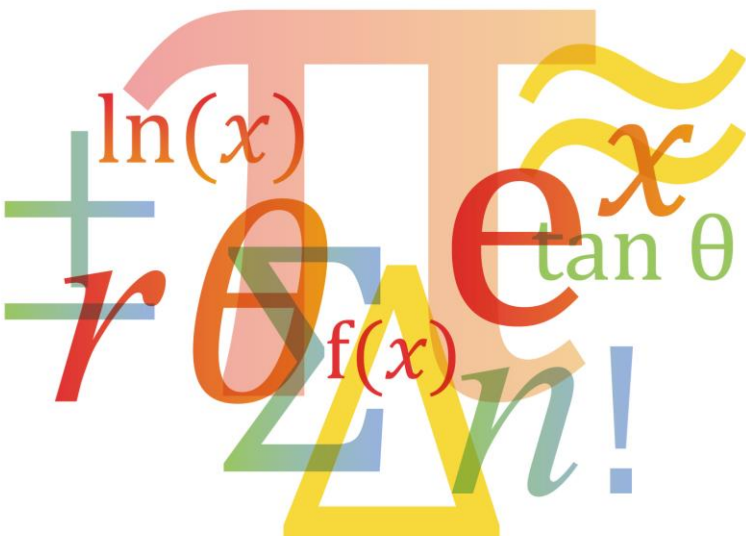
Cambridge Assessment International Education

Teacher Pack

Probability

Cambridge International AS & A Level

Mathematics 9709



Version 1

Cambridge Pathway

Contents

Icons used in this pack:

Teacher preparation

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Introduction

This pack will help you to develop your learners' skills in mathematical thinking and mathematical communication, which are essential for success at AS & A Level and in further education.

Mathematical thinking and communication will be developed by focussing on:

1. Conceptual understanding – the 'why' behind the 'what'
2. Strategic competence – forming and solving problems
3. Adaptive reasoning – explanations, justifications and deductive reasoning

Throughout all activities, the learners will also develop:

- Procedural fluency – know when, how and which rules to use
- Positive disposition – believe maths can be learned, applied and is useful
- Their skills in writing mathematically – writing working & proofs

These link to the course Assessment Objectives (AOs) which you can find in detail in the syllabus:

A01 Knowledge and understanding

A02 Application and communication

Each *Teacher Pack* contains one or more lesson plans and associated resources, complete with a section of preparation and reflection.

Each lesson is designed to be an hour long but you should adjust the timings to suit the lesson length available to you and the needs of your learners.

Important note

Our *Teacher Packs* have been written by **classroom teachers** to help you deliver topics and skills that can be challenging. Use these materials to supplement your teaching and engage your learners. You can also use them to help you create lesson plans for other topics.

This content is designed to give you and your learners the chance to explore a more active way of engaging with mathematics that encourages independent thinking and a deeper conceptual understanding. It is not intended as specific practice for the examination papers.

The *Teacher Packs* are designed to provide you with some example lessons of how you might deliver content. You should adapt them as appropriate for your learners and your centre. A single pack will only contain at most four lessons, it will **not** cover a whole topic. You should use the lesson plans and advice provided in this pack to help you plan the remaining lessons of the topic yourself.

Lesson preparation

This *Teacher Pack* will cover the following syllabus content.

Candidate should be able to:	Notes and examples
• evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations	e.g. the total score when two fair dice are thrown. e.g. drawing balls at random from a bag containing balls of different colours.
• use addition and multiplication of probabilities, as appropriate, in simple cases	Explicit use of the general formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is not required.
• understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $P(A \cap B)$ and $P(A) \times P(B)$.	
• calculate and use conditional probabilities in simple cases	e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ may be required in simple cases.

Dependencies

For all lesson plans in this *Teacher Pack*, knowledge from the following 9709 syllabus content is required.

Candidate should be able to:	Notes and examples
• Knowledge of the following probability notation is also assumed: P(A), $P(A \cup B)$, $P(A \cap B)$ P(A|B) and the use of A′ to denote the complement of A.	.

Prior knowledge and skills

For all lessons, it is assumed that learners have already completed Cambridge IGCSE™ Mathematics 0580, or a course at an equivalent level. See the syllabus for more details of the expected prior knowledge for taking Cambridge International AS & A Level Mathematics 9709.

When planning any lesson, make a habit of always asking yourself the following questions about your learners' prior knowledge and skills:

- Do I need to re-teach this or do learners just need some practice?
- Is there an interesting activity that will efficiently achieve this?

Key learning objectives

The following list represents the main underlying concepts that you should make sure your learners have understood by the end of this topic.

- Learners should be able to use statistical language with precision and confidence.
- It is important to determine whether events are mutually exclusive or independent before making calculations of probabilities.
- When working out probabilities, a tree diagram is an excellent way of accounting for the number of combinations that a particular event has.
- Given any set notation, learners should be able to work out what outcomes this refers to, and subsequently find the probability.

Why this topic matters

This topic is a key milestone in developing the probability skills. These skills underpin the basic grounding for many of the statistics topics that will follow. When you reach later probability topics, such as probability distributions, you will need to have a good grasp of the basic ideas around probability in order to understand those topics fully.

Key terminology and notation

Your learners will need to be confident with the following terminology and notation.

Mutually exclusive	Two events that cannot happen at the same time.
Independent	When the outcome of one event has no effect on another.
Complement	The probability that an event does *not* occur.
Union	The set of all outcomes in A **or** B.
Intersection	The set of all outcomes in A **and** B.

Lesson progression

Lesson 1 covers the basics of finding probabilities, and learners should already be comfortable with these questions. You might find that this lesson does not take the full hour if you class are already familiar with the material. Lesson 2 moves on to determining whether two events are mutually exclusive or independent, and how to calculate their probabilities as a result. This is vital for the probability trees covered in lesson 3. Lesson 4 moves to using probability set notation in order to calculate specific events, usually from a two-way table. You will need to ensure that your learners are secure with the basics of probability before attempting the last two lessons.

Going forward

This topic underpins most of the probability work that will be undertaken in the A level course. The probability tree topic links to the binomial distribution, where questions involving a larger number of combinations leads to probability trees becoming too large.

Lesson 1: Calculating Probabilities

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.

Resources	• PowerPoint #1 – Calculating Probabilities • Worksheet A

Learning objectives	By the end of the lesson: • ***all*** learners should be able to calculate basic probabilities from problems involving equally likely outcomes. • most learners should accurately calculate probabilities from *any* question involving equally likely outcomes.

Timings	Activity
10 mins	**Starter/Introduction** Bring up slide 2. There are 4 probability questions for learners to think through and determine whether they are true or false. They can discuss their answers in pairs if you wish. Once they have finished, click forward through the PowerPoint to look at the answers to each question one at a time. The questions are designed to highlight misconceptions learners may have around probability.
3 mins 5 mins	**Main lesson** Slide 7 introduces the concept of equally likely outcomes. Go through the definition with learners, and share the examples given. Ask the learners to think of their own examples of equally likely outcomes. Also share the examples of non-equally likely outcomes. Slide 8 and 9 have a range of quick-fire questions involving the probabilities for a dice. Bring the questions up on the screen individually and get learners to think of the answer. You can then either ask one randomly selected learner to share their answer, or get pairs of learners to share with each other. Work through all of the questions in this manner. Learners can now work through the worksheet A. *If your class are reasonably able, then you might find this lesson is finished quickly as this material is largely covered in Cambridge IGCSE™.

Reflection	Reflect on your lesson, use the **Lesson reflection** notes to help you.

Lesson plan 2: Mutually Exclusive and Independent Events

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.

Resources	• PowerPoint #2 – Mutually Exclusive and Independent Events • Worksheet B

Learning objectives	By the end of the lesson: • ***all*** learners should be able to recognise when 2 events are mutually exclusive or independent. • most learners should • some learners should be able to calculate harder probabilities from problems that involve combinations.

Dependencies

Learners need to know how to calculate basic probabilities from equiprobable events.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
Forgetting to consider the number of combinations for achieving a required outcome.	If learners forget this then their probabilities will be much lower than they need to be.	Use the two examples in PowerPoint to emphasise this point thoroughly.

Timings	Activity
8 mins	**Starter/Introduction** Give learners the definition for 'mutually exclusive' events. Whilst they may not remember the term, they will have extensively used the probability rules that work for mutually exclusive events at Cambridge IGCSE™.After sharing the examples, and 'non examples', there are 5 checkpoint questions to check if learners have understood the definitions. Once learners have read the statement, they should vote with their fingers to show the answer they believe to be correct. Press forwards on the PowerPoint to reveal which one is correct, and talk through the reasoning should any learners display the wrong answer.
5 mins	**Main lesson** On slide 4, give learners the probability addition law for mutually exclusive events. The slide then includes two examples of how to correctly use the law, and shows one example of a wrong answer you can achieve by using this rule if you fail to see that the two events are not mutually exclusive. This will demonstrate the importance to the learners the importance of checking for mutual exclusivity before applying the law.
5 mins	Slide 6 introduces the idea of complementary events, the notation used, and using the 'one minus' calculation to quickly work out the probability of an event 'not' happening. It then gives two examples of how to use the rule in calculations. You can ask your learners to share what they think the answers are before you reveal them.
5 mins	

Timings	Activity
5 mins 5 mins	On slide 7, give learners the definition for independent events. The first example is a straight forward use of the multiplication law for independent events. The second example is designed to help learners' awareness of the need to consider the number of combinations that a particular outcome may have. In this case, there are three combinations for achieving a score of four from a 3-sided dice. These need to be calculated individually and then added together. More able learners might spot that the calculation is the same for all three and that you just need to multiply the end result by three. Slide 10 includes a contextual example of the law for independence. Again, the first example is relatively straight forward. The second example is another question that involves the learners to consider the number of combinations before finding their final answers. You could ask your learners to share the answers before you bring them up on the screen via the PowerPoint. Slide 11 introduces the idea of conditional probability, where the outcome of an event is dependent on previous events. After listing some examples, there is then an activity very similar to the starter which requires the learners to vote with their fingers to indicate whether they believe a statement describes independent events or not. The learners can now work through the worksheet B. **Challenge: You could make a more challenging example by creating a question with many possible combinations. An example could be 'what is the probability of achieving exactly 3 sixes on a die in 6 throws. This work on combinations eventually leads to the binomial distribution, which is taught later on in the A level course.**
5 mins	**Plenary** On the final slide there are 3 quick questions for you to determine whether learners have understood the key points of the lesson. To make it more challenging you could insist that the learners name examples that have not yet been used in the lesson.

Reflection	Reflect on your lesson, use the **Lesson reflection** notes to help you.

Lesson plan 3: Tree Diagrams

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.

Resources	• PowerPoint #3 – Tree Diagrams • Worksheet C

Learning objectives	By the end of the lesson: • ***all*** learners should be able to accurately draw a probability tree • most learners should be able to accurately calculate probabilities from an independent probability tree • some learners should be able to accurately calculate probabilities from a conditional probability tree

Dependencies

Learners need to know how to find probabilities from both independent and dependent events.

Common misconceptions

Misconception	Problems this can cause	An example way to resolve the misconception
For dependent events, learners sometimes forget to adapt the probabilities of the second event after writing the outcome of the first.	Learners will incorrectly label the second branches of their tree and calculate their probabilities with those incorrect values.	Always read the question carefully, and after working on each branch of the tree think carefully about what changes as a result of that event happening, and the changes to the probabilities as a result.

Timings	Activity
3 mins	**Starter/Introduction** The first slide is a quick recap from Cambridge IGCSE™ reminding learners how to draw probability trees correctly. Make sure to give a quick reminder on how you calculate the probability of a particular outcome by multiplying the branches together.
5/10 mins 5 mins	**Main lesson** Slide 3 gives learners the first example of a probability tree that is based on two independent events, events where the outcome of the first trial does not affect the second. As you run through the PowerPoint describe how learners would draw this as the probability tree appears on screen. Learners sometimes find the second set of branches more difficult, so explain how to set these out properly. You might want learners to copy this one into their notes to give them some practise of correctly drawing a probability tree. Once the tree is drawn get learners to read the question carefully to determine which probabilities from the tree they actually need to calculate. Sometimes learners will calculate all the probabilities first without checking to see which ones they need. Once you've shown learners the individual calculations remind them that you add up the results to calculate your final answer. Slide 4 is another example of a probability tree with independent events. This time the events are not the same and therefore you have to lead with one of them. Stress to the learners that it

Timings	Activity
	doesn't matter if they had picked English for the first branch instead of statistics. Otherwise, the process is the same as the first example.
10 mins	Slide 5 is the first example of a conditional probability tree, where the outcome of the first event affects the second branches. When going through the 2nd branches take time to get learners to understand how the numerators change depending on which chocolate bar was taken in the first pick. Learners sometimes incorrectly assume that once they've done the top three second branches that these will be the same for the middle and the bottom branches. Remind them that this is not the case.
5 mins	Slide 6 is the final probability tree and a different example of a conditional tree. At this point the learners should be fairly confident with the concept, and you could even get them to complete this one before you go through it. Learners can now work through the worksheet C.
5 mins	**Plenary** There are 3 quick questions at the end of the PowerPoint to help you assess whether your learners have fully understood the lesson. Once the question has been displayed, give the learners some thinking time before you bring up the answers. When you bring up the answers the learners can quickly vote by raising the number of fingers that corresponds to the answer they believe is correct.

Reflection	Reflect on your lesson, use the **Lesson reflection** notes to help you.

Lesson plan 4: Set Notation

Preparation	Read through the PowerPoint and ensure that you are familiar with the worked examples.

Resources	• PowerPoint #4 – Set Notation • Worksheet C

Learning objectives	By the end of the lesson: • ***all*** learners should recognise and confidently write set notation • most learners should be able to calculate probabilities using basic set notation • some learners should be able to calculate *any* probability using set notation, including 'given that' problems.

Dependencies

Learners need to know how to find probabilities from a two-way table.

Timings	Activity
3 mins	**Starter/Introduction** Slide 2 has the four main set notations that the learners must know. The learners copy these carefully into their notes.
10-15 mins 5 mins	**Main lesson** Slide 3 contains 12 questions on finding probabilities using set notation. Go through these one at a time, getting learners to think about the answers before you reveal them on the screen. Learners typically find the 'given that' questions the hardest to understand. One way to help them remember is to remind them that the set after the 'given that' symbol gives you the denominator of the fraction that follows. The set before the symbol gives you the numerator. The questions get more challenging towards the end, and mix together two or more different set notation elements. Less able learners may struggle with these later questions, so be prepared to talk through each one carefully. Use slides 4 – 7 to check for learner understanding. There are 4 different questions based on a two-way table. When you first bring up the question, give learners time to think about what they think the answer might be. After 10 seconds, bring up the four possible options and give learners the opportunity to vote for the answer they think is correct using their fingers. If the majority are correct, you can get one of the other learners to explain why their answer is correct. If you get a large proportion of learners getting incorrect answers, you may need to re-explain the set notation again, and do another check before moving on. The learners can now work through the worksheet D.
5 mins	**Plenary** There is a table on the slide 9. Get learners to make up their own set notations and calculate the answers for them. When they are ready, get them to trade questions with their partner. If they both agree on the answer then they are likely to be correct. If they disagree, then they need to discuss the problem between them until they agree. If they cannot agree, then you may need to step in and help them find the correct answer.

Reflection	Reflect on your lesson, use the **Lesson reflection** notes to help you.

Lesson reflection

As soon as possible after the lesson you need to think about how well it went.

One of the key questions you should always ask yourself is:

Did all learners get to the point where they can access the next lesson? If not, what will I do?

Reflection is important so that you can plan your next lesson appropriately. If any misconceptions arose or any underlying concepts were missed, you might want to use this information to inform any adjustments you should make to the next lesson.

It is also helpful to reflect on your lesson for the next time you teach the same topic. If the timing was wrong or the activities did not fully occupy the learners this time, you might want to change some parts of the lesson next time. There is no need to re-plan a successful lesson every year, but it is always good to learn from experience and to incorporate improvements next time.

To help you reflect on your lesson, answer the most relevant questions below.

Were the lesson objectives realistic?
What did the learners learn today? Or did they learn what was intended? Why not?
What proportion of the time did we spend on the most important topics?
Were there any common misconceptions?
What do I need to address next lesson?
What was the learning atmosphere like?
Did my planned differentiation work well?
How could I have helped the lowest achieving learners to do more?
How could I have stretched the highest achieving learners even more?
Did I stick to timings?
What changes did I make from my plan and why?

Summary evaluation

What two things went really well? (Consider both teaching and learning.)

What two things would have improved the lesson? (Consider both teaching and learning.)

What have I learned from this lesson about the class or individuals that will inform my next lesson?

Worksheets and answers

	Worksheet	Answers
For use with *Lesson 1*:		
A: Calculating Probabilties	**x**	**x**
For use with *Lesson plan 2*:		
B: Mutually Exclusive and Independent Events	**x**	**x**
For use with Lesson plan 3:		
C: Tree Diagrams	**x**	**x**
For use with Lesson plan 4:		
D: Set Notation	**x**	**x**

Worksheet A: Calculating Probabilities

1 Three coins are flipped at the same time. Find the probability that the three coins all show the same outcome.

2 In a popular board game, two six-sided dice are rolled at the same time. The scores on the dice are then added together to determine the number of spaces a player can move.

a Draw a sample space diagram for the outcome of the two dice.

b Find the probabilities that the total score:

i = 10
ii >10
iii is odd
iv = 13

3 A box contains 20 counters, numbered 1, 2, 3, …, 20. A counter is taken out of the box. What is the probability that it:

a is the number 4

b is a multiple of 5

c is greater than 10

d has a 20 on it?

4 The days of the week are written on seven separate cards. One card is chosen. What is the probability that it is:

a Tuesday

b either Wednesday or Tuesday

c not Friday, Saturday or Sunday?

5 In a well-known word game, Callum has the letters C, E, E, R, Q, S and T. One letter accidentally falls on to the floor. What is the probability that it is:

a Q

b B

c E or S

d not an E?

6 There are 20 beads in a box. Seven are white, three are yellow, six are blue and four are green. If one bead is selected at random, what is the probability that it is:

a white

b green or yellow

c not blue

d brown,

e neither white nor yellow?

7 The lengths, in cm, of 480 Galapagos Giant Tortoises are contained in the grouped data table provided. Using the values in the table, find the probability that a randomly chosen tortoise:

a Has a length of more than 150cm

b Is male

c is between 130cm and 150cm long

d is a male between 130cm and 150cm.

Length (cm)	**Frequency (Male)**	**Frequency (Female)**
$120 \leq l < 130$	8	28
$130 \leq l < 140$	40	30
$140 \leq l < 150$	48	64
$150 \leq l < 160$	94	54
$160 \leq l < 170$	62	52

Worksheet A: Answers

1 HHH = $\frac{1}{8}$ TTT = $\frac{1}{8}$ Answer = $\frac{1}{4}$

2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2bi $\frac{3}{36}$ **2bii** $\frac{3}{36}$ **2biii** $\frac{18}{36}$ **2iv** 0

3a $\frac{1}{20}$ **3b** $\frac{4}{20}$ **3c** $\frac{10}{20}$ **3d** $\frac{1}{20}$

4a $\frac{1}{7}$ **4b** $\frac{2}{7}$ **4c** $\frac{4}{7}$

5a $\frac{1}{7}$ **5b** 0 **5c** $\frac{3}{7}$ **5d** $\frac{5}{7}$

6a $\frac{7}{20}$ **6b** $\frac{7}{20}$ **6c** $\frac{14}{20}$ **6d** 0 **6e** $\frac{10}{20}$

7a $\frac{262}{480}$ **7b** $\frac{252}{480}$ **7c** $\frac{182}{480}$ **7d** $\frac{88}{480}$

Worksheet B: Mutually Exclusive & Independent Events

1 The probability of Damien passing a practical driving test is 0.8. Write down the probability of him not passing the test.

2 The probability of a TV set requiring repair within 1 year is 0.22. Write down the probability of a TV set not requiring repair within 1 year.

3 When Rebecca rings her father the probability that the phone is engaged is 0.1, the probability that the phone is not engaged but no one answers is 0.5 and the probability that the phone is answered is 0.4.

Find the probability that:

(a) the phone is engaged or no one answers,

(b) the phone is engaged or it is answered,

(c) the phone is not engaged.

4 Dave shops exactly once per week. In a typical week the probability that Dave shops on a Monday is 0.1, on a Tuesday is 0.2 and on a Wednesday is 0.3.

Find the probability that he goes shopping on:

(a) Monday or Tuesday, **(b)** Monday or Wednesday,

(c) Monday or Tuesday or Wednesday, **(d)** not on Monday,

(e) Thursday or Friday or Saturday or Sunday.

5 There are 35 customers in a canteen, 12 are aged over 50, 15 are aged between 30 and 50, and five are aged between 25 and 29.

Find the probability that the next customer to be served is aged:

(a) 30 or over, **(b)** 25 or over, **(c)** under 25, **(d)** 50 or under.

6 Davina is expecting a baby:

A is the event that the baby will have blue eyes;

B is the event that the baby will have green eyes;

C is the event that the baby will have brown hair.

(a) Write down two of these events which are:

(i) mutually exclusive, **(ii)** not mutually exclusive.

(b) Define the complement of event *A*.

7 A firm employs 20 bricklayers. The Inland Revenue selects one for investigation:

A is the event that the bricklayer selected earned less than £20 000 last year;

B is the event that the bricklayer selected earned more than £20 000 last year;

C is the event that the bricklayer selected earned £20 000 or more last year.

(a) Which event is the complement of *C*?

(b) Are the events *A* and *B* mutually exclusive?

(c) Write down two of the events which are not mutually exclusive.

Worksheet B: Answers

1 0.2 **2** 0.78 **3a** 0.6 **3b** 0.5 **3c** 0.9

4a 0.3 **4b** 0.4 **4c** 0.6 **4d** 0.9 **4e** 0.4

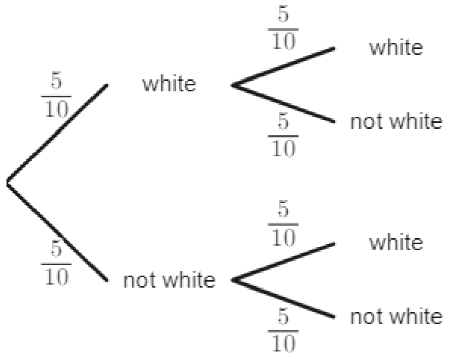
5a $\frac{17}{35}$ **5b** $\frac{32}{35}$ **5c** $\frac{3}{35}$ **5d** $\frac{23}{35}$

6ai A and B **6aii** A and C **6b** Does not have blue eyes

7a A **7b** Yes **7c** B and C

Worksheet C: Tree Diagrams

1 Louis has 5 white counters and 5 purple counters in a box.
Louis takes one counter at random from the box, puts it back, and takes another counter from the box.
A tree diagram is drawn to represent this information.



Find the probability that Louis takes two counters of different colours.

2 Christine has 3 green counters, 1 yellow counter and 1 white counter in a box.
Christine takes one counter at random from the box, puts it back, and takes another counter from the box.
Find the probability that Christine takes at least one green counter.

3 Kyle has 13 red counters and 7 green counters in a bag.
Kyle takes one counter at random from the bag, puts it back, and takes another counter from the bag. Find the probability that Kyle takes two counters of different colours.

4 Dave has 4 yellow counters, 5 blue counters and 1 purple counter in a box.
Dave takes one counter at random from the box, keeps it, and takes another counter from the box. Find the probability that Dave takes at least one yellow counter.

5 Louise has 4 blue balls and 1 yellow ball in a box.
Louise takes one ball at random from the box, keeps it, and takes another ball from the box.
Find the probability that Louise takes two balls of different colours.

6 Yasmine has 4 green socks and 16 red socks in a box.
Yasmine takes one sock at random from the box, keeps it, and takes another sock from the box.
Find the probability that Yasmine takes at least one green sock.

Worksheet C: Answers

1 $\frac{1}{2}$

2 $\frac{21}{25}$

3 $\frac{182}{400}$

4 $\frac{60}{90}$

5 $\frac{8}{20}$

6 $\frac{140}{380}$

Worksheet D: Set Notation

1 Two hundred and forty learners register for training course. After one year they are recorded as pass or fail. A table of the results, classified by age, is shown below:

	(Age Years)	
	Under 18	**18 and over**
Pass	94	66
Fail	56	24

A learner is selected at random from the list.
Y denotes the event that the selected learner is under 18.
P denotes the event that the selected learner passed.
Determine the value of:

(a) $P(Y)$, **(b)** $P(P)$, **(c)** $P(Y')$, **(d)** $P(Y \cap P)$, **(e)** $P(Y \cup P)$,

(f) $P(Y|P)$, **(g)** $P(P'|Y)$, **(h)** $P(Y\,|P')$, **(i)** $P(Y'|P)$, **(j)** $P(Y \cap P')$.

2 Last year, the employees of a large company received either no pay rise, a small pay rise or a large pay rise. In the table below the employees have been sorted into categories, classified by whether they were weekly paid or monthly paid.

	No pay rise	**Small pay rise**	**Large pay rise**
Paid weekly	23	83	3
Paid monthly	2	6	21

A tax inspector decides to investigate the tax affairs of an employee selected at random.
W is the event that a weekly paid employee is selected.
N is the event that an employee who received no pay rise is selected.
W' and *N*' are the events 'not *W*' and 'not *N*', respectively.
Find:

(a) $P(W)$, **(b)** $P(N')$, **(c)** $P(W|N)$, **(d)** $P(W \cup N)$, **(e)** $P(N \cap W)$, **(f)** $P(W \cap N')$,

(g) $P(N \cup W')$, **(h)** $P(W|N')$, **(i)** $P(N'|W')$.

3 A firm that allows customers to rent cars has warehouses in Southampton and Birmingham. The cars are categorised as small, medium or large according to the size of their engine. Shown in the table below is the number of cars in each class, based at each warehouse.

	Small	Medium	Large
Southampton	13	14	12
Birmingham	18	22	11

One of the 90 cars is selected at random for inspection.
S is the event that the selected car is based at Southampton.
M is the event that the selected car is medium.
L is the event that the selected car is large.
S', *M*' and *L*' are the events 'not *S*', 'not *M*' and 'not *L*', respectively. Evaluate:

(a) $P(S)$, **(b)** $P(M')$, **(c)** $P(S \cup L)$, **(d)** $P(S \cap M)$, **(e)** $P(S \cup M')$,

(f) $P(S\,|L)$, **(g)** $P(L|S')$, **(h)** $P(M'\,|S)$, **(i)** $P(M \cap L)$.

(j) By comparing your answers to **(a)** and **(f)** state whether or not the events *S* and *L* are independent.

Worksheet D: Answers

1a $\frac{150}{240}$	**1b** $\frac{160}{240}$	**1c** $\frac{90}{240}$	**1d** $\frac{94}{240}$	**1e** $\frac{216}{240}$	
1f $\frac{94}{160}$	**1g** $\frac{56}{150}$	**1h** $\frac{56}{80}$	**1i** $\frac{66}{160}$	**1j** $\frac{56}{240}$	
2a $\frac{109}{138}$	**2b** $\frac{113}{138}$	**2c**	**2d** $\frac{111}{138}$	**2e** $\frac{23}{138}$	**2f** $\frac{86}{138}$
2g $\frac{115}{138}$	**2h** $\frac{86}{113}$	**2i** $\frac{27}{29}$			
3a $\frac{39}{90}$	**3b** $\frac{54}{90}$	**3c** $\frac{50}{90}$	**3d** $\frac{14}{90}$	**3e** $\frac{60}{90}$	
3f $\frac{12}{23}$	**3g** $\frac{11}{51}$	**3h** $\frac{25}{39}$	**3i** 0	**3j** Not independent – as P(S) and P(S I L) are not equal	

Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom
t: +44 1223 553554
e: info@cambridgeinternational.org www.cambridgeinternational.org