions.

The lesson presented here is designed for learners that are already familiar with the concept of using a 'y =' cartesian graph in order to calculate coordinates and are ready to engage with ideas involving combining 2 or more functions together.

It is expected that learners should already understand the concept that substituting a value into a function (an input) will lead to you calculating an output. They should be able to link this to their knowledge of cartesian graphs in order to plot functions on a graph.

It would be useful if learners were able to work with composite functions that have elements such as surds and algebraic fractions. Some of these questions could be non-calculator, so some mastery of written methods will be crucial at times.

Important note

Our *Teaching Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other skills.

This content is designed to give you and your learners the chance to explore mathematical skills. It is not intended as specific practice for exam papers.

This is one of a range of Teaching Packs. Each pack is based on one mathematical topic with a focus on specific mathematical techniques. The packs can be used in any order to suit your teaching sequence.

In this pack you will find the lesson plans and worksheets you will need to successfully complete the teaching of this topic.

Syllabus links

This *Teaching Pack* links to the following syllabus content (see syllabus for detail):

E2.13 Understand functions, domain and range and use function notation.
Understand and find inverse functions $f^{-1}(x)$.
Form composite functions as defined by $gf(x) = g(f(x))$.

The pack covers mathematical skills, adapted from **AO1: Demonstrate knowledge and understanding of mathematical techniques** and **AO2: Reason, interpret and communicate mathematically when solving problems**.

Lesson Plan 1 – Function notation



Resources

- Lesson 1 PowerPoint presentation
- Worksheet 1

Learning objectives

By the end of the lesson:

- **all learners** will be able to substitute values into a function to find an outcome.
- **most learners** will be able to solve equations in order to find the value of missing inputs.
- **some learners** will be able to use algebra to describe the domain and range of different functions.

Timings	Activity
5 minutes	<p>Starter / Introduction</p> <p>Slide 2 Use this activity as a quick recap for how to substitute into a range of functions. One common area of misconception is the function: $y = 4x^3$, as learners often multiply by 4 before applying the power, which is incorrect.</p>
5 minutes	<p>Main lesson</p> <p>Slides 3–4 Begin by explaining how function notation is used. The example on slide 4 serves to show learners that are used to doing this with cartesian equations, and it is just the notation that is new to them.</p>
5 minutes	<p>Slides 5–6 There are 3 worked examples for the teacher to go through, and then 3 for the learners to work on afterwards to check for understanding.</p>
5 minutes	<p>Slides 7–8 These focus on more challenging teacher examples where learners have to substitute into more than one position. There are then two more for the students to complete to check their understanding.</p>
10 minutes	<p>Slides 9–11 These slides provide two examples of using functions backwards in order to find what inputs gave a certain output. The first example is a linear equation and learners should find it reasonably easy. The second example is quadratic, and highlights that there could be two different inputs that give the same output. On slide 11, learners have 2 examples to try themselves.</p>
5 minutes	<p>Slides 12–14 Explain the definition of domain, and show some examples of common functions that have an enforced domain. learners sometimes struggle with this concept initially. The pictures of the graphs should help overcome this.</p>
5 minutes	<p>Slides 15–18 This is very similar to the previous three slides, but instead refers to the range of functions. Some basic examples are provided for learners to get to grips with the concept.</p>
Rest of the lesson	<p>Work on Worksheet 1.</p>



Lesson Plan 2 – Composite functions

Resources

- Lesson 2 PowerPoint presentation
- Worksheet 2

Learning objectives

By the end of the lesson:

- **all learners** will be able to substitute values into composite functions.
- **most learners** will be able to write the simplified version of a composite function.
- **some learners** will be able to solve equations involving composite functions.

Timings	Activity
5 minutes	<p>Starter / Introduction</p> <p>Slide 2 This activity gets learners to practice the key skill that will be required for the lesson, substituting one function into another. This will help you determine how well learners are able to do this. You may need to cover some extra examples if the students struggle with this skill.</p>
3–5 minutes	<p>Main lesson</p> <p>Slides 3–4 Slide 3 starts with a quick explanation about what a composite function actually is, the notation used and then a quick example about how to substitute values into one. If your learners are confident in the skills learnt in the first lesson then they should find this reasonably easy to understand.</p>
3–5 minutes	<p>Slides 5–6 There are two worked examples where inputs have been substituted into composite functions. Emphasise to the learners that you tackle these functions one at a time.</p>
5–8 minutes	<p>Slide 7–8 Get learners to work through these three questions themselves. Once the majority have finished, go through the answers carefully, being sure to address any misconceptions.</p>
5 minutes	<p>Slide 9 This slide goes through how to form composite functions. It gives two examples, one with $fg(x)$ and the other $gf(x)$, and highlights the big difference in the answers generated.</p>
5–8 minutes	<p>Slide 10 Learners now have two questions to work on themselves. The second question is quite difficult, and the students will need to be precise with their algebra to arrive at the correct answer. The negative in front of the squared bracket may cause issues if the students do not multiply the whole bracket by -1.</p>
5 minutes	<p>Slide 11 This is one final example of learners to solve an equation involving composite functions. This is a challenging question and draws on the equation solving from the previous lesson. Be prepared to work slowly as you go through it.</p> <p>Work on Worksheet 2.</p>

Lesson Plan 3 – Inverse functions



Resources

- Lesson 3 PowerPoint presentation
- Worksheet 3

Learning objectives

By the end of the lesson:

- **all learners** will be able to find the inverse of simple functions.
- **most learners** will be able to find more complicated inverse functions involving brackets or indices.
- **some learners** will be able to equation solve with a wide array of challenging inverse functions.

Timings	Activity
5–10 minutes	<p>Starter / Introduction</p> <p>Slide 2 Use this activity as a quick recap for how to rearrange formulae to make a different variable the subject. It is important that any misconceptions are addressed here, as this is the main technique required for finding inverse functions.</p>
5–8 minutes	<p>Main lesson</p> <p>Slide 3 Use this slide as an explanation as to what an inverse does, and how it relates to the original function. Emphasise that it reverses the effect of the original function.</p> <p>Slide 4 This example with a specific function, and a specific input, should help show what an inverse function actually does. By substituting the original output as the input into the inverse function, you generate the original input. Learners take note of the notation for inverse functions, and make links to trigonometry, where they may have seen this notation before.</p>
5 minutes	<p>Slide 5 Demonstrate the process of how to find the inverse of a function. Be clear on each step as you work through the worked example. Learners often forget to label their final function with the proper inverse notation, so it is worth emphasising this point at the end of the example.</p>
5–8 minutes	<p>Slide 6 There are two questions for learners to complete themselves to demonstrate their understanding of the technique. The first is a linear function and should be reasonably straight forward. The second involves a square root, as well as knowledge of the correct order in which to rearrange the equation correctly.</p>
3–5 minutes	<p>Slide 7 Work through the more difficult example. Learners may initially find it difficult to understand what the question is asking for, however, once they have substituted both functions into the equation the solution will be reasonably simple.</p> <p>Work on Worksheet 3.</p>



Teacher's notes

Key words / concepts you could highlight during the lesson, or have pre-taught before the lesson:

Learners need a basic understanding of substitution and equation solving before tackling Lesson 1.

Key words

- Substitute
- Function
- Domain
- Range
- Composite
- Inverse

It is likely that learners will feel reasonably comfortable with the content in Lesson 1, as they have used these skills previously. You may find that Lesson 2 and Lesson 3 may take longer than a single lesson to cover, as the content is more difficult, and the questions will take learners longer to complete.



Lesson resources

Worksheet 1: Function notation

Worksheet 1: Function notation answers

Worksheet 2: Composite functions

Worksheet 2: Composite functions answers

Worksheet 3: Inverse functions

Worksheet 3: Inverse functions answers



Worksheet 1: Function notation

1. $f(x) = 4x^2$.

(a) Find $f(2)$.

(b) Find $f(10)$.

(c) Find $f(0.5)$.

(d) Find the value of a when $f(a) = 36$.

2. $g(x) = \frac{12}{x}$ $h(x) = 3x - 2$

(a) Find $g(2)$.

(b) Find $h(10)$.

(c) Find $g(4) \times h(4)$.

(d) Find $\frac{h(1)}{g(12)}$.

3. $f(x) = x^2 - 16$ Find the value of a when:

(a) $f(a) = -16$.

(b) $f(a) = 0$.

(c) $f(a) = -12$.

4. $f(x) = 2x(x + 3)$, $g(x) = (x - 2)(x + 4)$. Find the values of b when:

(a) $f(b) = 0$.

(b) $g(b) = 0$.

(c) $f(b) = 80$.

(d) $g(b) = -9$.

(e) $f(b) + g(b) = 20$.

Worksheet 1: Function notation answers



1.

(a) $f(2) = 4 \times 4 = 8.$

(b) $f(10) = 4 \times 100 = 400.$

(c) $f(0.5) = 4 \times 0.25 = 1$

(d) $4a^2 = 36 \quad a = 3 \text{ or } a = -3$

2.

(a) $g(2) = 6 .$

(b) $h(10) = 28.$

(c) $g(4) \times h(4) = 3 \times 10 = 30.$

(d) $\frac{h(1)}{g(12)} = 1 .$

3.

(a) $a = 0$

(b) $a = 4 \text{ or } -4$

(c) $a = 2 \text{ or } -2$

4.

(a) $b = -3 \text{ or } b = 0$

(b) $b = 2 \text{ or } b = -4$

(c) $b = 5 \text{ or } b = -8$

(d) $b = -1$

(e) $b = 2 \text{ or } b = -4.666$



Worksheet 2: Composite functions

1. $f(x) = 4x^2$ $g(x) = x^3$ $h(x) = x - 2$

(a) Find $fh(2)$.

(b) Find $gh(5)$.

(c) Find $hg(5)$.

(d) Find $gf(1)$.

(e) Find $fhg(2)$.

2. $f(x) = 4x^2$ $g(x) = x^3$ $h(x) = x - 2$. Find expressions for the following. Give your answers fully simplified.

(a) $fh(x)$

(b) $gh(x)$

(c) $hg(x)$

(d) $gf(x)$

(e) $fhg(x)$

3. Given that $f(x) = 4x^2$ and $g(x) = \frac{1}{x-1}$

Challenging

(a) Find an expression for $fg(x)$.

(b) Solve the equation $fg(x) = 36$.



Worksheet 2: Composite functions answers

1.

(a) $fh(2) = 0$

(b) $gh(5) = 27$

(c) $hg(5) = 123$

(d) $gf(1) = 64$

(e) $fhg(2) = 144$

2.

(a) $fh(x) = 4x^2 - 16x + 16$

(b) $gh(x) = x^3 - 6x^2 + 12x - 8$

(c) $hg(x) = x^3 - 2$

(d) $gf(x) = 64x^6$

(e) $fhg(x) = 4x^6 - 16x^3 + 16$

3.

(a) $fg(x) = \frac{4}{(x-1)^2}$

(b) $x = \frac{2}{3}$ or $x = \frac{4}{3}$



Worksheet 3: Inverse functions

1. For each of the following functions, find $f^{-1}(x)$.

(a) $f(x) = 3x + 9$

(b) $f(x) = \frac{x}{10} + 13$

(c) $f(x) = 7(x - 4)$

(d) $f(x) = x^2 + 19$

(e) $f(x) = 2x^2 + 3$

2. $g(x) = 12(x - 5)$

(a) $g(a) = 24$, find a .

(b) Find $g^{-1}(-36)$.

(c) $g^{-1}(b) = 0$, find b .

3. $f(x) = 7(x - 2)$ and $g(x) = 7(x + 2)$

(a) Find $f^{-1}(x)$.

(b) Find $g^{-1}(x)$.

(c) Find $f^{-1}(14)$.

(d) Find $g^{-1}(14)$.

(e) Find the value of c such that $f^{-1}(c) + g^{-1}(c) = 4$.

Worksheet 3: Inverse functions answers



1.

(a) $f^{-1}(x) = \frac{x-9}{3}$

(b) $f^{-1}(x) = 10(x - 13)$

(c) $f^{-1}(x) = \frac{x}{7} + 4$ or $\frac{x+28}{7}$

(d) $f^{-1}(x) = \sqrt{x - 19}$

(e) $f^{-1}(x) = \sqrt{\frac{x-3}{2}}$

2.

(a) $a = 7$

(b) $g^{-1}(-36) = 8$

(c) $b = -60$

3.

(a) $f^{-1}(x) = \frac{x}{7} + 2$

(b) $g^{-1}(x) = \frac{x}{7} - 2$

(c) $f^{-1}(14) = 4$

(d) $g^{-1}(14) = 0$

(e) $c = 14$

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